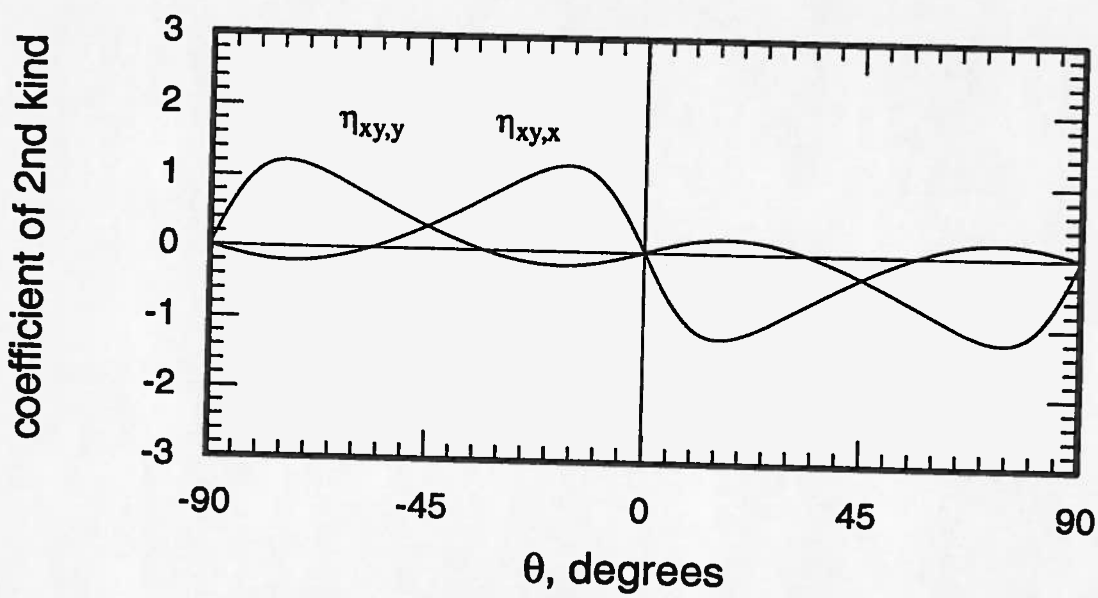
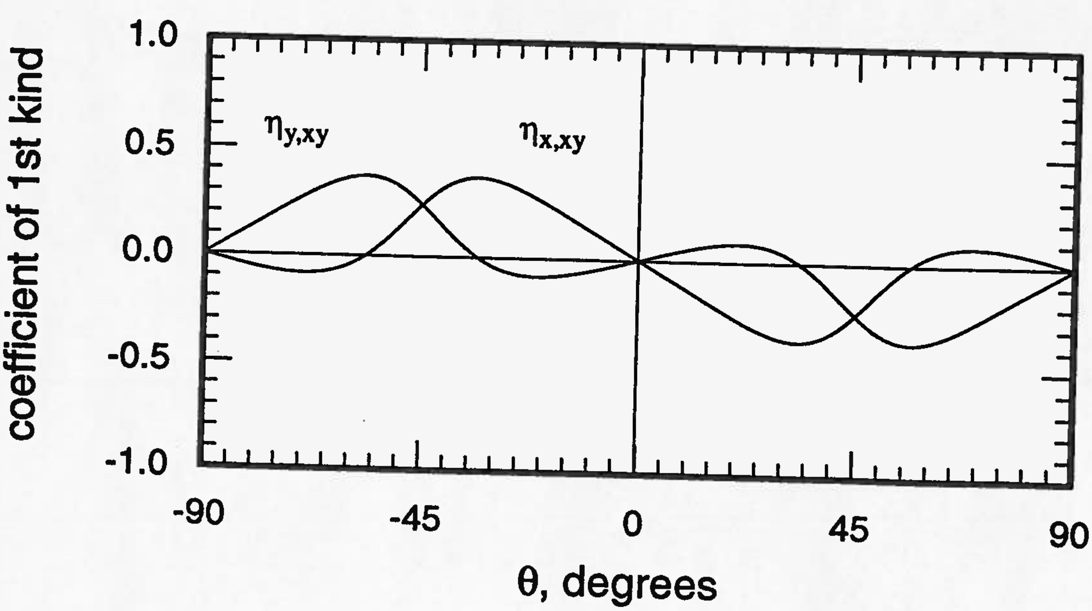
Exercises for Section 5.5, Exercise 2 (p. 198)

The magnitudes of $\eta_{xy,x}$ and $\eta_{xy,y}$ for the glass-reinforced material are about one-half the magnitudes of these coefficients for graphite-reinforced material. Also, for the glass-reinforced material, $\eta_{xy,x}$ has the largest magnitude near $\theta = \pm 15°$, and $\eta_{xy,y}$ has the largest magnitude near $\theta = \pm 75°$. For graphite-reinforced material, these two values are ±8° and ±82°, respectively.

The coefficients $\eta_{x,xy}$ and $\eta_{y,xy}$ have the largest magnitude at about the same value of θ as for graphite-reinforced materials, namely, ±35° for $\eta_{x,xy}$ and ±55° for $\eta_{y,xy}$. The magnitudes of these coefficients for glass-reinforced material are slightly less than the magnitudes of these coefficients for graphite-reinforced material. The calculations were done as part of program *sbar2d.f*, one of the programs available to adopters of the book.





Exercise for Section 5.7, Exercise 2 (p. 211)

For this situation, the stress-strain pairs are:

$$(\sigma_x = ?,\ \varepsilon_x = 0);\ (\sigma_y = 0,\ \varepsilon_y = ?);\ (\tau_{xy} = 0,\ \gamma_{xy} = ?)$$

The stress-strain relations, including free thermal strain effects, in terms of the compliances, eq. 5.160 with $\Delta M = 0$, become,

$$\begin{bmatrix} 0 - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \gamma_{xy} - \alpha_{xy} \Delta T \end{bmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix}$$

Expanding

$$-\alpha_x \Delta T = \bar{S}_{11} \sigma_x$$

$$\varepsilon_y - \alpha_y \Delta T = \bar{S}_{12} \sigma_x$$

$$\gamma_{xy} - \alpha_{xy} \Delta T = \bar{S}_{16} \sigma_x$$

Solving the first stress-strain relation for σ_x, and then the second and third relations for ε_x and γ_{xy} leads to

$$\sigma_x = \frac{-\alpha_x \Delta T}{\bar{S}_{11}} \qquad \varepsilon_y = \left(\alpha_y - \frac{\bar{S}_{12}}{\bar{S}_{11}} \alpha_x\right) \Delta T \qquad \gamma_{xy} = \left(\alpha_{xy} - \frac{\bar{S}_{16}}{\bar{S}_{11}} \alpha_x\right) \Delta T$$

Also for this case, from eq. 5.165,

$$\varepsilon_z = \alpha_z \Delta T + (S_{13} \cos^2\theta + S_{23} \sin^2\theta)\, \sigma_x$$

The material properties for $\theta = 45°$ are:

$$\bar{S}_{11} = 78.3 (\text{TPa})^{-1} \qquad \bar{S}_{12} = -35.3 \qquad \bar{S}_{16} = -38.1$$

$$\alpha_x = 12.14 x 10^{-6} / {}^oC \qquad \alpha_y = 12.14 x 10^{-6} \qquad \alpha_{xy} = -24.3 x 10^{-6}$$

As a result

$$\sigma_x = -7.75 \text{MPa} \qquad \varepsilon_y = 881 x 10^{-6} \qquad \gamma_{xy} = -0.000921\ (-0.0527^o) \qquad \varepsilon_z = 1368 x 10^{-6}$$

Since $\theta = 45°$, this problem and the problem of fig. 5.15 are practically identical, namely, a constraint in the direction perpendicular to the applied compressive stress, and the fibers making a 45° angle with that stress.