Homework # 5

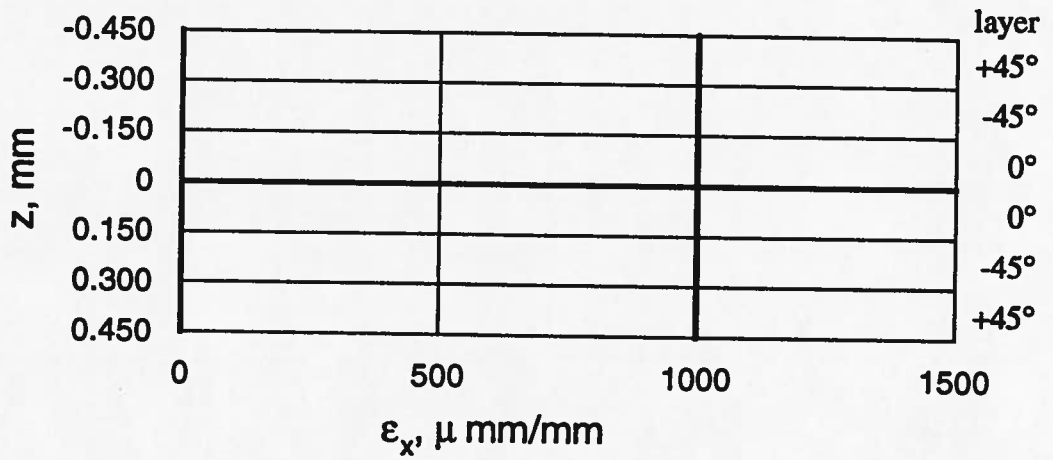
Question 1

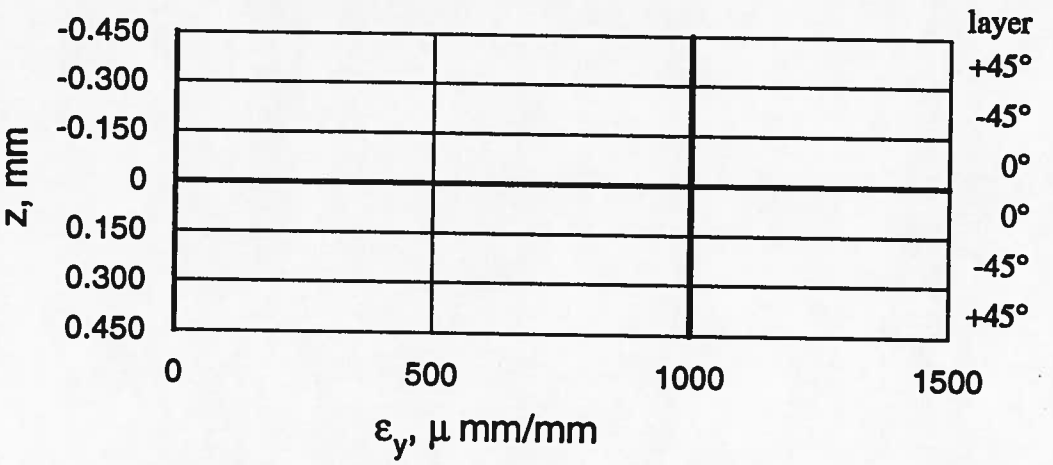
Exercises for Section 6.8, Exercise 3 (p. 271)

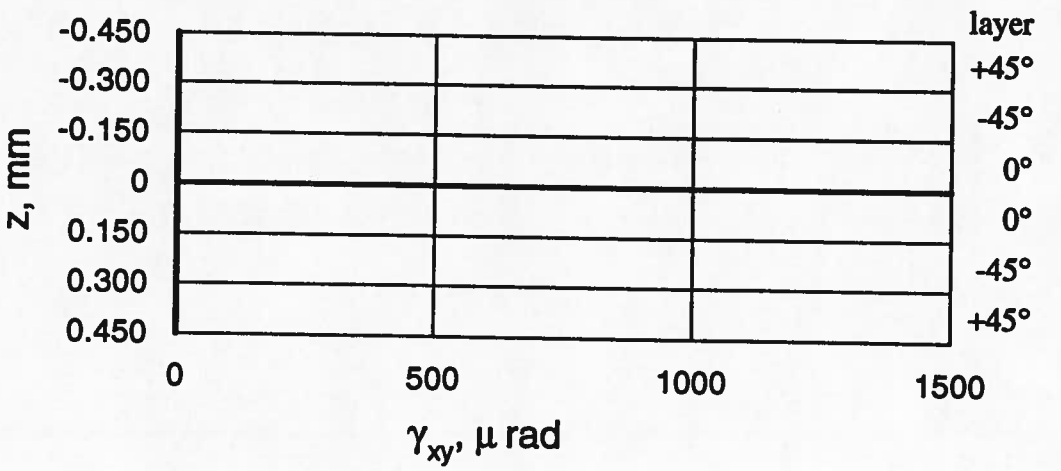
To solve this Exercise, the Computer Exercise was done first, as suggested. The solution to the problem will be presented in terms of the output from the suggested programing steps. In fact, the program used is the program that was developed over the course of the entire book, and is program *stress.f*, available to adopters of the book. That program is more fully developed than just the steps suggested in the current Computer Exercise. Presented will be a copy of the terminal session resulting from the interactive execution of the program, and a copy of the output from the program. Both have been cut and pasted somewhat to reflect the fact that all the theory in *stress.f* is not needed to solve this Exercise and has not been fully discussed yet . If you use *stress.f* it will ask for a temperature increment, ΔT, and the output will contain information that will be presented in ch. 11 when free thermal strains are discussed further. Enter zero when asked to enter the temperature increment, and ignore the output lines related to free thermal strain effects. Also, the program prints out the A, B, and D matricies for the laminate. These will be discussed in ch. 7. Ignore that ouput for the time being. To follow is a brief description of the output for this biaxial stretching exercise. The output can be found on subsequent pages

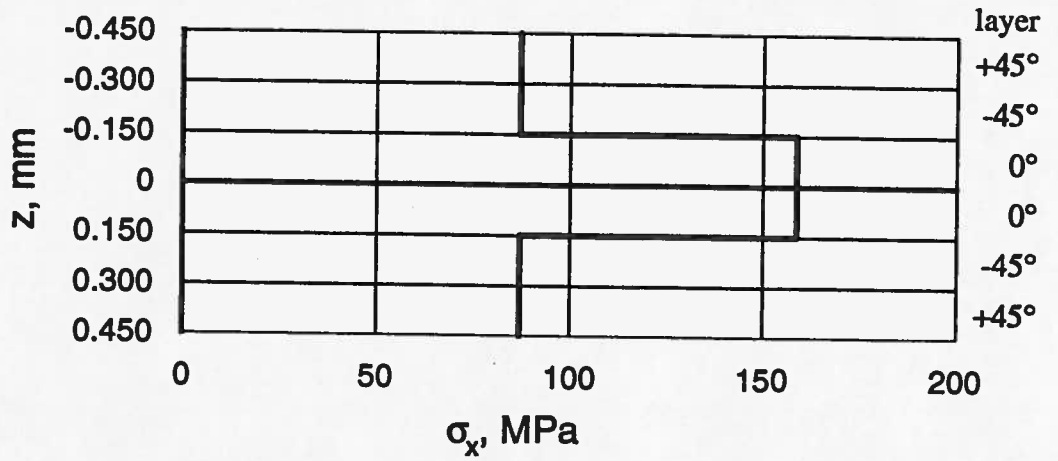
Terminal Session

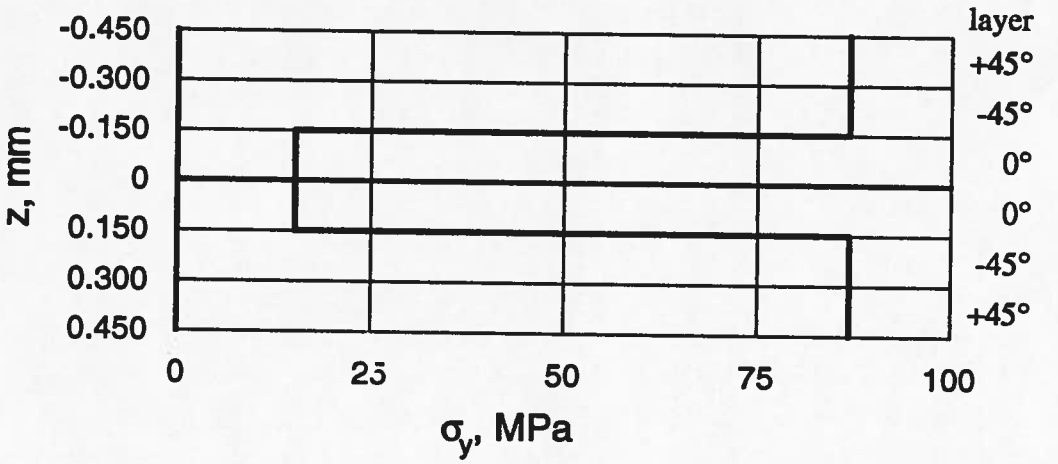
```
 ENTER NUMBER OF LAYERS AND INTEGER KK
  KK=0, ALL LAYERS SAME MATERIAL
  KK=1, EACH LAYER A DIFFERENT MATERIAL
6 0
 ENTER E11,E22,G12,NU12, AND THICKNESS
155.e9 12.1e9 4.4e9 0.248 150.e-6
 ENTER ALPHA1 AND ALPHA2
-0.018e-6 24.3e-6
 ENTER ORIENTATION (DEGREES) OF LAYER  1
45
 ENTER ORIENTATION (DEGREES) OF LAYER  2
-45
 ENTER ORIENTATION (DEGREES) OF LAYER  3
0
 ENTER ORIENTATION (DEGREES) OF LAYER  4
0
 ENTER ORIENTATION (DEGREES) OF LAYER  5
-45
 ENTER ORIENTATION (DEGREES) OF LAYER  6
45
 ENTER TEMPERATURE INCREMENT
0
 RESULTANTS OR REF SUFRACE STRAINS AND CURVATURES?
 1=RESULTANTS, 2=STRAINS AND CURVATURES
2
 ENTER STRAINS AND CURVATURES
 EPSX,EPSY,GAMXY,KX,KY,KXY
1000.e-6 1000.e-6 0 0 0 0

[Process completed]
```

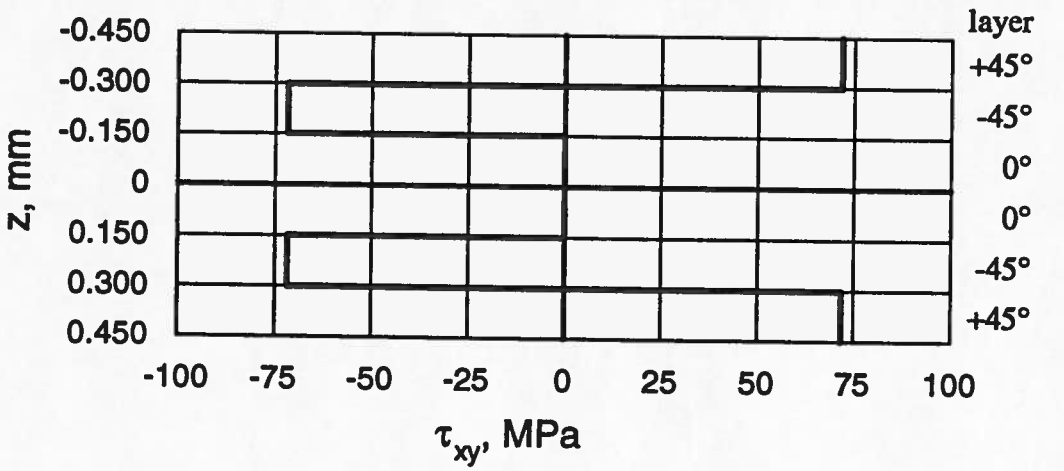

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
500
1000
1500
ε_x, μ mm/mm

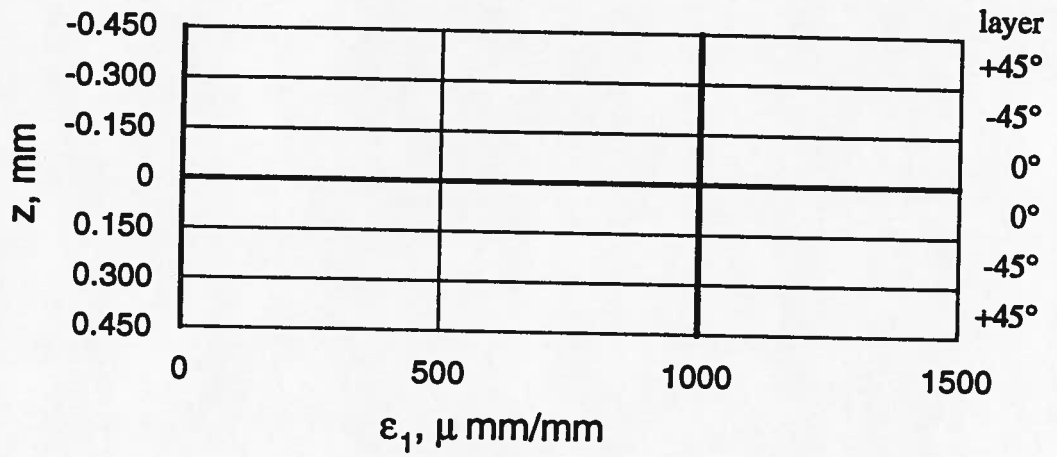

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
500
1000
1500
ε_y, μ mm/mm

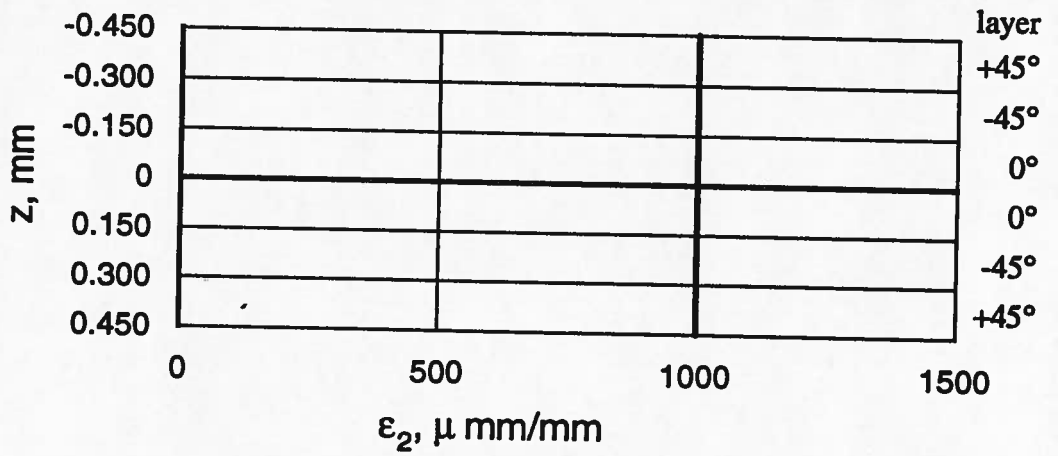

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
500
1000
1500
γ_{xy}, μ rad

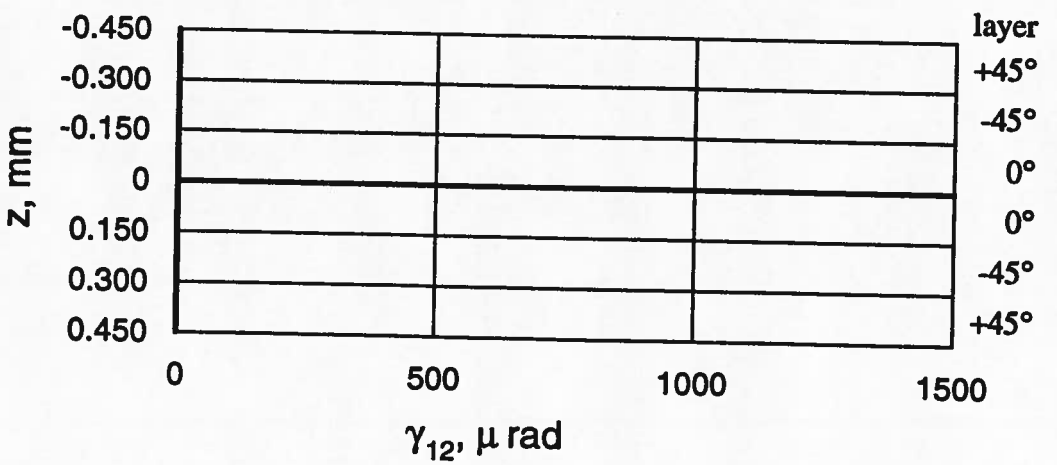

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
50
100
150
200
σx, MPa

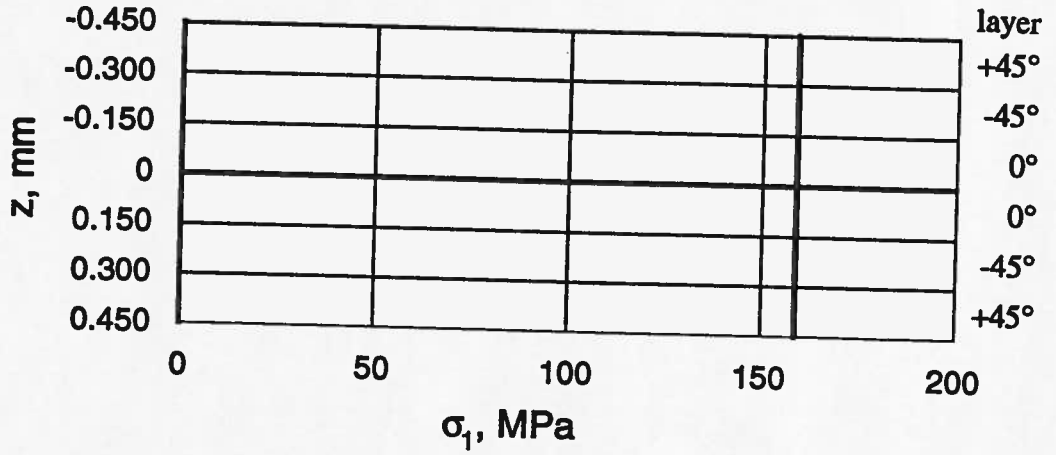

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
25
50
75
100
σy, MPa

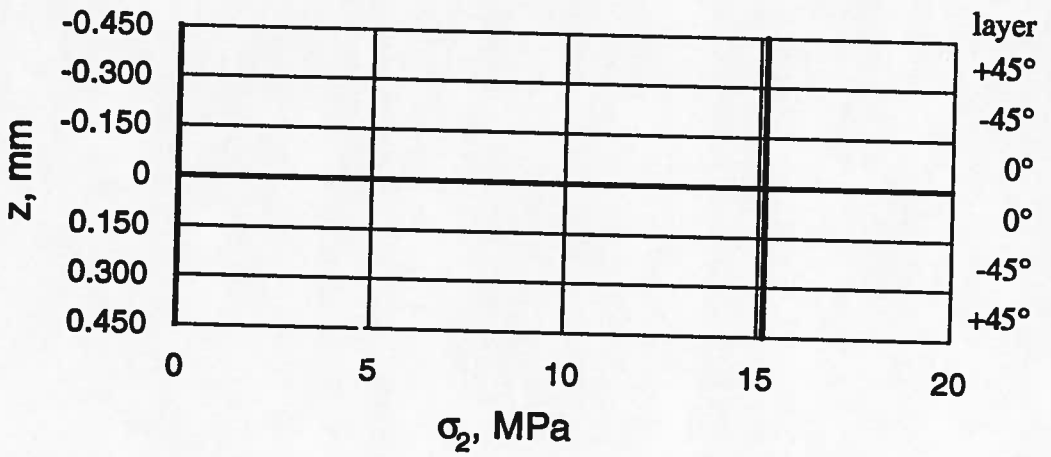

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-100
-75
-50
-25
0
25
50
75
100
τxy, MPa

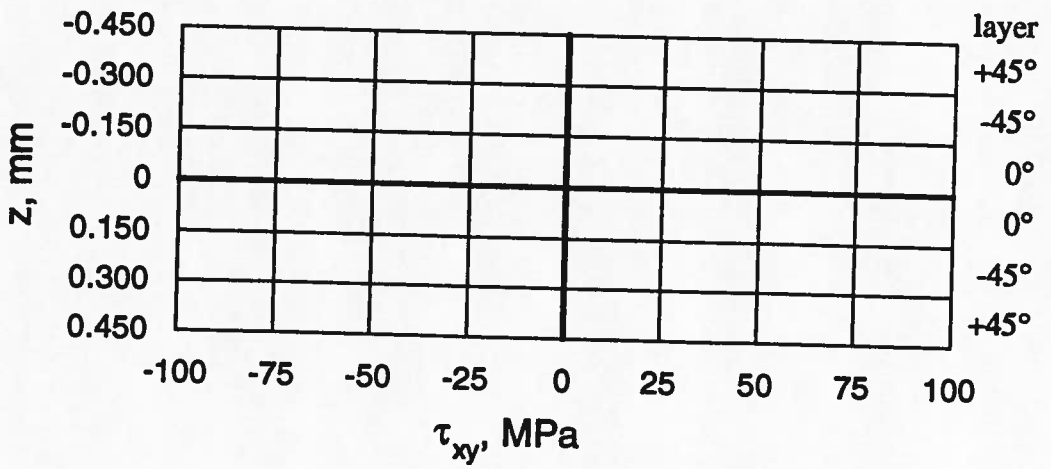

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
500
1000
1500
ε1, μ mm/mm


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
500
1000
1500
ε2, μ mm/mm


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
500
1000
1500
γ12, μ rad


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
50
100
150
200
σ1, MPa


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
0
5
10
15
20
σ2, MPa


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-100
-75
-50
-25
0
25
50
75
100
τxy, MPa

Question 2

Exercises for Section 6.8, Exercise 4 (p. 272)

(Note: In part (c) of the Exercise, the comment regarding M_{xy} is incorrect. For this problem M_{xy} is identically zero.)

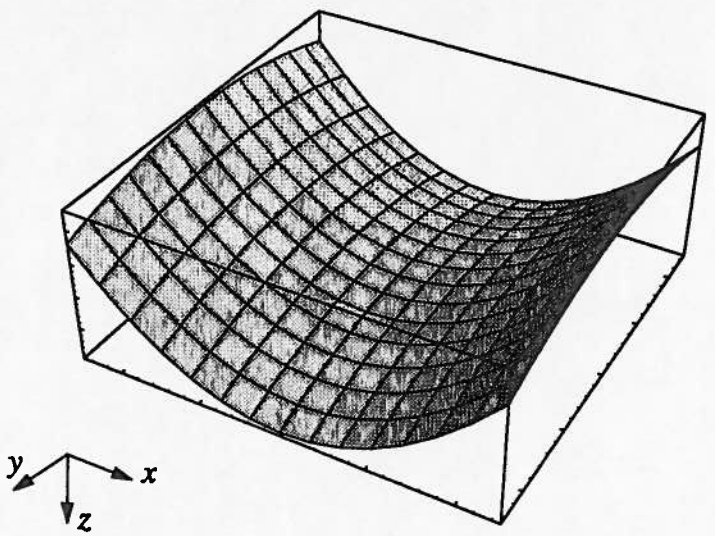
Below is the output cut and pasted from program *stress.f*. Following that are the answers to the questions in the Exercise, and the plots of the strains and stresses. This problem has unusual distributions of strains and stresses.

```
LAYER NO.  1   THETA =   45.0   THICKNESS =  0.15000E-03

         E1 =  0.1550E+12  G12 =  0.4400E+10  ALPHA1 = -0.1800E-07
         E2 =  0.1210E+11  NU12 =  .248       ALPHA2 =  0.24300E-04

Z( 0)=-0.45000E-03  Z( 1)=-0.30000E-03  (SEE FIG. 6.3)

        Q MATRIX, EQ. 4.17                    QBAR MATRIX, EQ. 5.84
 0.1557E+12  0.3015E+10  0.0000E+00    0.4788E+11  0.3908E+11  0.3590E+11
 0.3015E+10  0.1216E+11  0.0000E+00    0.3908E+11  0.4788E+11  0.3590E+11
 0.0000E+00  0.0000E+00  0.4400E+10    0.3590E+11  0.3590E+11  0.4047E+11

COEFFICIENTS OF THERMAL EXPANSION, EQ. 5.143

ALPHAX =  0.1214E-04   ALPHAY =  0.1214E-04   ALPHAXY = -0.2432E-04

*    *    *    *    *    *    *    *    *    *    *

LAYER NO.  2   THETA =  -45.0   THICKNESS =  0.15000E-03

         E1 =  0.1550E+12  G12 =  0.4400E+10  ALPHA1 = -0.1800E-07
         E2 =  0.1210E+11  NU12 =  .248       ALPHA2 =  0.24300E-04

Z( 1)=-0.30000E-03  Z( 2)=-0.15000E-03  (SEE FIG. 6.3)

        Q MATRIX, EQ. 4.17                    QBAR MATRIX, EQ. 5.84
 0.1557E+12  0.3015E+10  0.0000E+00    0.4788E+11  0.3908E+11 -0.3590E+11
 0.3015E+10  0.1216E+11  0.0000E+00    0.3908E+11  0.4788E+11 -0.3590E+11
 0.0000E+00  0.0000E+00  0.4400E+10   -0.3590E+11 -0.3590E+11  0.4047E+11

COEFFICIENTS OF THERMAL EXPANSION, EQ. 5.143

ALPHAX =  0.1214E-04   ALPHAY =  0.1214E-04   ALPHAXY =  0.2432E-04

*    *    *    *    *    *    *    *    *    *    *

LAYER NO.  3   THETA =     .0   THICKNESS =  0.15000E-03

         E1 =  0.1550E+12  G12 =  0.4400E+10  ALPHA1 = -0.1800E-07
         E2 =  0.1210E+11  NU12 =  .248       ALPHA2 =  0.24300E-04

Z( 2)=-0.15000E-03  Z( 3)= 0.29104E-10  (SEE FIG. 6.3)
```

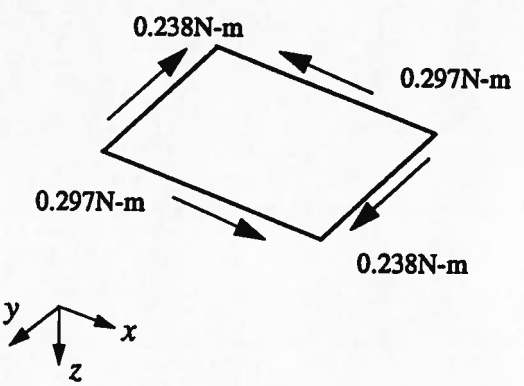
```
COMPUTED STRESS RESULTANTS

NXM= 0.00000E+00 NYM= 0.00000E+00 NXYM= 0.00000E+00
MXM= 0.19077E+01 MYM=-0.11897E+01 MXYM= 0.00000E+00
```

The magnitude of moment M_x is greater than the magnitude of moment M_y because of the presence of the 0° layers. Though the 0° layers are near the reference surface, and thus have minimal influence on bending response, they do have an influence, offering some resistance to bending because of the stiff fibers.

Deformed shape



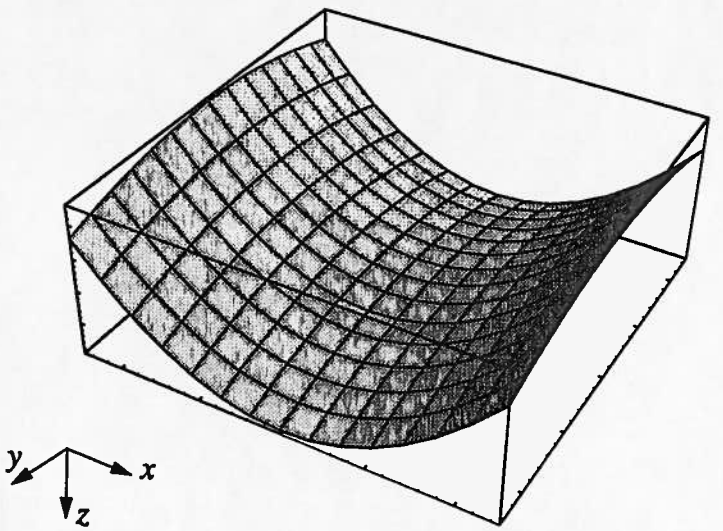
Required edge moments (all N's = 0)



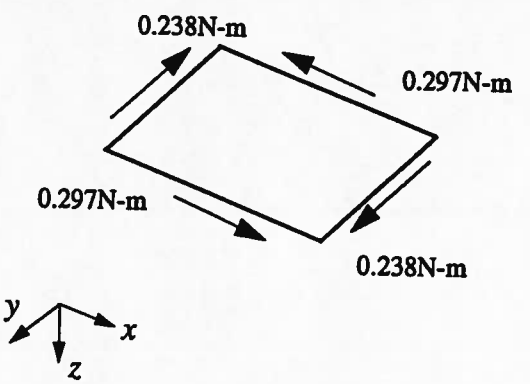
```
COMPUTED STRESS RESULTANTS

NXM= 0.00000E+00 NYM= 0.00000E+00 NXYM= 0.00000E+00
MXM= 0.19077E+01 MYM=-0.11897E+01 MXYM= 0.00000E+00
```

The magnitude of moment M_x is greater than the magnitude of moment M_y because of the presence of the 0° layers. Though the 0° layers are near the reference surface, and thus have minimal influence on bending response, they do have an influence, offering some resistance to bending because of the stiff fibers.

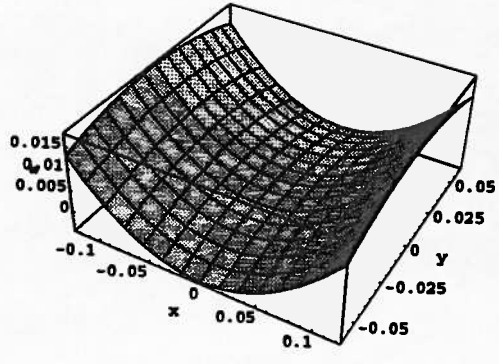
Deformed shape



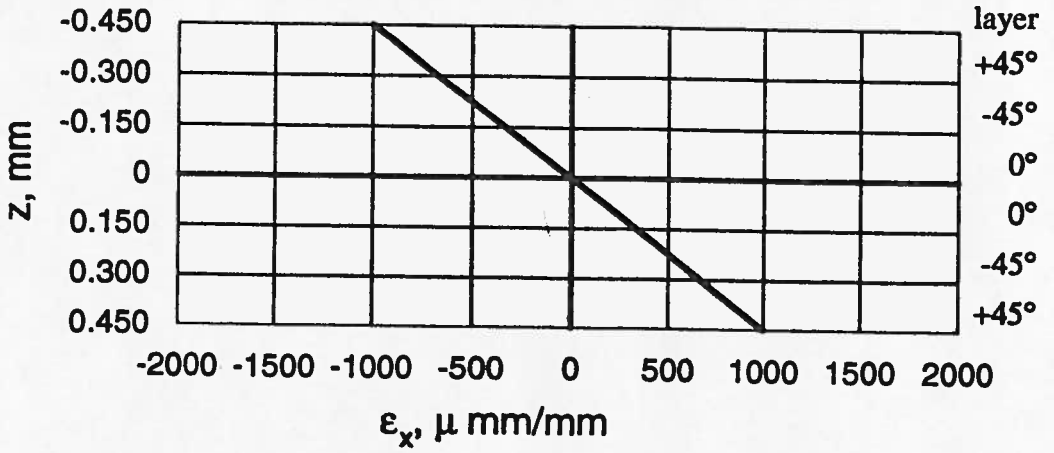
Required edge moments (all N's = 0)

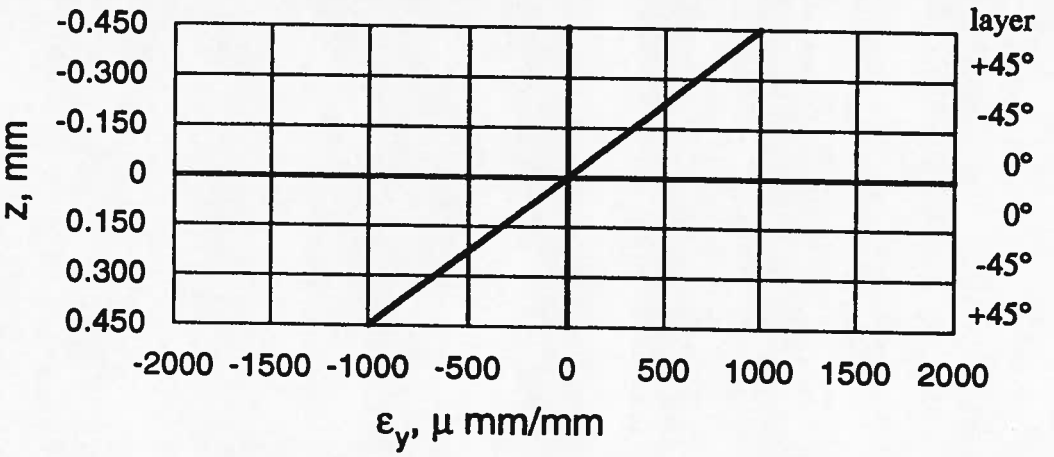


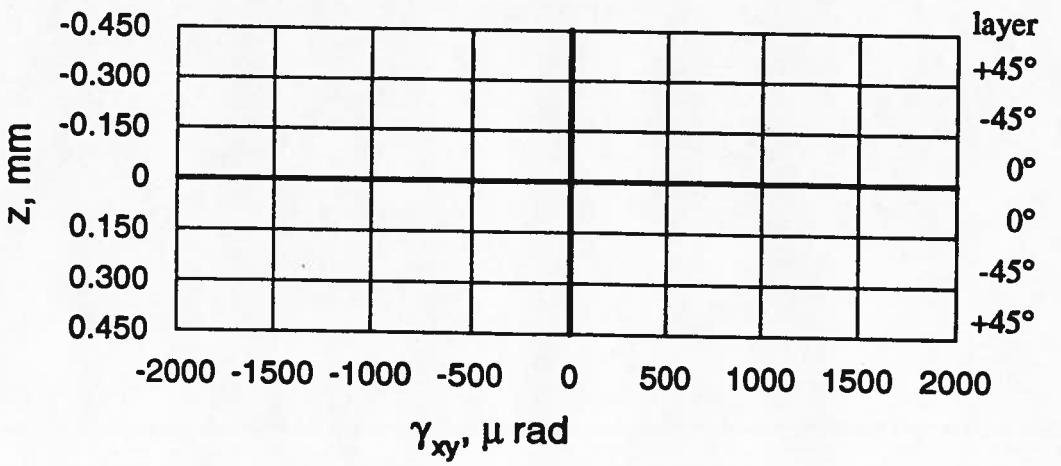
In[2]:=
```
kx = 2.222222
ky = -2.222222
```
Out[1]=
```
2.22222
```
Out[2]=
```
-2.22222
```
In[3]:=
```
w[x_,y_] := (kx*x*x + ky*y*y)/2
```
In[4]:=
```
Plot3D[w[x,y],{x,-0.125,0.125},{y,-0.0625,0.0625},
 AxesLabel-> {x,y,w}]
```

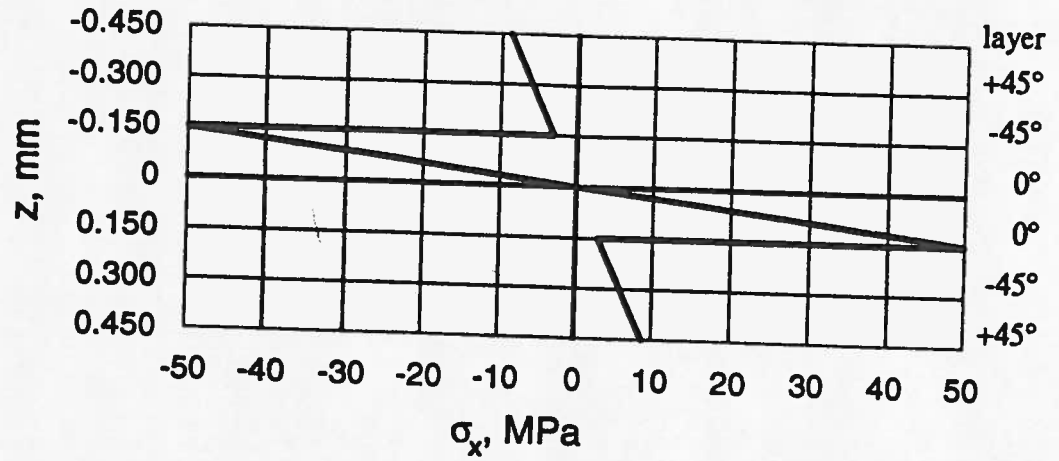


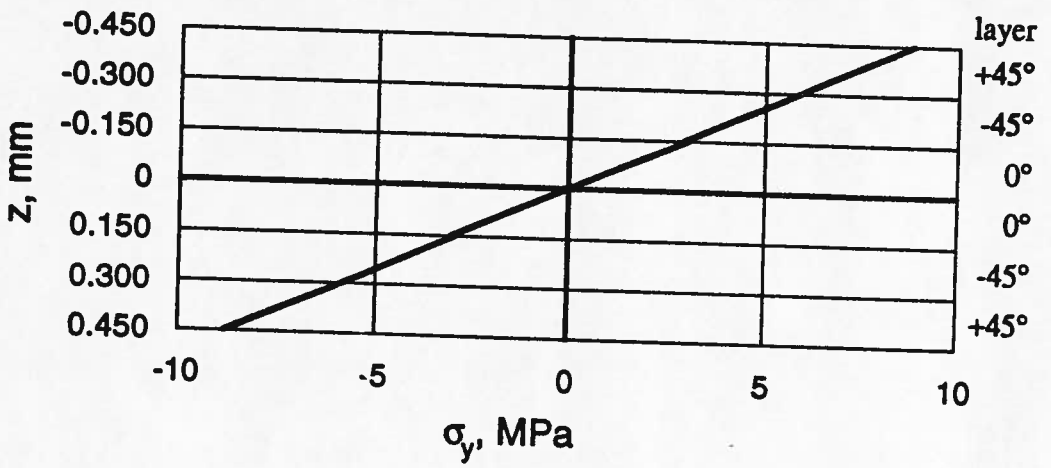
Out[4]=
```
-SurfaceGraphics-
```

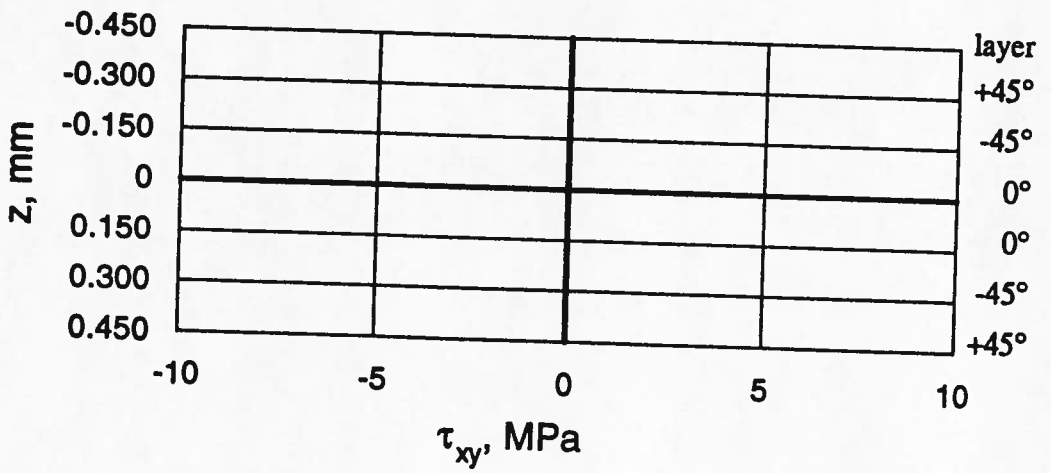

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-2000 -1500 -1000 -500 0 500 1000 1500 2000
ε_x, μ mm/mm

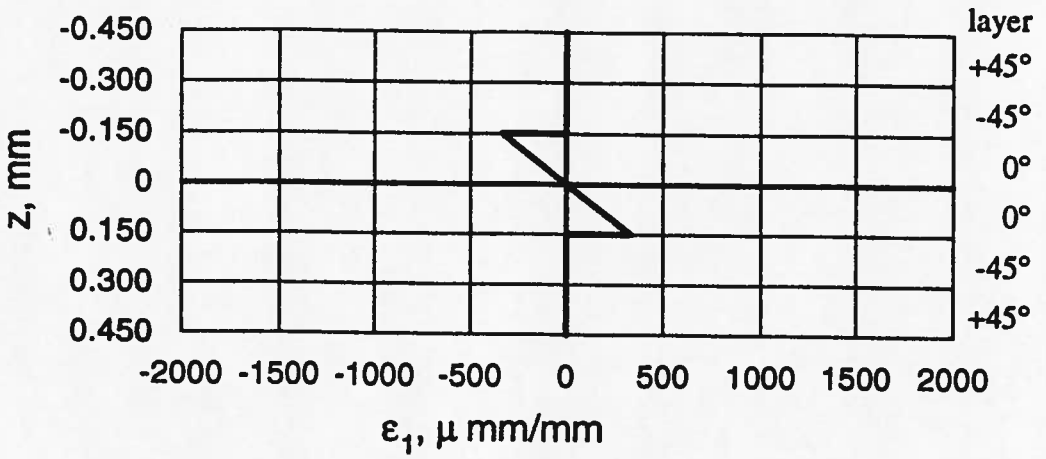

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-2000 -1500 -1000 -500 0 500 1000 1500 2000
ε_y, μ mm/mm

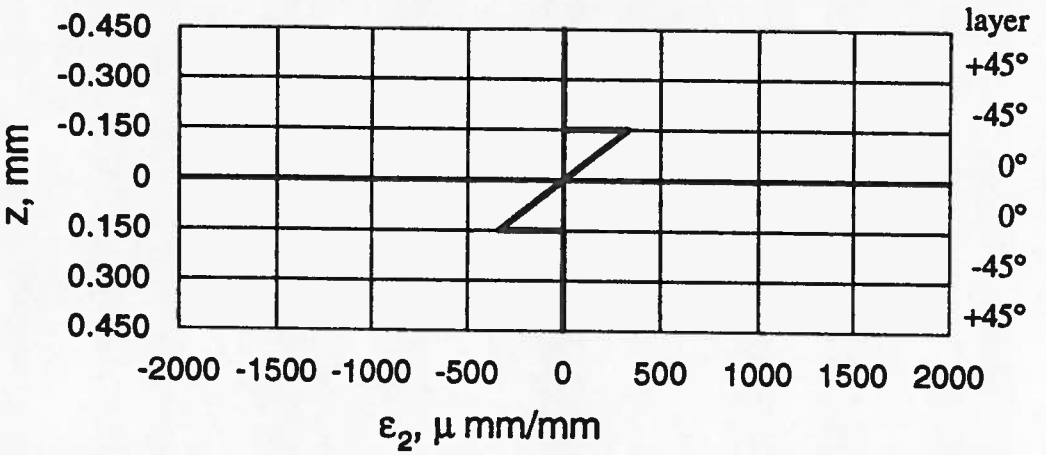

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-2000 -1500 -1000 -500 0 500 1000 1500 2000
γ_{xy}, μ rad

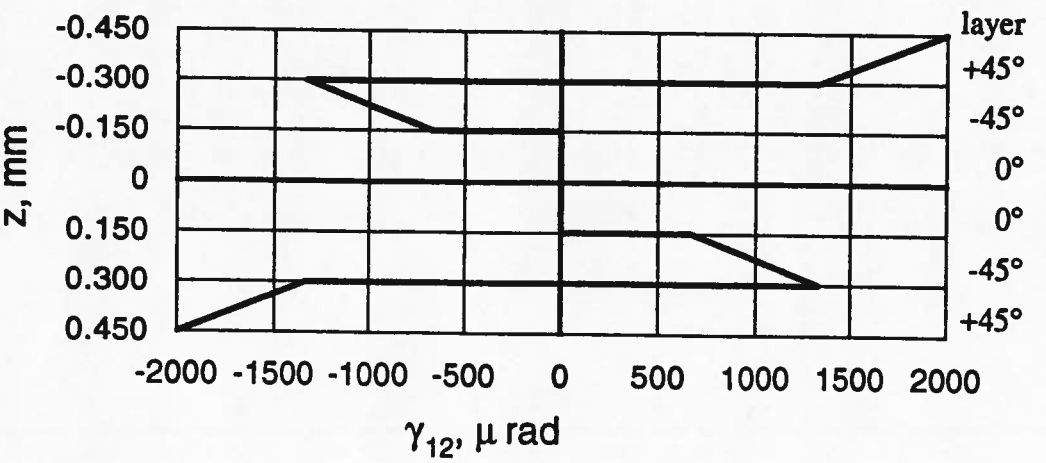

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
-50 -40 -30 -20 -10 0 10 20 30 40 50
σx, MPa
layer
+45°
-45°
0°
0°
-45°
+45°

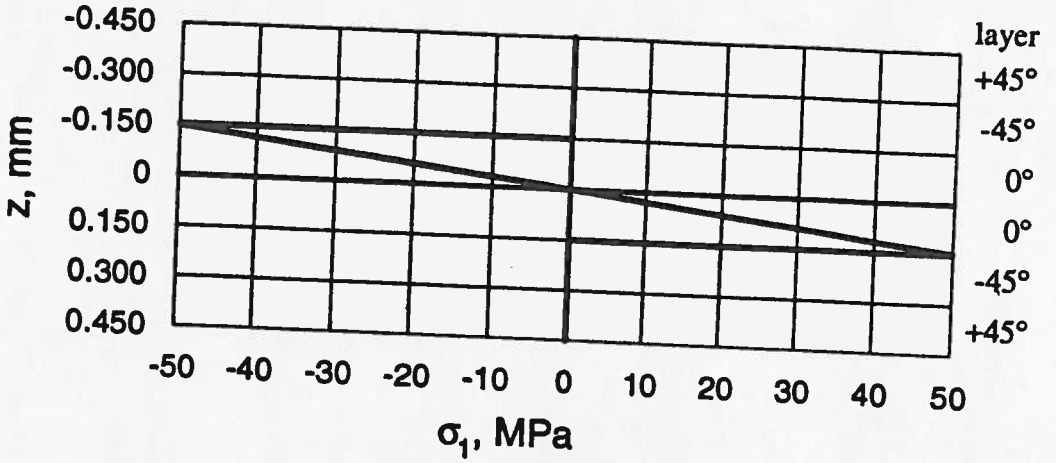

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
-10 -5 0 5 10
σy, MPa
layer
+45°
-45°
0°
0°
-45°
+45°

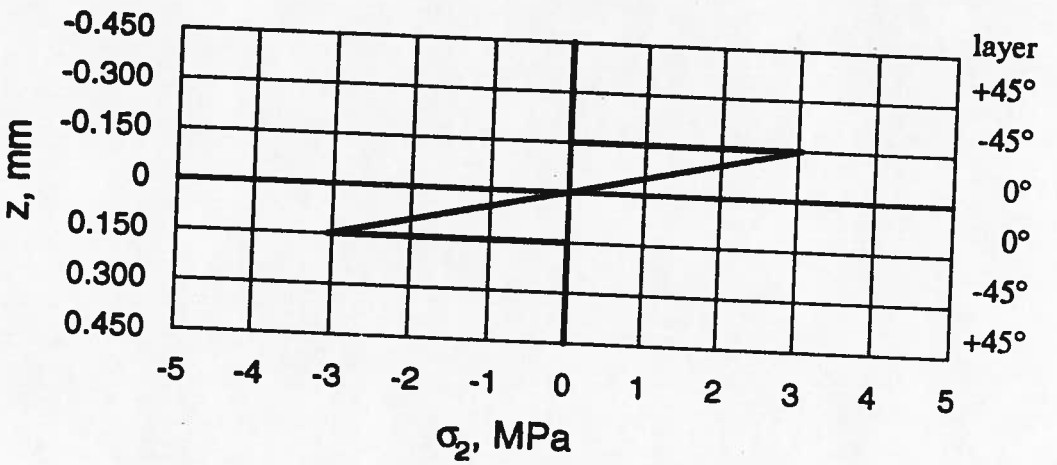

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
-10 -5 0 5 10
τxy, MPa
layer
+45°
-45°
0°
0°
-45°
+45°

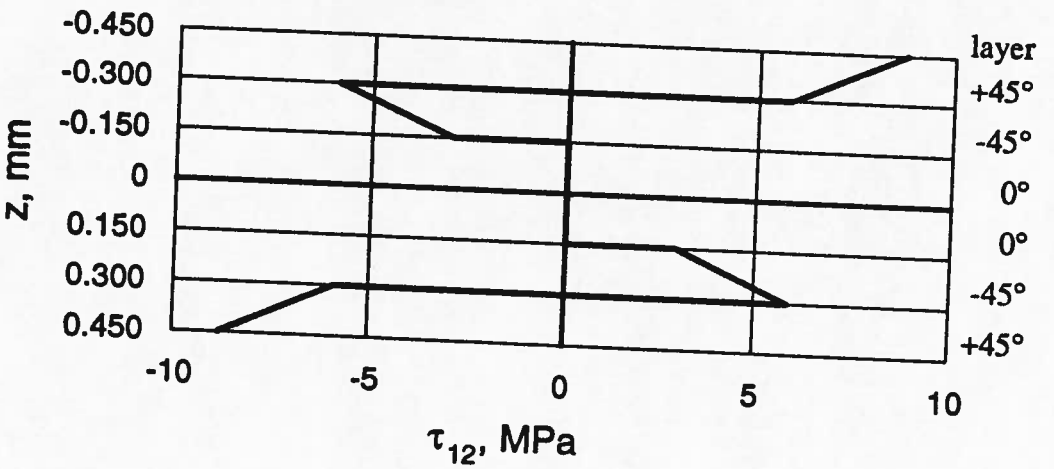

-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-2000 -1500 -1000 -500 0 500 1000 1500 2000
ε1, μ mm/mm


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-2000 -1500 -1000 -500 0 500 1000 1500 2000
ε2, μ mm/mm


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-2000 -1500 -1000 -500 0 500 1000 1500 2000
γ12, μ rad


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-50 -40 -30 -20 -10 0 10 20 30 40 50
σ1, MPa


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-5 -4 -3 -2 -1 0 1 2 3 4 5
σ2, MPa


-0.450
-0.300
-0.150
0
0.150
0.300
0.450
z, mm
layer
+45°
-45°
0°
0°
-45°
+45°
-10 -5 0 5 10
τ12, MPa