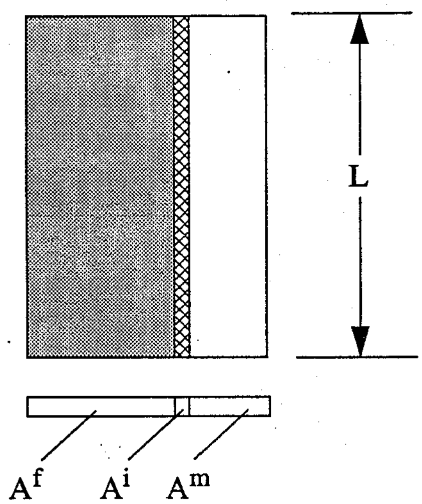
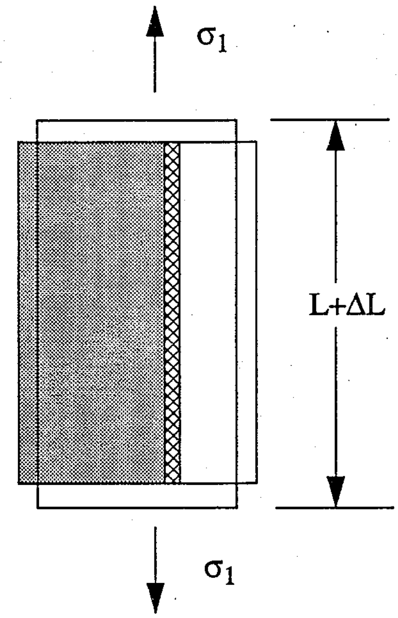
Exercises for Section 3.4, Exercise 3 (p. 145)



undeformed unit cell



stress is applied in 1 direction and all components change length the same amount, namely ΔL

Assuming one-dimensional states of stress, and assuming the fiber, interphase, and matrix are all perfectly bonded together and experience the same strain in the 1 direction, the stresses in those components are given by, in analogy to eq. 3.62,

$$\sigma_1^f = E_1^f \varepsilon_1^f = E_1^f \frac{\Delta L}{L} \qquad \sigma_1^i = E_1^i \varepsilon_1^i = E_1^i \frac{\Delta L}{L} \qquad \sigma_1^m = E_1^m \varepsilon_1^m = E_1^m \frac{\Delta L}{L}$$

The forces in the three constituents are

$$F_1^f = \sigma_1^f A^f = E_1^f \frac{\Delta L}{L} A^f \qquad F_1^i = \sigma_1^i A^i = E_1^i \frac{\Delta L}{L} A^i \qquad F_1^m = \sigma_1^m A^m = E_1^m \frac{\Delta L}{L} A^m$$

The total force in the 1 direction divided by the total cross sectional area A, where $A = A^f + A^i + A^m$, is the composite stress σ_1, namely,

$$\sigma_1 = \frac{F_1^f + F_1^i + F_1^m}{A} = \left(E_1^f \frac{A^f}{A} + E_1^i \frac{A^i}{A} + E_1^m \frac{A^m}{A}\right) \frac{\Delta L}{L}$$

By definition, for the composite,

$$\sigma_1 = E_1 \varepsilon_1 \qquad \text{where} \qquad \varepsilon_1 = \frac{\Delta L}{L}$$

Also, the volume fractions of the constituents are given by

$$V^f = \frac{A^f}{A} \qquad V^i = \frac{A^i}{A} \qquad V^m = \frac{A^m}{A}$$

So, the composite extensional modulus E_1 is given by the following rule-of-mixtures formula:

$$E_1 = E_1^f V^f + E_1^i V^i + E_1^m V^m$$

Solution-2

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$$V_f = 60\% = .6$$
$$V_m = .4$$

From (3.2-1) Material properties are

$\alpha_1^f = -0.540 \times 10^{-6}/K$ $\qquad$ $\alpha^m = 41.4 \times 10^{-6}/K$

$E_1^f = 233$ GPa $\qquad$ $E^m = 4.62$ GPa

$\nu_{12}^f = 0.200$ $\qquad$ $\nu^m = 0.360$

$\alpha_2^f = 10.10 \times 10^{-6}/K$

$$E_1 = E_1^f \nu^f + E^m \nu^m$$
$$= 233 \times 0.60 + 4.62 \times 0.40 = 141.648 \text{ GPa}$$

$$\alpha_1 = \frac{\alpha_1^f E_1^f V_f + \alpha^m E^m V_m}{E_1^f V_f + E^m V_m}$$

$$= \frac{(-0.540 \times 10^{-6}) \times 233 \times 0.6 + (41.4 \times 10^{-6}) \times 4.62 \times 0.4}{233 \times 0.6 + 4.62 \times 0.4}$$

$$= \underline{7.167 \times 10^{-9}}/K$$

$$\alpha_2 = \left[\alpha m + (\alpha_{2f} - \alpha m)V_f + \left(\frac{E_{1f}\nu m - Em\nu_{12f}}{E_1}\right)(\alpha m - \alpha_{1f})(1 - V_f)V_f\right]$$

$$= 41.4 \times 10^{-6} + (10.10 \times 10^{-6} - 41.4 \times 10^{-6}) + \left\{\left[\frac{233(0.36) - 4.62(0.20)}{233(0.6) + 4.62(0.4)}\right] \times (41.4 \times 10^{-6} + 0.54 \times 10^{-6})(1 - 0.6)\, 0.6\right\}$$

$$= \underline{2.85 \times 10^{-5}}/K$$

Solution-3

$\beta^m = 0.004 / \%$ moisture

$\Delta M = 0.5 \%$ moisture

$V_f = 0.6$

$\beta^f = 0$

From book $E_1^f = 233$ GPa

$E_m = 4.62$ GPa

$\nu_{12}^f = 0.2$

$\nu_m = 0.360$.

$$\beta_1 = \frac{\beta_{1f} E_{1f} V_f + \beta_m E_m V_m}{E_{1f} V_f + E_m V_m}$$

$$= \frac{0 + (0.004 \times 4.62 \times 0.4)}{233(0.6) + 4.62(0.4)} = 5.22 \times 10^{-5} / \%M$$

Now $\beta_1 = \frac{\%\text{ Expansion}}{\%\text{ Moisture}}$ $\Rightarrow$ $\%$ Expansion $= 5.22 \times 10^{-5} \times 0.5$

$= 2.6 \times 10^{-5}$

$$\beta_2 = \beta_m + (\beta_{2f} - \beta_m) V_f + \left[\frac{E_{1f} \nu_m - E_m \nu_{12f}}{E_1}\right](\beta_m - \beta_{1f})(1 - V_f) V_f$$

$$= 0.004 + (0 - 0.004)\, 0.6 + \left\{\left[\frac{233(0.36) - 4.62(0.2)}{233(0.6) + 4.62(0.4)}\right] \times (0.004 - 0)(1 - 0.6)\, 0.6\right\} = 2.16 \times 10^{-3} / \%M$$

$\%$ expansion (transverse) $= 2.16 \times 10^{-3} \times 0.5$

$= 1.08 \times 10^{-3}$