Hwk 2 #1

For this situation, the stress-strain pairs are:

(σ_1 =?, ε_1=0); (σ_2 = -50MPa, ε_2=?); (σ_3=?, ε_3=0); (τ_{23}=0, γ_{23}=?); (τ_{13}=0, γ_{13}=?); (τ_{12}=0, γ_{12}=?)

The stress-strain relations in terms of the compliances, eq. 2.45, become,

$$\begin{bmatrix} 0 \\ \varepsilon_2 \\ 0 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{bmatrix}$$

$$\gamma_{23} = S_{23}0 = 0 \qquad \gamma_{13} = S_{13}0 = 0 \qquad \gamma_{12} = S_{12}0 = 0$$

Solving the first and third equations for σ_1 and σ_3 results in

$$\sigma_1 = \frac{S_{23}S_{13} - S_{12}S_{33}}{S_{11}S_{33} - S_{13}S_{13}}\sigma_2 \qquad \sigma_3 = \frac{S_{13}S_{12} - S_{11}S_{23}}{S_{11}S_{33} - S_{13}S_{13}}\sigma_2$$

The interaction between stresses is clearly demonstrated with these relations, and they are somewhat more complicated than eqs. 2.75 and 2.83, cases with a single constraint on deformations. Substituting for σ_1 and σ_3 ,

$$\varepsilon_2 = \left(S_{22} - \frac{S_{11}S_{23}S_{23} + S_{33}S_{12}S_{12} - 2S_{12}S_{13}S_{23}}{S_{11}S_{33} - S_{13}S_{13}}\right)\sigma_2$$

The combination of compliances is a reduced compliance and is more complicated than eqs. 2.76 and 2.84 because there are two constraints on deformations.

a) - Numerically, using the compliances for graphite-reinforced material,

$$\varepsilon_2 = 64.5x10^{-12}\sigma_2$$

So the change in length in the 2 direction is

$$\delta\Delta_2 = \Delta_2\varepsilon_2 = \Delta_2 64.5x10^{-12}\sigma_2 = (50)\,64.5x10^{-12}(-50x10^6) = -0.1612\text{mm}$$

b) - The reduced compliance for this case, 64.5 $(\text{TPa})^{-1}$, is only slightly less than the reduced compliance for the case with a constraint only in the 3 direction (fig. 2.11c, the second equation of eq. 2.76, and the 65.3 $(\text{TPa})^{-1}$ of the second equation eq. 2.78) and considerable less than the case with a constraint only in the 1 direction (fig. 2.11d, the first equation of eq. 2.84, and the 82.2 $(\text{TPa})^{-1}$ of the second equation of eq. 2.86)

c) - Using the expressions for σ_1 and σ_3 from above,

$$\sigma_1 = -18.17\text{MPa} \qquad \sigma_3 = -23.3\text{MPa}$$

The stress σ_1 is larger than the stress required for the case of no contraint in the 3 direction, (-12.4 MPa of eq. 2.87)

 HwK # 2 - 2

Exercise for Section 2.7 (p. 77)

For this situation, the stress-strain pairs are:

($\sigma_1 = 0, \varepsilon_1 = ?$); ($\sigma_2 = ?, \varepsilon_2 = 0$); ($\sigma_3 = 0, \varepsilon_3 = ?$); ($\tau_{23} = 0, \gamma_{23} = ?$); ($\tau_{13} = 0, \gamma_{13} = ?$); ($\tau_{12} = 0, \gamma_{12} = ?$)

The stress-strain relations, including free thermal strain effects, in terms of the stiffnesses, eq. 2.108, become,

$$\begin{bmatrix} 0 \\ \sigma_2 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_1 - \alpha_1 \Delta T \\ 0 - \alpha_2 \Delta T \\ \varepsilon_3 - \alpha_3 \Delta T \end{bmatrix}$$

$$0 = C_{23}\gamma_{23} \qquad 0 = C_{13}\gamma_{13} \qquad 0 = C_{12}\gamma_{12}$$

Clearly, the shear strains are zero for this case. For ease of notation, define stress-like terms due to free thermal strain effects (see eq. 2.118) to be

$$\sigma_1^T = (C_{11}\alpha_1 + C_{12}\alpha_2 + C_{13}\alpha_3)\,\Delta T$$
$$\sigma_2^T = (C_{12}\alpha_1 + C_{22}\alpha_2 + C_{23}\alpha_3)\,\Delta T$$
$$\sigma_3^T = (C_{13}\alpha_1 + C_{23}\alpha_2 + C_{33}\alpha_3)\,\Delta T$$

Solving the first and third equations for ε_1 and ε_3, and then the second equation for σ_2 leads to

$$\varepsilon_1 = \frac{C_{33}\sigma_1^T - C_{13}\sigma_3^T}{C_{11}C_{33} - C_{13}C_{13}} \qquad \varepsilon_3 = \frac{C_{11}\sigma_3^T - C_{13}\sigma_1^T}{C_{11}C_{33} - C_{13}C_{13}}$$

$$\sigma_2 = \frac{(C_{12}C_{33} - C_{13}C_{23})\,\sigma_1^T + (C_{13}C_{13} - C_{11}C_{33})\,\sigma_2^T + (C_{11}C_{23} - C_{12}C_{13})\,\sigma_3^T}{C_{11}C_{33} - C_{13}C_{13}}$$

It is important to note that even though the total strain in the 2 direction is zero, i.e., $\varepsilon_2 = 0$, the free thermal strain in the 2 direction, $\alpha_2\Delta T$, has an impact on the problem. The interaction between stresses and the effects of free thermal strains is demonstrated with these relations.

Using the stiffnesses and coefficients of thermal expansion for graphite-reinforced material, numerical values can be obtained. Specifically,

$$\sigma_1^T = 271000\Delta T \qquad \sigma_2^T = 552000\Delta T \qquad \sigma_3^T = 552000\Delta T$$

a) - From eq. 2.103, the changes in dimensions due to $\Delta T = +50°$ are

$$\delta\Delta_1 = \Delta_1\varepsilon_1 = (50)\ (22.6x10^{-6}) = 0.001131\text{mm}$$
$$\delta\Delta_2 = \Delta_2\varepsilon_2 = (50)\ (0) = 0\text{mm}$$
$$\delta\Delta_3 = \Delta_3\varepsilon_3 = (50)\ (1771x10^{-6}) = 0.0886\text{mm}$$

b) - The stress to restrain the deformations in the 2 direction is

$$\sigma_2 = -14.70\text{MPa}$$

c) - Relative to the free thermal strain case, eqs. 2.101 and 2.102, the change in dimension in the 1 direction is opposite in sign and larger in magnitude. The change in dimension in the 3 direction is the same sign and about 50% larger. The partially constrained case results in different dimensional changes than the free case due to the compressive stress σ_2 causing deformations through Poisson effects that add to the deformations due to the free thermal strain effects.

d) - For the partially restrained case, the mechanical strains, eq. 2.109, are

$$\varepsilon_1^{mech} = \varepsilon_1 - \alpha_1\Delta T = 22.6x10^{-6} - (-0.018x10^{-6})\,50 = 22.6x10^{-6} + 0.9x10^{-6} = 23.5\mu\varepsilon$$
$$\varepsilon_2^{mech} = \varepsilon_2 - \alpha_2\Delta T = 0 - (24.3x10^{-6})\,50 = 0 - 1215x10^{-6} = -1215\mu\varepsilon$$
$$\varepsilon_3^{mech} = \varepsilon_3 - \alpha_3\Delta T = 1771x10^{-6} - (24.3x10^{-6})\,50 = 1771x10^{-6} - 1215x10^{-6} = 556\mu\varepsilon$$

The 1st and 3rd mechanical strains are quite different than for the fully restrained case, eq. 2.117, the second term in the above calculations.

HW 2-3

Exercises for Section 2.8, Exercise 1 (p. 81)

For this situation, the stress-strain pairs are:

(σ_1 =?, ε_1=0); (σ_2 =?, ε_2=0); (σ_3=0, ε_3=?); (τ_{23}=0, γ_{23}=?); (τ_{13}=0, γ_{13}=?); (τ_{12}=0, γ_{12}=?)

The stress-strain relations, including free moisture strain effects, in terms of the stiffnesses, eq. 2.125 with $\Delta T = 0$, become,

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ 0 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} \begin{bmatrix} 0 - \beta_1 \Delta M \\ 0 - \beta_2 \Delta M \\ \varepsilon_3 - \beta_3 \Delta M \end{bmatrix}$$

$$0 = C_{23}\gamma_{23} \qquad 0 = C_{13}\gamma_{13} \qquad 0 = C_{12}\gamma_{12}$$

Clearly, the shear strains are zero for this case. For ease of notation, define stress-like terms due to free moisture strain effects (see eq. 2.128) to be

$$\sigma_1^M = (C_{11}\beta_1 + C_{12}\beta_2 + C_{13}\beta_3)\,\Delta M$$
$$\sigma_2^M = (C_{12}\beta_1 + C_{22}\beta_2 + C_{23}\beta_3)\,\Delta M$$
$$\sigma_3^M = (C_{13}\beta_1 + C_{23}\beta_2 + C_{33}\beta_3)\,\Delta M$$

Solving the third stress-strain relation for ε_3 , and then the first and second relations for σ_1 and σ_2 leads to

$$\varepsilon_3 = \frac{\sigma_3^M}{C_{33}} \qquad \sigma_1 = \frac{C_{13}}{C_{33}}\sigma_3^M - \sigma_1^M \qquad \sigma_2 = \frac{C_{23}}{C_{33}}\sigma_3^M - \sigma_2^M$$

It is important to note that even though the total strains in the 1 and 2 directions are zero, i.e., $\varepsilon_1 = \varepsilon_2 = 0$, the free moisture strains in those directions, $\beta_1\Delta M$ and $\beta_2\Delta M$, have an impact on the problem. The interaction between stresses and the effects of free moisture strains is demonstrated in this problem.

Using the stiffnesses and coefficients of thermal expansion for graphite-reinforced material,

$$\varepsilon_3 = 0.00704\Delta M \qquad \sigma_1 = -37.1\Delta M\text{MPa} \qquad \sigma_2 = -58.4\Delta M\text{MPa}$$

Using $\Delta M = 1.5\%$

$$\varepsilon_3 = 10560\mu\varepsilon \qquad \sigma_1 = -55.7\text{MPa} \qquad \sigma_2 = -87.7\text{MPa}$$