Optimal Solution for A Joint Power Allocation and Relay Assignment Problem

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***Abstract*—The ultimate efficiency of the wireless network is achieved when the various resources are allocated in a joint optimization process. This paper proposes the optimal solution for a *joint* power allocation and relay-assignment (JPARA) problem subject to the sum-power constraint at the source node. This formulation is applicable to different system models. Specifically we assume a network consisting of one source node, one destination node and N relays, where the source node uses orthogonal frequency division multiplexing (OFDM) to transmit its information to the destination. Each relay should be assigned to one of the subcarriers and the source node should distribute its power among the subcarriers. We propose an algorithm to find the JPARA solution based on max-min criterion. The optimality of the proposed algorithm is analytically proved and its complexity is calculated to be $O(N^4)$. We show that our proposed algorithm offers significant improvement in the performance of the worst end-to-end link compared to the separate optimization case. In order to further improve the performance, a new approach is proposed in which each subcarrier is assigned a predetermined initial power and the remaining power is distributed through the proposed JPARA algorithm.**

Keywords*—*Joint power allocation and relay assignment, cooperative networks, max-min criterion, sum-rate criterion.

I. Introduction

COOPERATIVE communication has introduced itself as a promising technology to extend the coverage of future networks. In cooperative systems, some terminals share their antennas to create a virtual array through a distributed transmission and thus achieve spatial diversity [1]. A basic challenge in deploying a multi-relay network is how to assign the available resources (power, relays, subcarriers and bandwidth) to each terminal. The solution to this problem is mainly determined by two factors: the *system model* and the *optimization criterion*.

In this paper, different from the previous works, we aim at the *joint* optimization of two important resource-allocation processes: relay assignment and power allocation. The joint optimization of these variables significantly outperforms the separate optimization, since we can improve the bottleneck in one optimization problem by the proper use of variables in the other problem. Hence, we expect the highest performance of the network when we find the *jointly* optimal solution for different optimization problems. In this case, normally the optimization problem turns out to be nonlinear, because the optimization variables (power and permutation selection variable) are not simply added.

Two widely accepted and commonly used criteria for resource allocation are *max-min* [2] and *max-sum* [3]. According to max-min criterion, the goal is to maximize the minimum end-to-end (E2E) signal-to-noise ratio (SNR) of the network; whereas max-sum criterion requires that the permutation with the largest sum of E2E rate is selected. Compared to these standard criteria, our joint power allocation and relay assignment (JPARA) problem in this paper takes the form of max-max-min.

JPARA problems (and in general, resource allocation problems) in wireless networks are considered by many authors with diverse sets of assumptions including different system models, different optimization criteria and different constraints. We can roughly categorize this rich literature into three categories:

1) For some system models the problem has turned out to be in the form of the standard max-min or max-sum problems [4], [5] and can be solved using the standard solutions to these problems. For example in [4] a joint power control and resource allocation problem for co-channel deployed femtocells is considered, where the min-sum problem for a centralized system model is solved based on the Hungarian algorithm. In [5], interference-aware relay selection is formulated as a weighted bipartite matching problem (max-min), and it is solved using the Hungarian algorithm.

2) For some other system models, authors faced non-standard optimization problems and proposed heuristics or iterative algorithms (sometimes suboptimal) to solve that problem [6]–[11]. For example, in [6] an iterative algorithm for a distributed JPARA min-sum problem is proposed, which has low complexity and achieves a very good performance. Mallick et al. [7] proposed JPARA algorithms that can work robustly under imperfect channel knowledge for a decode-and-forward (DF) cellular relay network. They also derived a suboptimal solution for the original problem from the relaxed problem. Their objective is to minimize the uplink transmit power of the network, taking each user's target data rate as the quality-of-service (QoS) constraint. In [8], each user transmits its own data towards the base station and also serves as a relay for other users. The objective function of their JPARA problem is to minimize the sum power (min-sum), and an iterative algorithm is developed that jointly performs relay selection and optimally allocates source and relay powers to satisfy the sum-

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rate (max-sum) criterion. In [9] JPARA problem for orthogonal multiuser systems using amplify-and-forward (AF) relaying nodes in downlink is considered. Their optimization criterion is sum-rate and their problem is described to be non-convex without a known tractable solution. To tackle this, the authors in [9], proposed an algorithm using Markov chain Monte-Carlo with divergence minimization. In [10] a joint opportunistic subchannel and power-scheduling algorithm for transmission at both the source node and the relays is developed. They use the stochastic subgradient algorithm to solve the problem and they show that the results are asymptotically optimal. Hau et al. [11] has considered JPARA problem in a similar system model. Different from our objective function in this paper, their objective is to maximize the spectral efficiency under a total power constraint. They show that their problem can be decomposed into some sub-problems through dual relaxation.

$3)$ In some special cases, the problem structure permitted the authors to find analytical closed-from solutions for the problems at hand [12], [13]. For example, Talwar et al. [13] presented an optimal joint *single* relay selection and power allocation scheme for two-way relay networks. Their approach is based on the max-min criterion under a total transmit power budget where a closed-form solution was presented.

The main contributions of this paper are as follows.

1) This paper targets the joint optimization of power allocation and relay-assignment processes, which is expected to achieve a higher efficiency in the utilization of scarce wireless resources. Different from the previous optimization problems, our joint optimization problem turns out to be a new mixed integer programming. To the best of our knowledge, this problem formulation has not been investigated in the literature. We manage to find the optimal solution for this problem. We present a graphical illustration of the proposed optimization method, which can inspire the solution for some other joint optimization problems.
2) We prove the optimality of the proposed algorithm and analyze its achieved diversity order. We show that the complexity of this algorithm is upper bounded by $O(N^4)$.
3) Our simulation results reveal a significant improvement in the minimum received SNR of the network, compared to the *separate optimization* of each resource, e.g. the case where we employ the optimal max-min relay assignment and then, the optimal allocation of the power. Our simulation results unveil that the proposed algorithm can save up to 23% on the consumed power. We also analyze the probability density function (PDF) of the required power by each user, and we show that a diversity order equal to the number of relays is achieved. We investigate the performance of the proposed algorithm in two scenarios. In the first scenario, fixed relays are considered where the strength of the relay-destination (R-D) channels is much better than that of source-relay (S-R) channels [14], [15]. In the second scenario, mobile relays are assumed where the assumption on the reliability of R-D channels is relaxed.
4) Our results unveil that although the worst E2E performance improves with increasing the portion of the power that is distributed through the JPARA algorithm (P/P_{total}), the average E2E performance is not a monotonic function of this power. Furthermore, we show that the best strategy for the first scenario is to equally distribute nearly 30% of the total power to each subcarrier and distribute the remaining power through the proposed JPARA algorithm.

The rest of the paper is organized as follows. Section II introduces the system model and problem formulation. Some preliminaries are reviewed in Section III. The proposed algorithm for the first scenario is presented in Section IV. The optimality proof for the proposed algorithm and the complexity analysis follow in the subsequent subsections. The second scenario is investigated in Section V. Finally, the simulation results are presented in Section VI.

II. System Model

Consider the network illustrated in Fig. 1, which consists of one source node, one destination node, and N relays. The source node, the relay nodes, and the destination node are denoted by S, R_i, and D, respectively. Here, i denotes the relay index. The source node uses orthogonal-frequency-division multiplexing (OFDM) with L subcarriers to send its information to the destination. We assume $L \leq N$, because otherwise the problem simply converts to the best relay selection for each subcarrier. We assume no direct communication between the source and the destination due to the poor quality of the channels between them. The relays are assumed to be half-duplex. This network configuration has several applications in ad-hoc, cellular and wireless sensor networks. We assume that each relay can serve only one subcarrier. This assumption is usually made for two purposes: i) to reduce complexity and to avoid the expensive and power consuming analog radio frequency (RF) chains at relays [16]; and ii) to satisfy the power and energy limitations of the relays. This problem is equally applicable to different cooperation schemes, but to be more specific, we assume the AF scheme. A two-hop relay mode is employed. In the first hop, the source terminal transmits and relays receive. In the second hop, the relay terminals transmit and the destination receives. We assume that a centralized resource allocation is employed and the SNR values of all the channels are known to the resource allocator. We acknowledge that this assumption requires adding some overhead and the amount of this overhead increases with the size of the network. Therefore difficulty of the implementation of this scheme grows with the number of nodes, which is a known problem for such networks [17], [18].

We assume that there exists an amount of power P at the source node to be distributed among subcarriers. The question is how to find the jointly optimal solution for the following two problems: i) how to assign each subcarrier to one of the relay nodes; and ii) how to distribute the available power P among different subcarriers. As it is common to aim at improving the minimum SNR, minimum rate or minimum QoS which is offered by the network [2], [18], [19], we use the same criterion to maximize the minimum E2E SNR.

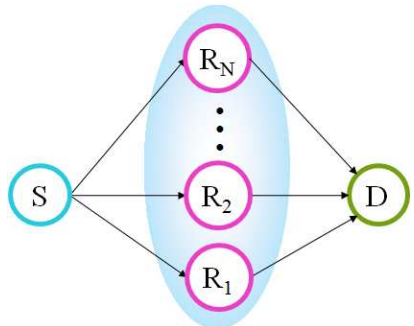

Fig. 1. System model

For this problem, consider the case in which at least one of the R-D channels in each permutation experiences deep fade. In this case, the E2E SNR of those links is mainly determined by the bottleneck in their R-D channels. Hence, amplifying their S-R channel does not have a linear effect on their E2E SNR. However, max-min criterion maintains that the weakest E2E channel has a higher priority to receive power amplification. To avoid overspending power in the mentioned links, we add an extra assumption to our optimization criterion, where we assume all subcarriers are assigned a predetermined initial power P_0, and the remaining power P is to be distributed among them. We will study the behavior of the system performance for different values of P_0/P (including $P_0 = 0$) in Section VI.

Let us denote by $x_{i,j}$ the equivalent SNR corresponding to subcarrier i when it is assigned to link S-R_j-D. The exact formulas to calculate $x_{i,j}$ in terms of the SNR values of the S-R_j and R_j-D links are given by [20]

$$x_{i,j} = \frac{x_{i,j}^{S-R} x_{i,j}^{R-D}}{x_{i,j}^{S-R} + x_{i,j}^{R-D} + 1},$$

where it is assumed that the amplification gain at the relay node equals $G^2 = 1/(|h_{i,j}|^2 + N_0)$. Here $h_{i,j}$ is the fading coefficient of subcarrier i on the S-R_j channel and N_0 is the power spectral density of the additive noise. There are LN E2E SNR values in each realization of the channels which can be written in a matrix format as $\mathbf{X} = [x_{i,j}]_{L\times N}$.

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,N} \\ \vdots & \ddots & \vdots \\ x_{L,1} & \cdots & x_{L,N} \end{bmatrix}. \tag{1}$$

Note that, assignment of relay j to subcarrier i corresponds to the selection of $x_{i,j}$ from the above matrix. In this manner, the relay assignment problem is transformed to the selection of some elements from $\mathbf{X}$. There are $K = N!/(N-L)!$ different permutations in $\mathbf{X}$. We denote each permutation by π_i, where $i = 1,\ 2, \ldots, K$. We sort the elements in each permutation and denote the nth smallest element of permutation π_i by $\pi_i(n)$. There are LN elements in $\mathbf{X}$. Now, let us arrange these elements in order of magnitude and denote the ith smallest element by $x_{i:LN}$. Hence, the smallest and the largest elements are denoted by $x_{1:LN}$ and $x_{LN:LN}$, respectively.

III. Standard Max-min and Max-sum Criteria

Before presenting our proposed algorithm, we first address the standard max-min and max-sum optimization problems, and then introduce water-filling formulation in the context of our work.

Max-min Criterion: The mathematical definition of this criterion which is also called *linear bottleneck assignment* is given by [2], [21]

$$\max_i \min_n \pi_i(n), \tag{2}$$

where each of the elements $\pi_i(n)$ is an element of an L-by-N cost/benefit matrix $\mathbf{X}$. It can be formulated also in a graph theoretical setup as finding a perfect matching in a bipartite weighted graph $H = (S \bigcup D, E)$ which maximizes the minimum weight of all matching edges [22]. Here, S and D denote the classes of the nodes and E denotes the edge set. Applying this criterion to a cooperative system where all of the E2E SNR values are independent achieves full diversity [23].

Max-sum Criterion: The mathematical definition takes the following form,

$$\max_i \sum_n \pi_i(n).$$

Obviously, if we have an algorithm to find the optimum max-sum permutation, we can employ it to find the optimum min-sum permutation [21]. Similar to the max-min criterion, the max-sum criterion achieves full diversity provided that it is applied to the rate values, and all E2E SNR values are independent [3].

Among the different algorithms for solving the max-min problem, the so-called threshold algorithm has attracted much attention [24]. A threshold-based algorithm alternates between two phases. In the first phase, a cost element x_{thr} (the threshold value) is chosen and a new matrix $\bar{\mathbf{X}}$ is formed.

$$\bar{\mathbf{X}} = \begin{cases} 1 & x_{i,j} \geq x_{thr} \\ 0 & x_{i,j} < x_{thr} \end{cases} \tag{3}$$

In the second phase, it is checked whether the bipartite graph with adjacency matrix $\bar{\mathbf{X}}$ contains a perfect matching or not. The biggest value x_{thr} for which the corresponding bipartite graph contains a perfect matching, is the optimum solution of the max-min problem. There are several methods to implement such a threshold algorithm. One possibility is to order the cost elements in an increasing order and to apply a binary search [21, p. 174] in the first phase. This leads to an $O(C(n)\log n)$ algorithm, where $C(n)$ is the time complexity for checking the existence of a perfect matching. An algorithm with the currently best time complexity is obtained by combining the threshold approach with augmenting paths [25]. This idea goes back to Gabow and Tarjan [26].

As for the max-sum criterion, the first Hungarian algorithm was presented in the mid-1950s by Kuhn [27]. This algorithm in its original formulation solves the problem in $O\left(N^4\right)$ time. The best time complexity for a Hungarian algorithm is $O(N^3)$ which was proposed by Lawler [28], however, the first $O(N^3)$ algorithm for max-sum problem appeared in [29].

Water-filling and Equalization: By *water-filling*, we mean to use the power P to increase the smallest element of the permutation at hand. As the name suggests, this is similar to water-filling in other applications. In our case, we use the power P to amplify the weakest channel ($\pi_i(1)$) up to the SNR of the second weakest channel ($\pi_i(2)$). Then we use the remaining power to amplify both ($\pi_i(1)$) and ($\pi_i(2)$) up to the SNR of ($\pi_i(3)$). We continue in the same way until we finish the extra power. If still some power remains, we use this power to amplify all channels with the same factor. If we have enough power to make all the SNRs equal, we say that power P is enough to equalize this permutation.

Using this definition for equalization, *the joint optimization* problem can be expressed in the following form.

$$\max_{u_{i,j}} \max_{p_i \in P} \min_{\{i,j|u_{i,j}\neq 0\}} u_{i,j} x_{i,j}\left(p_i\right) \tag{4a}$$

Subject to:

$$\sum_{i=1}^{L} p_i \leq P \tag{4b}$$

$$\sum_{i=1}^{L} u_{i,j} \leq 1 \tag{4c}$$

$$\sum_{j=1}^{N} u_{i,j} = 1 \tag{4d}$$

$$u_{i,j} \in \{0,1\}. \tag{4e}$$

By using the previously introduced permutation terminology, (4a) becomes:

$$\max_{\pi_k, k\in\{1:K\}} \max_{p_i \in P} \min_{n} \pi_k(n) \tag{5}$$

In this paper, we frequently refer to the above definition.

IV. Proposed Optimization Algorithm for The First Scenario

For this system model, we assume that the strength of the R-D channels is much better than the strength of the S-R channels. This case corresponds to the following scenarios: i) Fixed line-of-sight relays (the fading in the R-D channels is usually modeled as Rice distributed) [14], [15]. ii) Mobile relays where the relays are placed close to the destination, such that the R-D channel is much more reliable than the S-R channels. In this section, first the idea behind our proposed algorithm is explained and then the algorithm is introduced in detail. The optimality proof and the complexity analysis of the proposed algorithm are presented in the following subsections.

The idea behind the proposed algorithm is to split the problem into two simpler problems: for this purpose, we split the elements of $\mathbf{X}$ into two new matrices $\mathbf{Y} = [y_{i,j}]$ and $\mathbf{Z} = [z_{i,j}]$ such that $\mathbf{Y}$ contains the elements smaller than a threshold value x_{thr}, and $\mathbf{Z}$ contains the elements larger than x_{thr}.

$$y_{i,j} = \begin{cases} x_{i,j} & x_{i,j} \leq x_{thr} \\ 0 & x_{i,j} > x_{thr} \end{cases} \tag{6}$$

$$z_{i,j} = \begin{cases} 0 & x_{i,j} \leq x_{thr} \\ x_{i,j} & x_{i,j} > x_{thr} \end{cases} . \tag{7}$$

We will find this threshold such that P is enough to equalize the non-zero elements of the best permutation in $\mathbf{Y}$ (denoted by $\pi_{opt}(\mathbf{Y})$), but it is not enough to equalize "a permutation in $\mathbf{X}$ that contains $\pi_{opt}(\mathbf{Y})$ and any other element". As such, the problem is mapped into two simpler problems:

1) a simpler JPARA problem in $\mathbf{Y}$, because the non-zero elements of all of the permutations in $\mathbf{Y}$ can be equalized. In this way, all of the extra power is being spent to equalize the elements in $\mathbf{Y}$.
2) a standard max-min problem in $\mathbf{Z}$, where there is no extra power available.

In order to find the threshold value, we exploit an algorithm similar to threshold algorithms (Section III) and binary search. Let us denote the optimal permutation by π_{opt} and the equalized max-min SNR corresponding to this permutation by T_{opt}. Using the binary search algorithm, we examine different elements of the matrix to find the threshold value $x_{thr} = x_{M:LN}$ such that $x_{M:LN} \leq T_{opt} < x_{M+1:LN}$.

For this purpose, in this section we use the commonly accepted approximation from [2]

$$x_{i,j}^{E2E} = \min\left(x_{i,j}^{RD}, x_{i,j}^{RD}\right)$$

where $x_{i,j}^{SR}$ and $x_{i,j}^{RD}$ denote the SNR value of subcarrier i on S-R_j and R_j-D channels, respectively (we will relax this assumption in the next section and we will consider the general case). Without loss of generality, we assume $P_0 = 1$ to simplify the presentation of the algorithm. Then we achieve

$$P_{i,j}\left(\Gamma_{new}\right) = \frac{\Gamma_{new}}{x_{i,j}^{SR}} - 1, \tag{8}$$

where Γ_{new} is the target SNR value and $P_{i,j}$ is the required power. By assuming $\Gamma_{new} = x_{m:LN}$, and using (8), we can calculate the amount of *power* required to amplify each $x_{i,j}$ to $x_{m:LN}$. Let us denote the resulting matrix by $\mathbf{\Phi}_m = [\varphi_{i,j}]$. Thus we have

$$\varphi_{i,j} = \begin{cases} \dfrac{x_{m:LN}}{x_{i,j}} - 1 & x_{i,j} \leq x_{m:LN} \\ 0 & x_{i,j} > x_{m:LN} \end{cases}$$

Here since some of the elements of $\mathbf{X}$ are larger than $x_{m:LN}$, for those elements one needs no extra power and we have $\varphi_{i,j} = 0$. Let us denote the min-sum permutation in $\mathbf{\Phi}_m$ by $\pi_{min-sum(m)}$. If $\sum_n \pi_{min-sum(m)}(n) \leq P$, then the corresponding permutation can be equalized.

Algorithm 1 is the main proposed algorithm which employs two other sub-algorithms to find the jointly optimal solution.

Algorithm 2 is used to find the largest m for which $\sum_n \left(\pi_{min-sum(m)}(n)\right) \leq P$. This part of the optimization

Algorithm 1 The main algorithm

1) By using Algorithm 2, find the largest m for which $\sum_n \left(\pi_{min-sum(m)}(n)\right) \leq P$. Denote the result by M.
2) Run Algorithm 3 in order to find the jointly optimal permutation in $\mathbf{\Phi}_M$
3) Let $\mathbf{\Psi}_M = [\psi_{i,j}]$ where
$$\psi_{i,j} = \begin{cases} x_{i,j} & x_{i,j} > x_{M:LN} \\ 0 & x_{i,j} \leq x_{M:LN} \end{cases}$$
4) Find the max-min permutation in $\mathbf{\Psi}_M$

process can be described as:

$$\max_{m} \min_{u_{i,j}} \sum_{i=1}^{L} \sum_{j=1}^{N} u_{i,j}\varphi_{i,j}\left(x_{m:LN}\right) \tag{9a}$$

Subject to:

$$\sum_{i=1}^{L} \sum_{j=1}^{N} u_{i,j}\varphi_{i,j}\left(x_{m:LN}\right) \leq P \tag{9b}$$

$$\sum_{i=1}^{L} u_{i,j} \leq 1 \tag{9c}$$

$$\sum_{j=1}^{N} u_{i,j} = 1 \tag{9d}$$

$$u_{i,j} \in \{0,1\}. \tag{9e}$$

Algorithm 2 Finding $\mathbf{\Phi}_M$

1) Set $i_{min} = 1$ and $i_{max} = LN$
2) Repeat the following procedure until we find m such that $min\text{-}sum\left(\mathbf{\Phi}_m\right) > P$ and $min\text{-}sum\left(\mathbf{\Phi}_{m+1}\right) < P$
3) {Calculate $m =$ the midpoint between i_{min} and i_{max}
4) Find $\pi =$ the min-sum permutation in $\mathbf{\Phi}_m$
5) If $\sum_n \pi(n) < P$
6) Set $i_{min} = m$
7) Else
8) Set $i_{max} = m$
9) End}

For this purpose, one can perform the standard linear search. This can be done as all LN candidates for $\mathbf{\Phi}_m$ can be checked to find the required one. However by using the binary search algorithm, we can find M in at most $\log(LN)$ steps. For this purpose, among LN possible values for m, we select the middle value $m = LN/2$, and the algorithm checks $\mathbf{\Phi}_{\lfloor LN/2 \rfloor}$. Here $\lfloor . \rfloor$ is the floor function. If one can equalize the best permutation in this matrix, then the middle value in the right side of m can be selected as the new m, i.e.

$$m = \left\lfloor \frac{1}{2}\left(\frac{LN}{2} + LN\right)\right\rfloor.$$

Otherwise, we select the middle value in the left side of m as the new m. In other words, in each step of this algorithm, we reduce the search span into half. Thus, we can find M in at most $\log(LN)$ steps.

It is to be noted that the optimal permutation does not necessarily correspond to the min-sum permutation in $\mathbf{\Phi}_M$. This is because $\mathbf{\Phi}_M$ is calculated using a specific target SNR ($x_{M:LN}$), but the optimal target SNR can be larger than $x_{M:LN}$ (since $x_{M:LN} \leq T_{opt} < x_{M+1:LN}$). Hence, if $T_{opt} > x_{M:LN}$, then one should recalculate $\mathbf{\Phi}_M$ and it is possible that a different permutation is the min-sum permutation.

After $\mathbf{\Phi}_M$ is found, Algorithm 3 is used to find the jointly optimal permutation in $\mathbf{\Phi}_M$, where P is enough to equalize all of the permutations. Fig. 2 illustrates a graphical interpretation of the proposed Algorithm. In this figure, each curve represents a permutation and the figure shows the amount of required power for each permutation to achieve a minimum target SNR equal to T. This algorithm alternates between two phases. In the first phase, it starts with a prespecified target SNR $T_0 = x_{M:LN}$ and finds the permutation that achieves $T = T_0$ with the lowest required power (point ❶). This phase can be described by the following optimization process which is done using the standard min-sum algorithm.

$$\min_{u_{i,j}} \sum_{i=1}^{L} \sum_{j=1}^{N} u_{i,j}\varphi_{i,j}\left(x_{m:LN}\right) \tag{10a}$$

Subject to:

$$p_i s \text{ are fixed, } i = 1, \cdots, L \tag{10b}$$
$$(9c), (9d), \text{ and } (9e)$$

Algorithm 3 Finding the optimal permutation in $\mathbf{\Phi}_M$

1) Set $i = 1$
2) Repeat steps 3 to 6 until there is no change in the selected permutation
3) Find the min-sum permutation in $\mathbf{\Phi}_M$ and denote this permutation by π_i [21]
4) By using P calculate $\Gamma_{new} = Equ(\pi_i)$ (please note that $\Gamma_{new} \leq x_{M+1:LN}$). If J denotes the number of non-zero elements in $\mathbf{\Phi}_M$, we have
$$\Gamma_{new} = Equ(\pi_1) \triangleq \frac{P + J}{\sum_{n=1}^{J} \frac{1}{\pi_i(n)}}$$
5) By using Γ_{new} as the new target value, recalculate $\mathbf{\Phi}_M$
6) Set $i = i + 1$

In the second phase, water-filling algorithm is used to calculate the best achievable T for this permutation (point ❷ which corresponds to $T = T_1$). This new target SNR value is used as the target SNR for the first phase in the next iteration, i.e. this time we find the permutation that achieves T_1 with the lowest required power (point ❸). The same steps are repeated through the solid curves (points ❹, ❺, ❻). At ❻, by running min-sum algorithm we find that no other permutation can achieve $T = T_3$ with a lower power. Hence $T_{opt} = T_3$ and the optimal permutation is reached.

To better illustrate the operation of the proposed optimization algorithm, we consider the following example. Let us

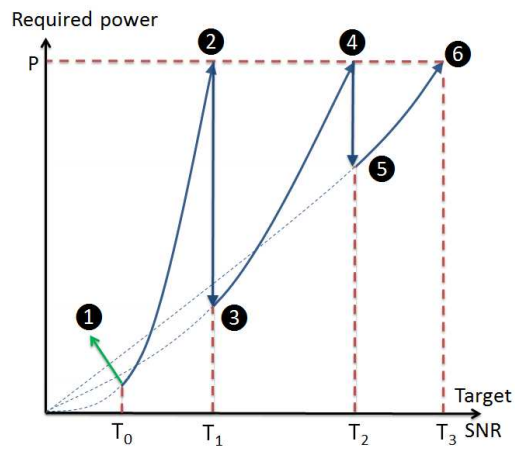

Fig. 2. A diagram that shows how Algorithm 3 converges to the optimal permutation.

consider the following matrix $\mathbf{X}$ where $L = N = 4$. We assume that the available power is $P = 2\ W$. Upon applying the Algorithm 2, we find $M = 13$ and $x_{M:LN} = x_{13:16} = 60$.

$\mathbf{X} =$

55	80	83	43
32	5	35	17
29	60	81	7
13	44	15	49

Now, we calculate the amount of power required to amplify every SNR to 60. The resulting $\mathbf{\Phi}_M$ is shown here, where the min-sum permutation is highlighted.

$\mathbf{\Phi}_M =$

0.091	0	0	0.395
0.875	11	0.714	2.529
1.069	0	0	7.571
3.615	0.364	3	0.225

One can see that the sum of the specified elements is 1.03. That is by using $P_{used} = 1.03\ W$, one can amplify all of the elements in the selected permutation to 60. However, the available power $P = 2\ W$ is not reached. If one uses this remaining power to further amplify the above selected permutation, we can achieve a higher target SNR of 71.58 (this value is smaller than $x_{M+1:LN} = 80$). Using this new target SNR, we recalculate $\mathbf{\Phi}_M$ and find the min-sum permutation (first iteration of Algorithm 3) to yield

$\mathbf{\Phi}_M =$

0.301	0	0	0.664
1.237	13.315	1.045	3.21
1.468	0.193	0	9.225
4.506	0.627	3.772	0.461

From the above, one can see that the sum of the specified elements is 1.6974 (i.e., $P_{used} = 1.6974\ W$) and we can amplify the smallest selected element to 71.58. However, we still have not consumed all the available power P. The interesting point is that the number of nonzero elements in the new min-sum permutation is 2 (in the last iteration it was 3, hence decreased by 1). Using the remaining power to further amplify the above selected permutation, we can achieve a higher target SNR equal to 77.4321 (this value is still smaller than $x_{M+1:LN} = 80$). Using this new target SNR, again we recalculate $\mathbf{\Phi}_M$ and we find the min-sum permutation (second iteration of Algorithm 3) to yield

$\mathbf{\Phi}_M =$

0.408	0	0	0.801
1.42	14.486	1.212	3.555
1.67	0.29	0	10.062
4.956	0.76	4.162	0.58

This time, the same permutation is returned as the min-sum permutation and all the available power is consumed. Thus, the algorithm is converged and the optimal permutation is found.

A. Optimality Proof

Using Algorithm 2, a target SNR value equal to $x_{M:LN}$ or larger is guaranteed (this is checked in Step 4 of Algorithm 2). Hence, in order to prove the optimality, we only need to prove that Algorithm 3 converges to the optimal permutation. This is proved by Theorem 1.

Before presenting the proof, we first introduce some new variables. The min-sum permutation which is found in the ith iteration of Algorithm 3 is denoted by π_i. The number of nonzero elements in π_i is denoted by $\mathcal{N}_{nonzero}(\pi_i)$, and the optimal permutation is denoted by π_{opt}. Before presenting the optimality proof, we introduce the following lemmas.

Lemma 1: In any iteration of Algorithm 3, if

$$\mathcal{N}_{nonzero}(\pi_{opt}) = \mathcal{N}_{nonzero}(\pi_i),$$

then we have $\pi_{opt} = \pi_i$

Proof: Suppose that in iteration j of Algorithm 3, we have

$$\mathcal{N}_{nonzero}(\pi_{opt}) = \mathcal{N}_{nonzero}(\pi_j), \tag{11}$$

Since, π_j is the min-sum permutation using the target SNR value T_j, we can conclude that

$$\sum_{i=1}^{n_j}\left(\frac{T_j}{\pi_j(i)} - 1\right) < \sum_{i=1}^{n_{opt}}\left(\frac{T_j}{\pi_{opt}(i)} - 1\right), \tag{12}$$

where $n_j = \mathcal{N}_{nonzero}(\pi_j)$ and $n_{opt} = \mathcal{N}_{nonzero}(\pi_{opt})$. This yields

$$T_j \sum_{i=1}^{n_j}\left(\frac{1}{\pi_j(i)} - \frac{1}{\pi_{opt}(i)}\right) < 0. \tag{13}$$

However π_{opt} is the optimal permutation, thus

$$\sum_{i=1}^{n_j}\left(\frac{T_{opt}}{\pi_j(i)} - 1\right) > \sum_{i=1}^{n_{opt}}\left(\frac{T_{opt}}{\pi_{opt}(i)} - 1\right), \tag{14}$$

which in turn yields

$$T_{opt} \sum_{i=1}^{n_j}\left(\frac{1}{\pi_j(i)} - \frac{1}{\pi_{opt}(i)}\right) > 0. \tag{15}$$

Now, since T_j and T_{opt} are both positive values, (13) and (15) cannot hold true in the same time. Hence, both permutations are identical. ■

Lemma 2: In any iteration of Algorithm 3, if a new permutation is found (i.e. if π_j is different from π_{j+1}), then this new permutation has fewer nonzero elements.

Proof: T_j is the target SNR value in the jth iteration of Algorithm 3, which is found in the $j-1^{th}$ iteration. Using this target SNR value, π_j is found as the min-sum permutation, i.e.

$$\sum_{i=1}^{n_j}\left(\frac{T_j}{\pi_j(i)}-1\right)<\sum_{i=1}^{n_{j+1}}\left(\frac{T_j}{\pi_{j+1}(i)}-1\right). \tag{16}$$

In the same manner, T_{j+1} is the target SNR value in the $j+1^{th}$ iteration of Algorithm 3, and we have

$$\sum_{i=1}^{n_j}\left(\frac{T_{j+1}}{\pi_j(i)}-1\right)>\sum_{i=1}^{n_{j+1}}\left(\frac{T_{j+1}}{\pi_{j+1}(i)}-1\right). \tag{17}$$

From (16) it is concluded that

$$T_j\left(\sum_{i=1}^{n_j}\frac{1}{\pi_j(i)}-\sum_{i=1}^{n_{j+1}}\frac{1}{\pi_{j+1}(i)}\right)<n_j-n_{j+1}, \tag{18}$$

and similarly from (17), we can conclude that

$$T_{j+1}\left(\sum_{i=1}^{n_j}\frac{1}{\pi_j(i)}-\sum_{i=1}^{n_{j+1}}\frac{1}{\pi_{j+1}(i)}\right)>n_j-n_{j+1}. \tag{19}$$

Now, by comparing (18) and (19),

$$T_j\left(\sum_{i=1}^{n_j}\frac{1}{\pi_j(i)}-\sum_{i=1}^{n_{j+1}}\frac{1}{\pi_{j+1}(i)}\right) < T_{j+1}\left(\sum_{i=1}^{n_j}\frac{1}{\pi_j(i)}-\sum_{i=1}^{n_{j+1}}\frac{1}{\pi_{j+1}(i)}\right). \tag{20}$$

(20) implies that one of the following three options holds true:

- If $n_j < n_{j+1}$, then since $T_{j+1} > T_j$, there exists a contradiction between this result and that from Step 2 of Algorithm 3 (similar to (13) and (15)).
- If $n_j = n_{j+1}$, then "$<$" in (16) and (18) is meaningless.
- The only remaining choice is $n_j > n_{j+1}$ and the lemma is proved.

■

Lemma 3: In any iteration of Algorithm 3, if a new permutation is not found (i.e. π_j is not different from π_{j+1}), then this new permutation is the optimal permutation.

Proof: In this case one needs to prove that either $\pi_j = \pi_{j+1}$ is the optimal permutation or a contradiction exists. Let us assume that π_{j+1} is not the optimal permutation and permutation π_{opt} is the optimal one. Since π_j is the min-sum permutation, found in the jth and $j+1^{th}$ iteration, we have:

$$\sum_{i=1}^{n_j}\left(\frac{T_{j+1}}{\pi_j(i)}-1\right)<\sum_{i=1}^{n_{opt}}\left(\frac{T_{j+1}}{\pi_{opt}(i)}-1\right). \tag{21}$$

On the other hand, T_{j+1} is the target SNR value which is achieved through equalization of π_j by using power P, i.e.

$$P=\sum_{i=1}^{n_j}\left(\frac{T_{j+1}}{\pi_j(i)}-1\right). \tag{22}$$

Combining (21) and (22) yields

$$T_{j+1}>\frac{P+n_{opt}}{\sum_{i=1}^{n_{opt}}\frac{1}{\pi_{opt}(i)}}. \tag{23}$$

However for the optimal permutation we have

$$\sum_{i=1}^{n_{opt}}\left(\frac{T_{opt}}{\pi_{opt}(i)}-1\right)=P, \tag{24}$$

where T_{opt} is the optimal target SNR achieved by equalizing π_{opt}. By rearranging (24), we have

$$T_{opt}=\frac{P+n_{opt}}{\sum_{i=1}^{n_{opt}}\frac{1}{\pi_{opt}(i)}} \tag{25}$$

By comparing (23) and (25), we can conclude that

$$T_{j+1}>T_{opt}$$

which is contradicting to the optimality of π_{opt} and the lemma is proved. ■

Theorem 1: Algorithm 1 finds the optimal permutation in $\mathbf{\Phi}_M$.

Proof: Suppose that $\mathcal{N}_{nonzero}(\pi_1) = n_1$ and $\mathcal{N}_{nonzero}(\pi_{opt}) = n_{opt}$. Let us consider the next n_x+1 iterations where $n_x = n_1 - n_{opt}$. If in any of these n_x+1 iterations the same permutation is found, then according to Lemma 3 this permutation is the optimal permutation. Otherwise in at most n_x+1 iterations, we will reach a new permutation π_x for which $\mathcal{N}_{nonzero}(\pi_x) = \mathcal{N}_{nonzero}(\pi_{opt})$. Then according to Lemma 1, π_x should be the optimal permutation. ■

B. Complexity Analysis

Here we show how the proposed algorithm responds to changes in input size N in terms of the processing time and the memory requirements. The complexity of each sub-algorithm which is employed in our algorithm is shown in Table 1 where we have also shown the maximum number of times that each sub-algorithm is called. As seen, the worst case complexity of the proposed algorithm is upper bounded by $O(N^4)$. That is the proposed optimization algorithm is just one order above the complexity of the standard max-sum algorithm. We should note that this is only the worst case performance of the algorithm and our extensive simulations show that most of the time, Algorithm 3 converges in a few iterations.

Algorithm name	Complexity	# of times it is called	Total
Min-sum algorithm to find M	$O(N^3)$	$2\log N$	$O(N^3 \log N)$
Min-sum algorithm called in Algorithm 3	$O(N^3)$	N	$O(N^4)$
Max-min algorithm in $\mathbf{\Psi}$	$O(\frac{N^{2.5}}{\log N})$	1	$O(\frac{N^{2.5}}{\log N})$
Total			$O(N^4)$

Table 1. The complexity corresponding to each part of the proposed algorithm and the total complexity.

V. Proposed Optimization Algorithm for The Second Scenario

In this section, we assume general mobile relays where the assumption on the strength of the R-D channels is relaxed. Again, suppose that we add power $P_{i,j}$ to amplify the SNR of the subcarrier i on the link S-R_j-D. The new E2E SNR for AF cooperation is [20]

$$\Gamma_{new} = \frac{(P_0 + P_{i,j})x_{i,j}^{SR} x_{i,j}^{RD}}{1 + (P_0 + P_{i,j})x_{i,j}^{SR} + x_{i,j}^{RD}}$$

Assuming $P_0 = 1$, we achieve

$$P_{i,j} = \frac{\Gamma_{new}(1 + x_{i,j}^{RD})}{x_{i,j}^{SR}(x_{i,j}^{RD} - \Gamma_{new})} - 1 \tag{26}$$

To find the jointly optimal solution, we can apply the previously proposed algorithms with one modification. This modification is in Step 2 of Algorithm 3, where we used water-filling to calculate the minimum amplified value (Γ_{new}) for a specific permutation π_1. This calculation needed simple addition and division micro-operations. In the current case, one needs to solve the following equation

$$\sum_{n=1}^{J} P_i = P,$$

which in turn maintains that

$$\sum_{n=1}^{J} \left(\frac{\Gamma_{new}(1 + x_{i,j}^{RD})}{x_{i,j}^{SR}(x_{i,j}^{RD} - \Gamma_{new})} - 1 \right) = P. \tag{27}$$

In (27), one needs to determine the roots of a polynomial equation of order J which requires numerical methods. As a result, the complexity of this algorithm is higher than that of the previous case (i.e., reliable R-D links) and it is determined by the accuracy of finding the roots of (27).

A. Optimality Proof

Theorem 2: The modified version of Algorithm 3 finds the optimal permutation in $\mathbf{\Phi}_M$.

Proof: Let us assume that our algorithm has converged to a target SNR value T_{j+1} and permutation π_j, but the jointly optimal target SNR value is T_{opt} and the corresponding permutation is π_{opt}. By considering (27) in Algorithm 3, we have

$$P = \sum_{i=1}^{n_j} \left(\frac{T_{j+1}(1 + x_j^{RD}(i))}{x_j^{SR}(i)(x_j^{RD}(i) - T_{j+1})} - 1 \right) < \sum_{i=1}^{n_j} \left(\frac{T_{j+1}(1 + x_{opt}^{RD}(i))}{x_{opt}^{SR}(i)(x_{opt}^{RD}(i) - T_{j+1})} - 1 \right). \tag{28}$$

Since $T_{opt} > T_{j+1}$ is a feasible target SNR, the required power for π_{opt} to achieve T_{j+1} should be less than P, which is in contradiction with the above result. Thus, the algorithm could not stop at T_{j+1}. ■

VI. Simulations

In this section, we present simulation results to confirm the validity of the proposed optimization algorithm and to investigate its performance under different system settings.

The validity of the proposed algorithm for 5-by-5 and 6-by-6 matrices and for tens of thousands of times has been checked. For this purpose, we calculated the equalized version of all permutations (total 120 permutations for 5-by-5 matrices and 720 permutations for 6-by-6 matrices) and then selected the permutation with the largest minimum. We compared the results with that of the proposed algorithm and in all trials, both algorithms reach the same result. It is interesting to know that in more than 95% of the trials, Algorithm 3 converges in the first iteration. In other word, the min-sum permutation in $\mathbf{\Phi}_M$ using T_1 is the optimal permutation.

In the first investigation, we consider a fixed relay network with $L = 3$ subcarriers and $N = 3$ relays. The channels are assumed to be Rayleigh flat fading channels and uncoded BPSK modulation is used for transmission. The total available power at the source node is set to $6\ W$. Two different settings are being considered. In the first setting each subcarrier is assigned an initial power of $P_0 = 1\ W$ and the remaining $P = 3\ W$ is available to be distributed by our proposed algorithm. In the second setting we assumed $P_0 = 1.7\ W$ and $P = 0.9\ W$. Fig. 3 shows the Monte-Carlo simulation of the average probability of error (P_E) for the E2E performance versus E_b/N_0 (the energy per bit to noise power spectral density ratio). The average SNR value of the links in the first hop are assumed to be equal. In the same figure, we present the results obtained using the max-min relay assignment with equal power distribution (EPD) among subcarriers. As it is obvious from this figure, the proposed jointly optimal scheme significantly outperforms the max-min criterion with EPD. These results

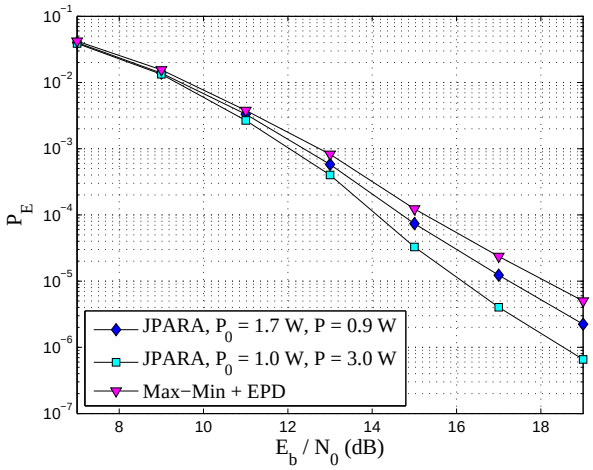

Fig. 3. E2E average probability of error for the scenario with fixed relays

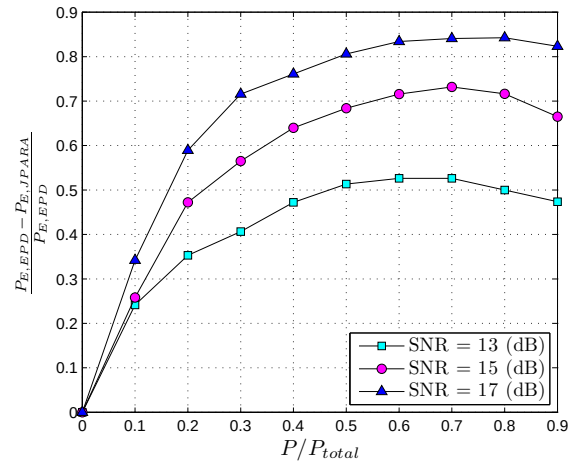

Fig. 4. The amount of improvement in probability of error versus $\frac{P}{P+NP_0}$ for the scenario with fixed relays

show that both schemes achieve the same diversity (equal to the number of relays) and the improvement offered by the proposed scheme appears in SNR gain. From the same simulation we can conclude that by increasing the portion of the power to be distributed by the proposed algorithm (From $P = 0.9\ W$ to $P = 3\ W$), the performance of the system improves. It is to be noted that EPD is actually an extreme case for our algorithm, where all of the power is equally distributed among different subcarriers.

In the second simulation we consider the same system model as before where we examine the performance improvement of the system over "max-min relay assignment with equal-power distribution" as a function of P/P_{total}. The results are shown in Fig. 4 for different average SNRs (13, 15, and 17 dB). As it is seen from this figure, the results are not monotonic curves. In other words, although the average performance of the worst E2E link increases with increasing P/P_{total}, the average E2E performance is not a monotonic function of this parameter and there exists an optimized value $P/P_{total} \simeq 0.7$ where the best improvement over EPD is achieved.

In Fig. 5 we examine the behavior of the network performance as a function of the number of relays assuming that the number of subcarriers is fixed at $L = 2$. As evident from these results the diversity order of the system is determined by the number of relays and hence, the performance of the system significantly improves for larger number of relays. This fact is more obvious in the next simulation (Fig. 6) where we express the performance improvement versus the number of relays, assuming that the number of subcarriers is fixed at $L = 2$. In this figure, different from the previous simulations, the average SNR of the channels are not the same, rather they are uniformly distributed between 0.8 and 1.2 of average SNR (horizontal axis). It is to be noted that the vertical axis is still in logarithmic scale, i.e. the performance of the system is mostly determined by the number of relays in the network.

Next, we consider the scenario with classical relays (where also R-D channels experience Rayleigh fading) and we simu-

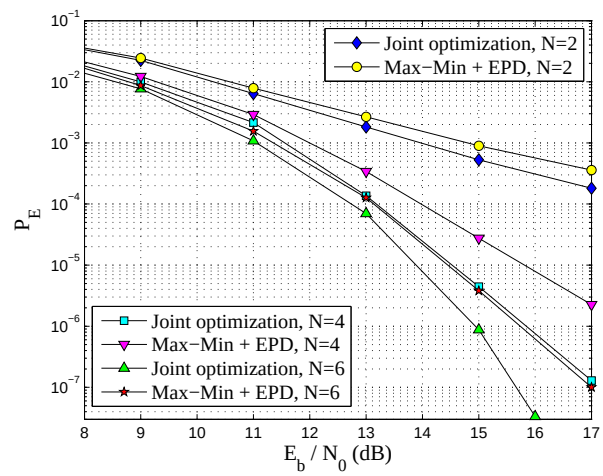

Fig. 5. The amount of improvement in probability of error for networks with different numbers of relays where $L = 2$

late the outage probability of the worst E2E link versus E_b/N_0. The result of using the proposed algorithm (joint optimization) is compared to that of separate optimization; that is employing max-min relay assignment and then to perform water-filling on the resulted permutation. Two networks with different network sizes are compared: the first network with $L = 3$, $N = 3$ and $P = 3\ W$ and the second network with $L = 4$, $N = 4$ and $P = 4\ W$. For both networks, we assume the same initial power for each subcarrier $P_0 = 1\ W$. Fig. 7 shows the results of such an investigation where it is seen that the network with larger number of relays achieves a higher diversity order (as it is expected for max-min relay selection [23]). From the same figure one can see that the network with larger number of relays attains a higher improvement over the case of separate optimization of relay-assignment and power allocation. This is a very important property of the proposed JPARA algorithm

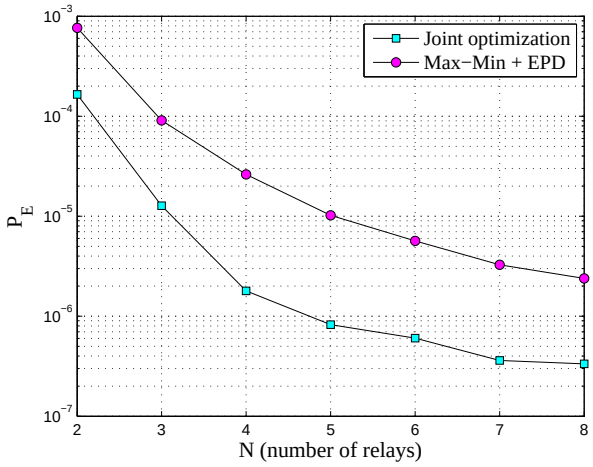

Fig. 6. The amount of improvement in probability of error versus different numbers of relays for the scenario with fixed relays

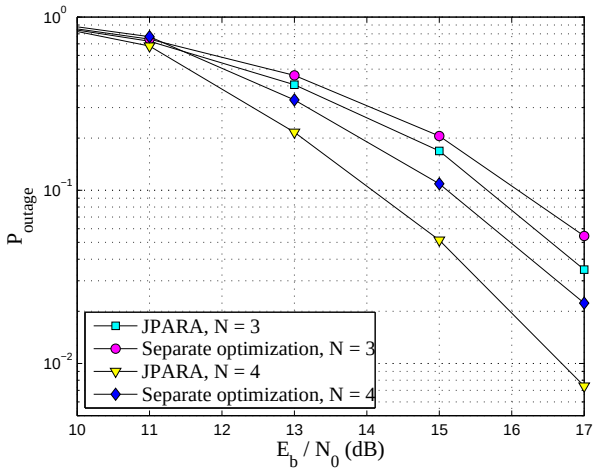

Fig. 7. Outage probability of the worst E2E link for the scenario with classical relays

that shows itself in networks with a larger number of relays.

Finally, we examine the effect of P/P_{total} on the performance of the system with classical relays over the case of separate optimization. The same network setting in Fig. 7 is assumed and the results are shown in Fig. 8. As shown from this figure, the results are fairly monotonic curves. In other words, the amount of improvement over EPD scheme and separate optimization scheme increases with increasing P/P_{total}. From this figure, and similar to the case of reliable R-D links, we can also conclude that the amount of improvement over other mentioned schemes increases with the increase in the number of relays.

VII. Conclusion

This paper proposed an optimal solution for a JPARA problem subject to the sum-power constraint at the source

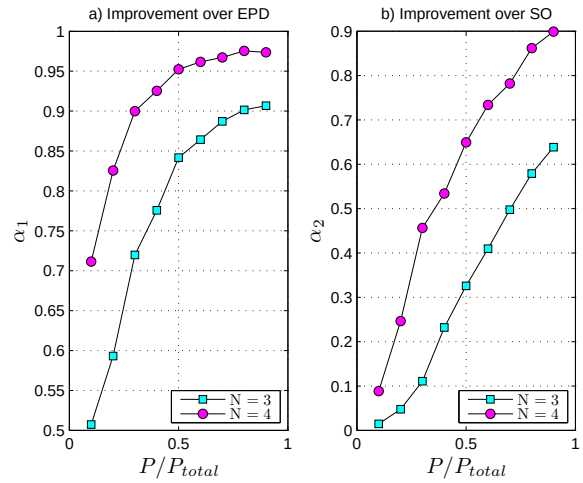

Fig. 8. a) The amount of improvement in outage probability over EPD scheme $\alpha_1 = (P_{EPD} - P_{JPARA})/P_{EPD}$. b) The amount of improvement in outage probability over separate optimization scheme $\alpha_2 = (P_{SO} - P_{JPARA})/P_{SO}$.

node, where the optimization criterion is to maximize the minimum E2E SNR. We assumed a network consisting of one source node, one destination node and N relays. The source node uses OFDM to transmit its information to the destination, where each subcarrier is transmitted through one of the relay nodes. We proposed an algorithm to find the JPARA solution based on max-min criterion. This algorithm uses the solutions of the standard max-min and max-sum algorithms. The optimality of the proposed algorithm is analytically proved and its complexity is calculated to be upper bounded by $O(N^4)$. We showed that by using this criterion, although the worst E2E link is significantly improved, the average performance of all links does not see the same improvement. This is due to the fact that the worst E2E link can absorb most of the power. An alternative approach is proposed in which each subcarrier is assigned a predetermined initial power and the remaining power is distributed through the JPARA algorithm. This alternative approach brought a significant improvement to the error performance of the network.

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