Outage Analysis of Cooperative CDMA Systems in Nakagami-m Fading Channels

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***Abstract*—The performance of uplink cooperative code-division multiple-access (CDMA) systems using adaptive decode-and-forward (DF) relaying is investigated over Nakagami-m fading channels. A closed-form expression for the outage probability for a multirelay system with relay selection is derived. Our method is based on the moment-generating function (MGF) for the total signal-to-noise ratio (SNR) at the base station where the cumulative density function (cdf) is obtained. We also examine the asymptotic performance of the system at high SNR from which we evaluate the achievable diversity gain for different system parameters. Both simulations and analytical results are presented for performance evaluation.**

***Index Terms*—Asynchronous code-division multiple access (CDMA), cooperative diversity, Nakagami-m fading, outage probability.**

I. Introduction

MULTIPATH fading, interference, and scarcity of power and bandwidth are the main limitations in any wireless communication system. The multipath fading problem can be solved by applying spatial transmit/receive diversity techniques. Transmit/receive diversity can be achieved by implementing multiple antennas at the transmitter/receiver as in multiple-input–multiple-output systems. However, employing multiple antennas at the mobile terminal may be impractical due to cost, size, and power limitations. The potential solution for these limitations is to apply user cooperation techniques by which mobile terminals share their physical resources to create virtual antennas, and therefore, transmit diversity can be achieved [1], [2]. Cooperative diversity with multiple relays is an auspicious solution by which the reliability of wireless communication systems and high data rate can be achieved [3].

The cooperation among users can be classified into two basic cooperative schemes, namely, amplify-and-forward (AF) and decode-and-forward (DF). In AF, the relay (i.e., cooperative user) transmits an amplified version of the received partner's signal. On the other hand, in the DF mode, relays decode and regenerate the received partner's signal for retransmission to the base station [3].

One common network of user cooperation diversity is direct-sequence code-division multiple access (DS-CDMA) as it represents many of the existing technologies for wireless systems such as multicarrier CDMA (MC-CDMA) and optical CDMA. In these CDMA systems, Rayleigh fading channels are commonly adopted for study, and orthogonality between users is assumed [4]. In CDMA systems, to overcome the effect of multiple-access interference (MAI) arising from the nonorthogonality of users' spreading codes, multiuser detectors such as the minimum mean square error (MMSE) and decorrelator have been presented. In [5], the authors studied the effect of MAI on the performance of asynchronous DS-CDMA cooperative systems over multipath fading channels. In their work, the authors considered the multirelay coded cooperation case, wherein the bit-error-rate (BER) performance of the system was analyzed for the two cases, namely, the perfect and imperfect interuser links over Rayleigh fading channels.

The outage probability over Rayleigh fading channels was investigated at high SNR in [6] for cooperative asynchronous DS-CDMA networks. In [7], a closed-form expression for the outage probability in DF cooperative networks is determined, assuming an independent and nonidentically distributed (i.n.i.d.) Rayleigh fading channel. In practice, the Rayleigh fading model is not realistic as it cannot represent the statistical characteristics of the complex indoor environments. In that sense, the Nakagami-m fading model is well known as a generalized distribution where many fading environments can be modeled. It can be used to model fading conditions, namely, severe, light, and no fading, by changing fading parameter m. In a Nakagami fading channel, in [8], a closed-form expression for the outage probability of cooperative relay networks, assuming an independent and identically distributed (i.i.d.) fading scenario, is obtained. In addition, in [9], the BER of a cooperative downlink transmission scheme for DS-CDMA systems over Nakagami-m fading channels to achieve relay diversity is evaluated, and transmitter zero-forcing (TZF) at the base station for suppressing the downlink multiuser interference is proposed. Moreover, in [10], the performance of downlink multiuser relay networks using a single AF relay is studied, and a closed-form expression for the outage probability at high SNR is derived. Furthermore, in [11], a multirelay network in Nakagami-m fading channels where the outage probability and average BER of AF-based relaying are derived is considered. The performance analysis for opportunistic DF with a selection combining receiver at the destination has been evaluated in terms of the outage probability in [12]. In [12], a closed-form expression for the outage of the system over a nonidentical Nakagami fading channel is derived. A closed form for the outage probability in DF over i.n.i.d. flat Nakagami-m

Manuscript received March 27, 2011; revised September 26, 2011; accepted November 22, 2011. Date of publication December 8, 2011; date of current version February 21, 2012. This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grant N008861. The review of this paper was coordinated by Dr. C.-C. Chong.

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Digital Object Identifier 10.1109/TVT.2011.2178870

channels using the moment-generating function (MGF) was introduced in [13]–[16].

All previous works, however, did not consider multiuser scenarios as in cooperative CDMA systems. In this paper, we investigate the outage performance of CDMA systems over Nakagami-m fading channels. We derive a closed-form expression for the MGF of the received SNR at the base station. This expression is then used to obtain the outage probability of the system. In our work, we employ an MMSE detector to suppress the effect of MAI at both the base station and the relay sides. We show that the full diversity gain is achieved at high SNR.

The rest of this paper is organized as follows: The system model for cooperative DS-CDMA over a Nakagami-m fading channel is presented in Section II. In Section III, we analyze the outage probability for multirelay cooperation. Section IV provides the simulation and numerical results of our system. Finally, conclusions are reported in Section V.

II. System Model

A. Received Signal Model

We consider an uplink K-user asynchronous nonorthogonal DS-CDMA system transmitting over a Nakagami-m fading channel. In our model, we consider a set of available cooperating users $\mathbf{s} \in \{1, \ldots, K\}$ from which a set $\ell \in \{1, \ldots, L\}$ of decodable relays (i.e., cooperating users that have correctly decoded the source messages) are able to transmit to base station $\mathbf{b}$, where $L \in \{1, \ldots, K-1\}$. A half-duplex system is assumed, and each user is equipped with a single antenna. The cooperative communication process can be divided into two phases.

1) Phase I: Each user transmits its own DS-CDMA modulated data to the base station and to L-relays. During this phase, the received signal at the base station can be written as

$$r_b^I(t) = \sum_{i=0}^{f-1} \sum_{s=1}^{K} x_s(i) C_s(t - \tau_s - iT_b) h_{sb} + n_b^I(t) \quad (1)$$

where f is the frame length, $x_k(i) \in \{1, -1\}$ is the ith data symbol of user $\mathbf{s}$, $C_s(t)$ is the spreading code of user $\mathbf{s}$ with spreading gain $N = (T_b/T_c)$, T_b is the bit period, T_c is the chip period, and τ_s is the random transmit delay of the sth user, which is assumed to be uniformly distributed along the symbol period. $n_b^I(t)$ is the additive white Gaussian noise with zero mean and variance $\sigma_n^2 = N_o/2$. In (1), h_{sb} denotes the channel coefficient between user $\mathbf{s}$ and the base station that is drawn from a Nakagami-m fading channel with $\Omega_{sb} = E(|h_{sb}|^2) = 1$ with parameter m_{sb}. The received signal at user (r) can be written as

$$r_r(t) = \sum_{i=0}^{f-1} \sum_{s=1,\, s \neq r}^{K} x_s(i) C_s(t - \tau_s - iT_b) h_{sr} + n_r(t) \quad (2)$$

where h_{sr} is the channel coefficient between user $\mathbf{s}$ and partner (r) and is drawn from a Nakagami-m fading channel with $E(|h_{sr}|^2) = 1$ over path p with parameters m_{sr} and Ω_{sr}. $n_r(t)$ is a Gaussian noise with zero mean and variance $\sigma_n^2 = N_o/2$.

2) Phase II: In this phase, each cooperating user transmits the received signal to the base station, which is expressed as

$$r_b^{II}(t) = \sum_{i=0}^{f-1} \sum_{s=1}^{K} \sum_{\ell=1}^{L} \widetilde{x}_s(i) C_{r_\ell}(t - D_{s,r_\ell} - \tau_{r_\ell} - i\ell T_b) h_{r_\ell b} + n_2(t) \quad (3)$$

where r_ℓ is the ℓth relay cooperating with user $\mathbf{s}$, $n_b^{II}(t)$ is a Gaussian noise with zero mean and variance $\sigma_n^2 = N_o/2$, and $D_{k,\ell}$ is the transmission delay during the second transmission period. $h_{r_\ell b}$ is the channel coefficient between user r_ℓ and the base station that is modeled as a Nakagami-m random variable (RV) with $E(|h_{r_\ell b}|^2) = 1$ and parameters $m_{r_\ell b}$ and $\Omega_{r_\ell b}$ [5].

B. Cooperative DS-CDMA Protocol

Here, the effect of asynchronous transmission on the outage probability of DF CDMA systems over i.n.i.d. Nakagami-m fading channels is studied. We consider repetition-based cooperative diversity. In this scheme, the number of available relays (cooperating users or partners) and the decoding set are denoted by L and $D(s)$, respectively, where $D(s) \subset L$. Decoding set $D(s)$ is defined as the set of relays that have the ability to fully decode the source information (i.e., no decoding error). Because of the half-duplex constraint, cooperation is performed in two time slots. In the first time slot, each user $\mathbf{s}$ transmits its signal to the set of relays L and to base station $\mathbf{b}$ and keeps silent in the second time slot. In the second time slot, only relays from decoding set $D(s)$ forward the source signal to the base station.

The independent fading channel coefficients between users themselves and the base station are represented by source–relay h_{sr_ℓ}, source–base station h_{sb}, and relay–base station $h_{r_\ell b}$, which are all modeled as i.n.i.d. Nakagami-m RVs. The instantaneous SNRs are given by $\gamma_{sr_\ell} = |h_{sr_\ell}|^2 (E_s/N_o)$, $\gamma_{sb} = |h_{sb}|^2 (E_s/N_o)$, and $\gamma_{r_\ell b} = |h_{r_m b}|^2 (E_s/N_o)$, where $|h_{sr_\ell}|^2$, $|h_{sb}|^2$, and $|h_{r_\ell b}|^2$ are gamma-distributed RVs. The probability density function (pdf) of γ_{ij} is given by

$$p_{\gamma_{ij}}(\gamma) = \frac{B_{ij}^{m_{ij}}}{\Gamma(m_{ij})} \gamma^{m_{ij}-1} \exp(-\gamma B_{ij}) \quad (4)$$

where $m_{ij} > 0.5$ is the Nakagami-m fading parameter, $\Gamma(\cdot)$ is the gamma function [17, eq. (8.310.1)], $B_{ij} = (m_{ij}/\bar{\gamma_{ij}})$ with average SNR, and $\bar{\gamma_{ij}} = E\langle h_{ij}^2 \rangle E_s/N_o$, where $E\langle \cdot \rangle$ denotes expectation. The cumulative density function (cdf) of γ_{ij} is given by

$$F_{\gamma_{ij}}(\gamma) = \frac{\gamma(m_{ij}, \gamma B_{ij})}{\Gamma(m_{ij})} \quad (5)$$

where $\gamma(a, x)$ is the lower incomplete gamma function [17, eq. (8.350.1)]. Note that our cooperative scheme employs MMSE detection to suppress the MAI at both the base station and the relay sides.

III. Outage Probability Analysis

Here, the repetition-based cooperative diversity protocol is considered [3], where the partners (relays) fully decode the received signals and repeat the information to the base station in the second time slot. The available degrees of freedom $K/2N$ [6] is a function of the total number of users, i.e., K, and the length of the available spreading code, i.e., N, and 1/2 stands for the bandwidth expansion needed for relaying due to the half-duplex constraint.

Here, the performance of the cooperative diversity protocol under diversity combining is studied. Let us consider the direct link between source $\mathbf{s}$ and base station $\mathbf{b}$, where the mutual information is given by

$$I_{sb} = \frac{K}{2N}\log\left(1+\frac{2N\gamma_{sb}}{K^2[M]_{s,s}}\right) \tag{6}$$

with $[M]_{s,s}$ in the case of MMSE detection is given by $[M]_{s,s} = [(R+\mathrm{SNR}^{-1}I)^{-1}]_{s,s}$, where $\mathrm{SNR} \triangleq (E_s/N_o)$ is the signal-to-noise-ratio in the absence of fading, R is a function of cross correlation between delayed signature waveforms, where R_s is the $(Kf \times Kf)$ cross-correlation matrix at the base-station receiver, and $R_{r,s\neq r}$ is the $((Kf-1)\times(Kf-1))$ cross-correlation matrix at the relay, which is defined as [5]

$$\mathbf{R}_{\mathbf{s},\mathbf{r}} = \begin{bmatrix} \rho_{1,1}(1,1) & \cdots & \rho_{1,1}(1,f) & \cdots & \rho_{1,K}(1,f) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ \rho_{1,1}(f,1) & \cdots & \rho_{1,1}(f,f) & \cdots & \rho_{1,K}(f,f) \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ \rho_{K,1}(f,1) & \cdots & \rho_{K,1}(f,f) & \cdots & \rho_{K,K}(f,f) \end{bmatrix} \tag{7}$$

where ρ_{ij} is the cross-correlation value between any two spreading codes C_i and C_j. In (6), $2N/K^2$ is the normalized discrete-power constraint [6]. Then, the outage probability occurs when I_{sb} fails to achieve a target rate of $\Re$, which can be written as

$$P_{\mathrm{out}} = P_r[I_{sb} < \Re]. \tag{8}$$

From (6) and (8), we have

$$\begin{aligned} P_{\mathrm{out}} &= P_r\left[\frac{K}{2N}\log\left(1+\frac{2N\gamma_{sb}}{K^2[M]_{s,s}}\right) < \Re\right] \\ &= P_r\left[\gamma_{sb} < \gamma_{\mathrm{th}-sb}\right] \end{aligned} \tag{9}$$

where

$$\gamma_{\mathrm{th}-sb} = \frac{2^{\frac{2N\Re}{K}}-1}{\frac{2N}{K^2[M]_{s,s}}}. \tag{10}$$

From (9), we can notice that $P[\gamma_{sb} < \gamma_{\mathrm{th}-sb}]$ is the cdf of γ_{sb}, which is given as (5), i.e.,

$$P_{\text{out-noncoop}} = P_r\left[\gamma_{sb} < \gamma_{\mathrm{th}-sb}\right] = \frac{\gamma\left(m_{sb}, B_{sb}\gamma_{\mathrm{th}-sb}\right)}{\Gamma(m_{sb})}. \tag{11}$$

In what follows, a closed-form expression for the MGF of the received SNR at the base station is developed, where the MGF is used to obtain the outage probability of the system. Now, the mutual information between the source and the $\mathcal{K}$th relay (partner) is given by

$$I_{sr_{\mathcal{K}}} = \frac{K}{2N}\log\left(1+\frac{2N\gamma_{sr_{\mathcal{K}}}}{K^2[M]_{r_{\mathcal{K}},r_{\mathcal{K}}}}\right). \tag{12}$$

Outage probability P_{out} can be defined as the probability that mutual information I_{DF} falls below certain rate $\Re$. Note that the partners' set $r_{\mathcal{K}} \in D(s)$ is a random set. Now, the outage probability of the system between user $\mathbf{s}$ and base station $\mathbf{b}$ is given by

$$P_{\mathrm{out}} = P_r[I_{\mathrm{DF}} < \Re] = \sum_{D(s)} P_r\left[D(s)\right] P_r\left[I_{\mathrm{DF}} < \Re | D(s)\right]. \tag{13}$$

We noted that it is difficult to find a closed-form expression for the outage probability of the DF cooperative network, particularly for the case of i.n.i.d. Nakagami-m fading. This is due to the difficulty of finding the pdf of the sum of gamma RVs given in (13). Similar to [7], we simplify the computation of the outage probability by indicating that the cooperation diversity can be visualized as a system that has effectively $L+1$ paths between source $\mathbf{s}$ and base station $\mathbf{b}$. Let us denote path (0) as the direct path between $\mathbf{s}$ and $\mathbf{b}$ and the $\mathcal{K}$th cascaded path between $\mathbf{s} \to r_{\mathcal{K}} \to \mathbf{b}$, where $\mathcal{K} = 1, 2, \ldots, L$. Let us also define $\gamma_{\mathcal{K}}$ as the received SNR for link $\mathbf{s} \to r_{\mathcal{K}}$ and link $r_{\mathcal{K}} \to \mathbf{b}$. The pdf of $\gamma_{\mathcal{K}}$ is then defined as

$$\begin{aligned} f_{\gamma_{\mathcal{K}}}(\gamma) &= f_{\gamma_{\mathcal{K}}|\text{link down}}(\gamma)\Pr\left[\text{link down}\right] \\ &\quad + f_{\gamma_{\mathcal{K}}|\text{link active}}(\gamma)\Pr\left[\text{link active}\right] \end{aligned} \tag{14}$$

where $f_{\gamma_{\mathcal{K}}|\text{link down}}$ and $f_{\gamma_{\mathcal{K}}|\text{link active}}$ are conditional pdf's, and $\Pr[\text{link down}]$ and $\Pr[\text{link active}]$ are the probabilities of inactive and active relays (i.e., ones that fully decode without errors). If the link is down, then pdf $f_{\gamma_{\mathcal{K}}|\text{link down}}(\gamma) = \delta(\gamma)$, where $\delta(\cdot)$ is the Dirac delta function. The probability of occurrence of this event is defined as in (11), i.e.,

$$U_{\mathcal{K}} = P_r\left[\gamma_{\mathcal{K}} < \gamma_{\mathrm{th}}\right] = F_{\gamma_{\mathcal{K}}}\left(\gamma_{\mathrm{th}}\right) \tag{15}$$

where $F_{\gamma_{\mathcal{K}}}(\cdot)$ is the cdf of $\gamma_{\mathcal{K}}$. Note that (15) represents the probability that the $\mathcal{K}$th relay (partner) does not belong to decoding set $D(s)$. Note also that link $\mathbf{s} \to \mathbf{b}$ is not connected to any relay, i.e., $U_0 = 0$. The probability that the link is active is equal to $1 - U_{\mathcal{K}}$, with the conditional pdf given by

$$f_{\gamma_{\mathcal{K}}|\text{link active}}(\gamma) = \frac{B_{\mathcal{K}}^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})}\gamma^{m_{\mathcal{K}}-1}\exp(-\gamma B_{\mathcal{K}}). \tag{16}$$

Using (16) in (14), we have

$$f_{\gamma_{\mathcal{K}}}(\gamma) = U_{\mathcal{K}}\delta(\gamma) + (1-U_{\mathcal{K}})\frac{B_{\mathcal{K}}^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})}\gamma^{m_{\mathcal{K}}-1}\exp(-\gamma B_{\mathcal{K}}). \tag{17}$$

Equation (17) represents the pdf of the $\mathcal{K}$th cascaded path from the source to the base station. Therefore, the total outage probability can be written as

$$P_{\mathrm{out}}(\gamma_{\mathrm{th}}) = P_r\left[\gamma_{sb} + \sum_{\mathcal{K}=1}^{L}\gamma_{\mathcal{K}} < \gamma_{\mathrm{th}}\right]. \tag{18}$$

From (18), we can notice that the mathematical probability model for the CDMA system is the cdf of the sum of $U_{\mathcal{K}}$'s. Using the MGF approach, the cdf of the sum of RVs can be extracted, i.e., $M_{\gamma_{\mathcal{K}}}(s) = E\{e^{-s\gamma_{\mathcal{K}}}\}$. Using (17), the MGF of the cascaded link is given by

$$M_{\gamma_{\mathcal{K}}}(s) = U_{\mathcal{K}} + (1 - U_{\mathcal{K}}) \left(1 + \frac{s}{B_{\mathcal{K}}}\right)^{-m_{\mathcal{K}}}. \quad (19)$$

Since $\gamma_{\mathcal{K}}$s are assumed to be independent, then the total MGF of the sum is expressed as the multiplication of MGFs, i.e.,

$$M_{\gamma_{\text{total}}}(s) = M_{\gamma_{sb}}(s) \prod_{\mathcal{K}=1}^{L} M_{\gamma_{\mathcal{K}}}(s) \quad (20)$$

where $M_{\gamma_{sb}}(s)$ is the MGF for direct link $\mathbf{s} \to \mathbf{b}$, which is given by

$$M_{\gamma_{sb}}(s) = \left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}}. \quad (21)$$

By applying (19) and (21) into (20), a product of $L+1$ terms $\prod_{\mathcal{K}=0}^{L}(1 + U_{\mathcal{K}})$ can be expanded as in [16], i.e.,

$$\prod_{\mathcal{K}=0}^{L}(1+U_{\mathcal{K}}) = 1 + \sum_{\mathcal{K}=0}^{L} \sum_{\lambda_0=0}^{L-\mathcal{K}} \sum_{\lambda_1=\lambda_0+1}^{L-\mathcal{K}+1} \cdots \sum_{\lambda_{\mathcal{K}}=\lambda_{\mathcal{K}-1}+1}^{L} \prod_{n=0}^{\mathcal{K}} U_{\lambda_n}. \quad (22)$$

Using (22), $M_{\gamma_{\text{total}}}(s)$ can be written as

$$\begin{aligned} M_{\gamma_{\text{total}}}(s) &= \left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}} \left(\prod_{\mathcal{K}=1}^{L} U_{\mathcal{K}}\right) \\ &\quad \times \prod_{\mathcal{K}=1}^{L} \left(1 + \frac{1 - U_{\mathcal{K}}}{U_{\mathcal{K}}} \left(1 + \frac{s}{B_{\mathcal{K}}}\right)^{-m_{\mathcal{K}}}\right) \\ &= \left(\prod_{\mathcal{K}=1}^{L} U_{\mathcal{K}}\right) \left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}} \\ &\quad + \left(\prod_{\mathcal{K}=1}^{L} U_{\mathcal{K}}\right) \sum_{\mathcal{K}=1}^{L} \sum_{\lambda_1=1}^{L-\mathcal{K}+1} \sum_{\lambda_2=\lambda_1+1}^{L-\mathcal{K}+2} \cdots \sum_{\lambda_{\mathcal{K}}=\lambda_{\mathcal{K}-1}+1}^{L} \\ &\quad \times \left(\prod_{n=1}^{\mathcal{K}} \frac{1 - U_{\lambda_n}}{U_{\lambda_n}}\right) \prod_{n=0}^{\mathcal{K}} \left(1 + \frac{s}{B_{\mathcal{K}_{\lambda_n}}}\right)^{-m_{\mathcal{K}_{\lambda_n}}}. \end{aligned} \quad (23)$$

Finally, the outage probability is expressed as

$$P_{\text{out(CDMA)}} = \mathcal{L}^{-1}\left\{M_{\gamma_{\text{total}}}(s)/s; t\right\}_{|t=\gamma_{\text{th}}} \quad (24)$$

where $\mathcal{L}^{-1}\{.;.\}$ is the inverse Laplace transform.

Using [18, eq. (21), p. 223], the inverse Laplace of the first term in (23) is given by

$$\mathcal{L}^{-1}\left\{\frac{\left(1 + \frac{s}{B_{sb}}\right)^{-m_{sb}}}{s}; t\right\}_{|t=\gamma_{\text{th}}} = \frac{\gamma\left(m_{sb}, B_{sb}\gamma_{\text{th}}\right)}{\Gamma(m_{sb})}. \quad (25)$$

Using [19], we can get the pdf of the sum of i.n.i.d. gamma variables $X_n \sim \mathcal{G}(m_n, B_n)$, as the pdf of $Y = \sum_{n=1}^{N} X_n$ can be expressed as

$$\begin{aligned} p_Y(\gamma) &= \prod_{n=1}^{N} \left(\frac{B_1}{B_n}\right)^{m_n} \\ &\quad \times \sum_{k=0}^{\infty} \frac{\delta_k \gamma^{\sum_{n=1}^{N} m_n + k - 1} e^{-\gamma/B_1}}{B_1^{\sum_{n=1}^{N} m_n + k} \Gamma\left(\sum_{n=1}^{N} m_n + k\right)} \times U(\gamma) \end{aligned} \quad (26)$$

where $B_1 = min_n\{B_n\}$, and coefficients δ_y are obtained as

$$\begin{cases} \delta_0 & = 1 \\ \delta_{k+1} & = \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^{N} m_j \left(1 - \frac{B_1}{B_n}\right)^i\right] \delta_{k+1-i}, \\ & k = 0, 1, 2, \ldots. \end{cases} \quad (27)$$

Finally, the outage probability P_{out} of the cooperative DF DS-CDMA over Nakagami-m fading channels with arbitrary $m_{\mathcal{K}}$ can be obtained after many algebraic manipulations, with the help of [17]–[20] as

$$\begin{aligned} &P_{\text{out(CDMA)}} \\ &= \left(\prod_{\mathcal{K}=1}^{L} U_{\mathcal{K}}\right) \frac{\gamma\left(m_{sb}, B_{sb}\gamma_{\text{th}}\right)}{\Gamma(m_{sb})} + \left(\prod_{\mathcal{K}=1}^{L} U_{\mathcal{K}}\right) \\ &\quad \times \sum_{\mathcal{K}=1}^{L} \sum_{\lambda_1=1}^{L-\mathcal{K}+1} \sum_{\lambda_2=\lambda_1+1}^{L-\mathcal{K}+2} \cdots \sum_{\lambda_{\mathcal{K}}=\lambda_{\mathcal{K}-1}+1}^{L} \left(\prod_{n=1}^{\mathcal{K}} \frac{1 - U_{\lambda_n}}{U_{\lambda_n}}\right) \\ &\quad \times \left[\prod_{i=0}^{\mathcal{K}} \left(\frac{B_1}{B_i}\right)^{m_{\mathcal{K}_{\lambda_i}}}\right] \sum_{k=0}^{\infty} \delta_k \frac{\gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}_{\lambda_i}} + k, B_1\gamma_{\text{th}}\right)}{\Gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}_{\lambda_i}} + k\right)}. \end{aligned} \quad (28)$$

It is worth mentioning that for the special case of an integer Nakagami-m fading channel, the infinite sum in (28) is removed as in [21]. Let us now investigate the diversity behavior of (28) when the SNR is sufficiently large, i.e., $(\bar{\gamma}_{ij} \to \infty)$. According to [17, eq. (8.351.2)], we have

$$\begin{aligned} \frac{\gamma\left(m_{sb}, B_{sb}\gamma_{\text{th}}\right)}{\Gamma(m_{sb})} &\approx \frac{(m_{sb})^{m_{sb}-1}(\gamma_{\text{th}})^{m_{sb}}}{\Gamma(m_{sb})} (\bar{\gamma_{sb}})^{-m_{sb}} \\ U_{\mathcal{K}} &= \frac{\gamma\left(m_{\mathcal{K}}, B_{\mathcal{K}}\gamma_{\text{th}}\right)}{\Gamma(m_{\mathcal{K}})} \\ &\approx \frac{(m_{\mathcal{K}})^{m_{\mathcal{K}}-1}(\gamma_{\text{th}})^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})} (\bar{\gamma_{\mathcal{K}}})^{-m_{\mathcal{K}}}. \end{aligned}$$

Using [17, eq. (8.350.5)]

$$\frac{\gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}_{\lambda_i}} + k, B_1\gamma_{\text{th}}\right)}{\Gamma\left(\sum_{i=0}^{\mathcal{K}} m_{\mathcal{K}_{\lambda_i}} + k\right)} = 0.$$

Finally, the outage probability in (28) can be asymptotically expressed as

$$\begin{aligned} P_{\text{out(CDMA)}} &\approx \left(\prod_{\mathcal{K}=1}^{L} \frac{(m_{\mathcal{K}})^{m_{\mathcal{K}}-1}(\gamma_{\text{th}})^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})} (\bar{\gamma_{\mathcal{K}}})^{-m_{\mathcal{K}}}\right) \\ &\quad \times \frac{(m_{sb})^{m_{sb}-1}(\gamma_{\text{th}})^{m_{sb}}}{\Gamma(m_{sb})} (\bar{\gamma_{sb}})^{-m_{sb}}. \end{aligned} \quad (29)$$

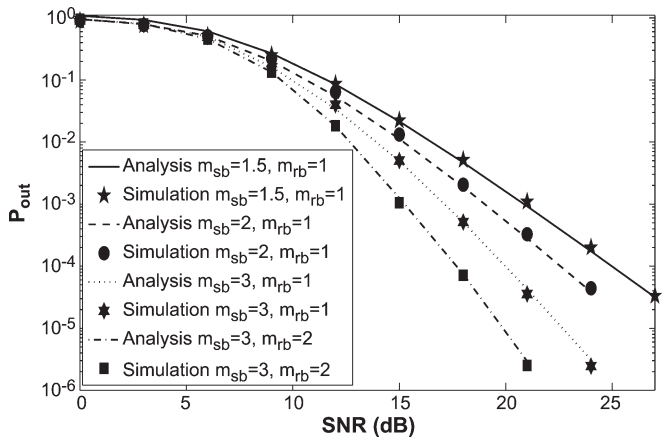

Fig. 1. Outage probability for cooperative DF DS-CDMA with a single relay, i.e., $L = 1$, and different m.

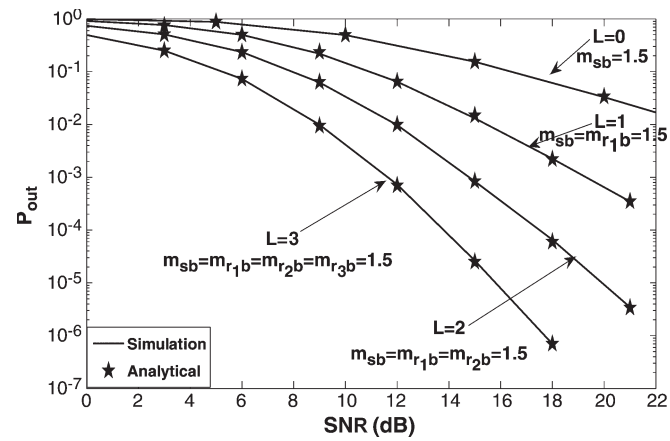

Fig. 2. Outage probability for cooperative DF DS-CDMA with different numbers of relays L over i.i.d. Nakagami-m fading channels.

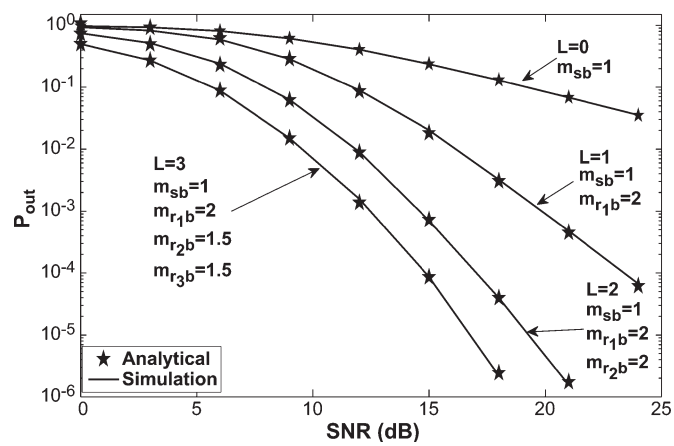

Fig. 3. Outage probability for cooperative DF DS-CDMA with different L over i.n.i.d. Nakagami-m fading channels.

Without loss of generality, let $\bar{\gamma}_{\rm th} = \bar{\gamma}_{\mathcal{K}} = \Delta$. Then, the outage probability in (29) is reduced to

$$P_{\rm out(CDMA)} \approx \frac{(m_{\mathcal{K}})^{m_{\mathcal{K}}-1}(\gamma_{\rm th})^{m_{\mathcal{K}}}}{\Gamma(m_{\mathcal{K}})} \cdot \frac{(m_{sb})^{m_{sb}-1}(\gamma_{\rm th})^{m_{sb}}}{\Gamma(m_{sb})} \times (\Delta)^{-m_{sb}-\sum_{\mathcal{K}=1}^{L} m_{\mathcal{K}}}. \quad (30)$$

The expression in (30) shows that the achievable diversity order of the system is $m_{sb} + \sum_{\mathcal{K}=1}^{L} m_{\mathcal{K}}$.

IV. Numerical Results

In what follows, we present some numerical results for the outage probability for both single and multirelay cooperation techniques. We build a Monte Carlo link-level simulation to verify these results with the derived analytical model. We assume asynchronous cooperative DS-CDMA with a fully loaded system, where $N = K$ over i.n.i.d. Nakagami-m fading channels. All simulations are based on the Nakagami simulator presented in [22]. The channels are modeled as block-fading channels, where the fading coefficients are considered fixed for the duration of one frame and independently change from one frame to another. Without loss of generality, we assume that spectral efficiency $\Re = 1$ bit/s/Hz and that fading parameters $m_{sr_\ell} = m_{r_\ell b}$. An MMSE detector is used to mitigate the effect of MAI.

Fig. 1 shows the outage probability of the single-relay cooperative system with different fading severity m_{ij} in the case of a nonidentical Nakagami-m fading channel (i.e., $m_{sb} \neq m_{rb}$).

The outage probability for the cooperative DS-CDMA system with different numbers of relays $L = \{0, 1, 2, 3\}$ over i.i.d. channels, where $m_{sb} = m_{r_{\mathcal{K}}b} = m_{sr_{\mathcal{K}}}$, is shown in Fig. 2. The results show the perfect matching between the analysis and simulations. The outage probability of the system over different fading channels parameters $m_{sb} \neq m_{r_{\mathcal{K}}b} \neq m_{sr_{\mathcal{K}}}$ is also shown in Fig. 3. Clearly, from Figs. 1–3, $P_{\rm out}$ of the underlying system is improved, and the full diversity gain is achieved with increasing L and/or m_{ij}. Figs. 1–3 also show that the derived closed-form expression of the outage probability in (28) can be applied for any arbitrary value of m, both single- and multirelay scenarios, and different channel environments (i.e., i.i.d. or i.n.i.d. channels). In Fig. 1, we examine the performance considering a single relay in an i.n.i.d. fading scenario. The effect of the fading parameter on the BER performance is evident from these results, where larger m improves the system performance. In Fig. 2, we consider a multirelay case where the fading statistics of the relay–base station and source–base station are identical. The results show the diversity gain achieved for different fading environments and numbers of relays. Finally, Fig. 3 shows the same results as in Fig. 2 but with nonidentical Nakagami distributions.

Fig. 4 shows a comparison of the outage probability of the cooperative DF asynchronous DS-CDMA system when using the conventional matched-filter detector and MMSE. The results are shown for $L = 1$ and $L = 3$ relays. The results in this figure show that the MMSE is able to achieve the full system diversity, whereas the conventional detector exhibits an error floor due to multiuser interference. That is, the diversity advantage of the cooperative system cannot be reached without mitigating the effect of multiuser interference.

V. Conclusion

In this paper, we have analyzed the outage performance of cooperative diversity in a DS-CDMA setting under diversity combining of the relayed information at the base station over Nakagami-m fading channels. Our cooperative system

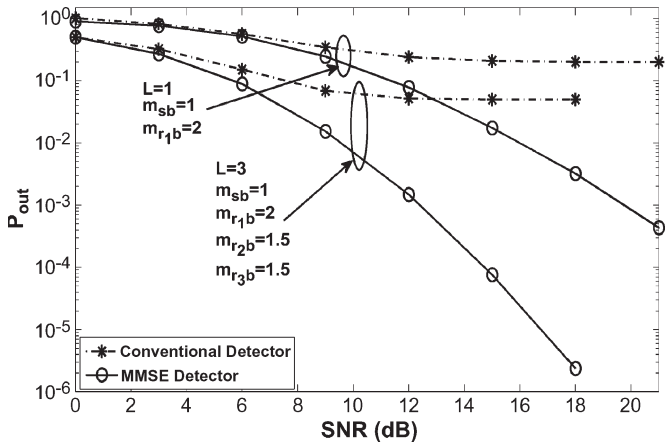

Fig. 4. Outage probability of a cooperative DF asynchronous DS-CDMA system using the conventional detector and MMSE for $L = 1$ and $L = 3$.

employed the MMSE detector to suppress the multiuser interference at both the base station and the relay sides. A closed-form expression for the outage probability of the DF cooperative system was derived for a multirelay scenario and for different fading parameters. We showed that the system is able to achieve the full diversity gain by combating the effect of multiuser interference.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their comments, which helped to improve this paper.

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