HW 3 #1

Exercises for Section 5.2, Exercise 2 (p. 181)

For this situation, the stress-strain pairs are:

$$(\sigma_x = 0, \varepsilon_x = ?);\ (\sigma_y = -25\text{MPa}, \varepsilon_y = ?);\ (\tau_{xy} = ?, \gamma_{xy} = 0)$$

The stress-strain relations in terms of the transformed, or off-axis, reduced compliances, eq. 5.25, become,

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} 0 \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

The third equation becomes

$$\tau_{xy} = -\left(\frac{\bar{S}_{26}}{\bar{S}_{66}}\right)\sigma_y$$

And using this in the first and second equations,

$$\varepsilon_x = \left(\bar{S}_{12} - \frac{\bar{S}_{16}\bar{S}_{26}}{\bar{S}_{66}}\right)\sigma_y$$

$$\varepsilon_y = \left(\bar{S}_{22} - \frac{\bar{S}_{26}\bar{S}_{26}}{\bar{S}_{66}}\right)\sigma_y$$

where it is understood that the transformed reduced compliances are evaluated at θ=30°. Using numerical values,

$$\bar{S}_{11} = 50.8(\text{TPa})^{-1} \qquad \bar{S}_{22} = 88.9$$

$$\bar{S}_{12} = -26.9 \qquad \bar{S}_{26} = -3.77$$

$$\bar{S}_{16} = -62.2 \qquad \bar{S}_{66} = 126.0$$

The answers to parts (a) and (b) are:

$$\varepsilon_x = 719\mu\varepsilon$$

$$\varepsilon_y = -2220\mu\varepsilon$$

$$\tau_{xy} = -0.748\text{MPa}$$

The element expands in the x direction and contracts in the y direction, and a small negative shear stress is required to enforce the shear constraint.

HW 3 #2

Exercises for Section 5.3, Exercise 2 (p. 191)

For this situation, the stress-strain pairs are:

$$(\sigma_x = ?,\ \varepsilon_x = -0.050/50.);\ (\sigma_y = ?,\ \varepsilon_y = -0.050/50.);\ (\tau_{xy} = ?,\ \gamma_{xy} = 0)$$

The stress-strain relations in terms of the transformed reduced stiffnesses, eq. 5.83, become,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{bmatrix}$$

where

$$\varepsilon_x = \varepsilon_y = \frac{-0.050}{50} = -0.001000$$

Using numerical values, as in eq. 5.97,

$$\bar{Q}_{11} = 92.8\text{GPa} \qquad \bar{Q}_{22} = 21.0$$
$$\bar{Q}_{12} = 30.1 \qquad \bar{Q}_{26} = 15.47$$
$$\bar{Q}_{16} = 46.7 \qquad \bar{Q}_{66} = 31.5$$

and the required stresses are

$$\sigma_x = -122.9\text{MPa} \qquad \sigma_y = -51.1 \qquad \tau_{xy} = -62.2$$

HWK 3 #3

Exercise for Section 5.7, Exercise 2 (p. 211)

For this situation, the stress-strain pairs are:

$$(\sigma_x = ?,\ \varepsilon_x = 0);\ (\sigma_y = 0,\ \varepsilon_y = ?);\ (\tau_{xy} = 0,\ \gamma_{xy} = ?)$$

The stress-strain relations, including free thermal strain effects, in terms of the compliances, e
5.160 with $\Delta M = 0$, become,

$$\begin{bmatrix} 0 - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix}$$

Expanding

$$-\alpha_x \Delta T = \bar{S}_{11} \sigma_x$$

$$\varepsilon_y - \alpha_y \Delta T = \bar{S}_{12} \sigma_x$$

$$\gamma_{xy} - \alpha_{xy} \Delta T = \bar{S}_{16} \sigma_x$$

Solving the first stress-strain relation for σ_x, and then the second and third relations for ε_x and γ_{xy}
leads to

$$\sigma_x = \frac{-\alpha_x \Delta T}{\bar{S}_{11}} \qquad \varepsilon_y = \left(\alpha_y - \frac{\bar{S}_{12}}{\bar{S}_{11}} \alpha_x\right) \Delta T \qquad \gamma_{xy} = \left(\alpha_{xy} - \frac{\bar{S}_{16}}{\bar{S}_{11}} \alpha_x\right) \Delta T$$

Also for this case, from eq. 5.165,

$$\varepsilon_z = \alpha_z \Delta T + (S_{13} \cos^2\theta + S_{23} \sin^2\theta)\, \sigma_x$$

The material properties for $\theta = 45°$ are:

$$\bar{S}_{11} = 78.3(\mathrm{TPa})^{-1} \qquad \bar{S}_{12} = -35.3 \qquad \bar{S}_{16} = -38.1$$

$$\alpha_x = 12.14 x 10^{-6} / {}^oC \qquad \alpha_y = 12.14 x 10^{-6} \qquad \alpha_{xy} = -24.3 x 10^{-6}$$

As a result

$$\sigma_x = -7.75\mathrm{MPa} \qquad \varepsilon_y = 881 x 10^{-6} \qquad \gamma_{xy} = -0.000921\ (-0.0527^o) \qquad \varepsilon_z = 1368 x 10^{-6}$$

Since $\theta = 45°$, this problem and the problem of fig. 5.15 are practically identical, namely, a co
straint in the direction perpendicular to the applied compressive stress, and the fibers making a
45° angle with that stress.