

# INCORPORATING SECOND-ORDER DERIVATIVES INTO REAL-TIME REGULARIZED ULTRASOUND ELASTOGRAPHY

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**Background:** Ultrasound elastography [1] involves imaging tissue while it undergoes deformation, and estimating its mechanical properties from the deformation pattern. Many elastography methods have been developed, commercialized and successfully tested on patients. At the heart of all elastography techniques is estimation of tissue deformation, where more accurate and fast estimates are required to obtain high quality elastography in real-time. We focus on methods that optimize a regularized cost function to obtain the displacement estimates. An excellent review of these techniques is provided in [2].

**Aims:** Our first aim is to improve our previous work on real-time optimization of a regularized cost function to obtain 2D displacement estimates [3,4]. A first-order Taylor expansion of the cost function is used in [3,4] to obtain a linear system of equations that can be solved in real-time. We modify this expansion to incorporate second-order derivative terms into the cost function to improve the quality of tracking estimates. Our second goal is to keep the method computationally efficient and real-time.

**Methods:** Let  $I_1$  and  $I_2$  be RF data from two ultrasound frames acquired from the same but deformed tissue. Let  $i = 1 \dots m$  denote indices of all samples of RF-line  $j$ , and assume that the current estimate of axial and lateral displacement of all samples in RF-line  $j$  are respectively  $a_i$  and  $l_i$ . The goal is to find update values  $\Delta a_i$  and  $\Delta l_i$ , so that  $(a_i + \Delta a_i, l_i + \Delta l_i)$  is a better approximation of the 2D displacements. The algorithm can then iterate until convergence is achieved. To find  $\Delta a_i$  and  $\Delta l_i$ , we optimize the following regularized cost function with two terms: data  $D$  and regularization  $R$ .

$C = D + R$ , where  $D = \sum [I_1(i, j) - I_2(i + a_i + \Delta a_i, j + l_i + \Delta l_i)]^2$  and  $R = f(a_1 + \Delta a_1, \dots, a_m + \Delta a_m) + f(l_1 + \Delta l_1, \dots, l_m + \Delta l_m)$  (1)  
The summation in  $D$  is performed over samples  $i = 1 \dots m$ , and  $f$  is a simple quadratic function that specifies the regularization (see [3] for more details). To optimize  $C$ , we linearized  $D$  using Taylor expansion [3]:

$$I_2(i + a_i + \Delta a_i, j + l_i + \Delta l_i) \approx I_2(i + a_i, j + l_i) + \Delta a I'_{2,a} + \Delta l I'_{2,l} \quad (2)$$

where  $I'_{2,a}$  and  $I'_{2,l}$  are respectively axial and lateral derivatives of  $I_2$  at sample  $(i + a_i, j + l_i)$ . Inserting Eq. (2) in Eq. (1) will make Eq. (1) quadratic, and its derivative linear, meaning that Eq. 1 can be solved efficiently by solving the linear system of equations. The issue is that if we include higher order derivatives in Eq. (2), the derivative of Eq. (1) will not become linear. To overcome this, we propose the novel cost function:

$$D = \sum w [I_1(i, j) - I_2(i + a_i + \Delta a_i, j + l_i + \Delta l_i)]^2 \quad w = 1 / (\alpha + |I'_{2,a}| + |I'_{2,l}|) \quad (3)$$

where  $\alpha$  is a small positive constant to prevent the denominator to become zero, and  $|I'_{2,a}|$  and  $|I'_{2,l}|$  are the absolute values of second-order derivatives in the axial and lateral directions respectively. The weight  $w$  reduces the contribution of highly nonlinear parts of the RF-data (where Eq. (2) does not hold).

**Results:** We have tested the new technique on simulated and patient data. The simulation data consists of 2D compression of a uniform phantom of fully developed speckles by 2%. The ultrasound images are simulated using Field II software. The new method improved the signal to noise ratio (SNR) of the resulting strain image. We also test the algorithm on a patient data with liver cancer. The results show that, compared to [3], the new method improves contrast to noise ratio (CNR).

**Conclusions:** We showed that direct incorporation of second-order derivatives into the cost function makes the optimization computationally intractable. We therefore proposed the novel cost function in Eq. (3), which penalizes areas with large second derivative. The increased computational complexity is negligible and the code runs in real-time.

**Acknowledgements:** We thank Drs. Boctor and Choti for sharing the patient data. H. R. is supported by NSERC DG.

## References:

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