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# Two-Level Lot-Sizing with Raw-Material Perishability and Deterioration

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**Abstract** In many production systems it is common to face significant rates of product deterioration, referring to physical exhaustion, loss of functionality and volume, or even obsolescence. This deterioration property, known as *perishability*, prevents such products from being used after their expiration time. We present lot-sizing problems that incorporate raw-material perishability and analyze how these considerations enforce specific constraints on a set of fundamental decisions, particularly for multi-level structures. We study three variants of the two-level lot-sizing problem incorporating different types of raw-material perishability: (a) *fixed shelf-life*, (b) *functionality deterioration*, and (c) *functionality-volume deterioration*. We propose mixed integer programming formulations for each of these variants and perform computational experiments with sensitivity analyses. We analyze the added value of explicitly incorporating perishability considerations into production planning problems. For this, we compare the results of the proposed formulations with those obtained by implementing a sequential approach that adapts a standard two-level lot-sizing solution with a Silver-Meal based rolling-horizon algorithm.

**Keywords** Raw-material perishability · Lot-sizing · Shelf-life · Deterioration · Production planning · Two-level lot-sizing · Mixed-integer programming · Batch ordering

## 1 Introduction

A common assumption in most of the production planning literature is that finished and intermediate products involved in the production process have unlimited *lifespans*, meaning they can be stored and used indefinitely. In practice, most items deteriorate over time, referring not only to physical exhaustion, functionality and volume (or quantity) loss, but also to obsolescence. Often, the rate of deterioration is low and there is little need for considering it in the planning process. However, in many industries it is common to deal with items that are subject to significant rates of deterioration. These items are referred to as *perishable products*.

*Perishability* relates to items that cannot be stored infinitely without deterioration or devaluation Billaut (2011). Clear cases of this type of products are found in food or pharmaceutical industries Farahani et al (2012); Vila-Parrish et al (2008). For instance, in the dairy industry perishability is found at different stages

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of the production process: from raw-material (milk) that enters the factories to finished products, which are stamped with a best-before-date Entrup et al (2005). As mentioned in Amorim et al (2011b), perishability enforces specific constraints on a set of crucial production planning decisions. This is especially so for multi-level production structures where two or more items are produced and at least one item is required as an input (raw-material). These intermediate products are often inventoried, allowing one to produce and consume them at different moments and rates in time Pochet (2001). When dealing with perishability, the data associated with inventories should be able to trace the age and usability status of items with specific time-stamps. Besides the amount of inventory kept in stock, we need to know when the material was acquired and to what level it has deteriorated, as well as the impact that such deterioration may have on the production process.

Production planning decisions determine the size and timing of production lots and therefore the frequency of setups. Meanwhile, setups affect lead times of items waiting in line to be processed which consequently increase deterioration. To reach acceptable quality levels and production yields, using deteriorated materials may consume more resources and therefore it may also have an effect on waiting times. If a perishable item reaches the end of its *shelf-life* and becomes unsuitable, it may have to be discarded. Thus, additional costs are incurred, as disposed materials may need certain treatment and be transported to disposal sites.

Although there exists a large amount of literature considering perishability in production and supply chain planning, to the best of our knowledge, raw-material (and/or intermediate products) perishability and the way it affects the production of higher level items in multi-level product structures have not been studied extensively. We focus on the case with raw-material batch ordering (or batch delivery), e.g. pallets, containers, or packages; and even though the batch lot-sizing problem has been well-studied, as far as we know this is not the case in the context of multi-level perishable product structures.

The contribution of this paper is to study how raw-material perishability and deterioration can be incorporated into multi-level classical lot-sizing problems and its impact on production processes regarding: manufacturing, inventory, and disposal costs, and capacity consumption for reaching the required quality of the finished products that are not perishable. The initial motivation for this study comes from composite manufacturing in fiber reinforced polymer composites production and other applications. These processes use fibers, such as glass or graphite, impregnated with polyimide monomeric reactants and other materials which have limited shelf-life, are sensitive to premature aging, and affect the production process in several ways (see Alston and Gahn (2000), and Alston and Scheiman (1999), for reports on this type of applications). However, once these materials are used in production, the final products are generally stable and no longer deteriorate. We study three variants of a two-level lot-sizing problem involving different types of perishability and deterioration, and present mixed integer programming (MIP) formulations to model them. We perform computational experiments to evaluate the added value of integrating raw-material perishability in the classical lot-sizing problems using our models, and their computational performance when used with a general purpose solver. We also analyze the impact of certain key parameters on the structure of solutions, such as shelf-life and the size of raw-material batches.

The paper is organized as follows: In Section 2 we review different characteristics of product perishability and deterioration. We discuss the most relevant approaches for integrating perishability in production planning and related problems. In Section 3 we present the three considered problem variants: *fixed shelf-life*, *functionality deterioration*, and *functionality and volume deterioration*. In Section 4 we show computational results with detailed sensitivity analyses. We perform a comparison between the proposed MIP formulations and a classical two-level lot-sizing model within a sequential approach in which production and raw-material related decisions are made independently using a rolling-horizon algorithm that follows the basic idea of the Silver-Meal heuristic (Silver and Meal, 1973). Finally, Section 5 brings final conclusions and future research directions.

## 2 Characteristics of perishability and modeling approaches

A general definition of perishability presented in Wee (1993) is the decay, damage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of an item resulting in decreasing usefulness from the original one. Most authors working in this field use deterioration, perishability, and depreciation

interchangeably. A perishable item has a fixed maximum lifetime, referred to as *shelf-life*. Shelf-life is the maximum length of time during which the material is considered of satisfactory quality and can be stored, remaining suitable for use or consumption. Shelf-life is usually considered from the moment the product is produced or the material is acquired.

### 2.1 Characteristics and classification of perishability

We distinguish three main viewpoints in classifying product perishability: (a) utility or functionality of the product, (b) physical state of the product, and (c) mathematical modeling point of view of perishability.

When the interest is in the functionality (Raafat, 1991; Pahl and Voß, 2010), and based on the value of inventory as a function of time, perishability is classified in: (1) constant-utility: items undergo decay but face no considerable decrease in value (prescription drugs), (2) decreasing-utility: items lose functional value throughout their shelf-life (milk, fruits), and (3) increasing-utility: items increase in value (wines, cheese, antiques). Similarly, but with an interest in how the customer perceives the functional value of items (Ferguson and Koenigsberg, 2007), we make two distinctions: (1) items whose functionality deteriorates over time (fresh produce), and (2) items whose functionality does not degrade, but their usefulness perceived by customers does (fashionable clothing and high-technology products).

The second perspective emphasizes the physical state of the product and can be found in several early inventory control papers. Ghare and Schrader (1963) characterize perishability regarding the type of deterioration: (1) direct spoilage (fresh food), (2) physical depletion (gasoline and alcohol), and (3) decay and obsolescence (newspapers). Lin et al (2006) take into account age-related characteristics and distinguish them between: (1) age-dependent deterioration (milk, fruits), and (2) age-independent deterioration (volatile liquids, radioactive and other chemicals). This perspective refers to the volume (quantity) loss of product, but not necessarily to the loss of functionality.

The third perspective refers to the treatment of perishability from a modeling viewpoint. Nahmias (1982) divides perishable products as: (1) with fixed shelf-life: cases where the shelf-life is known *a priori*, and (2) with random shelf-life: cases where the product shelf-life is a random variable with a specified probability distribution.

A detailed classification of perishability was proposed in Amorim et al (2011b), considering three dimensions: (1) physical product deterioration, (2) authority limits: represents external regulations or other conventions that influence perishability, and (3) customer value: reflects customer willingness to pay for a certain good.

### 2.2 Modeling approaches to perishability

We distinguish two different approaches in which perishability and deterioration are studied in inventory control, scheduling, and production and distribution planning. The first approach assumes a loss of a portion of inventory, determined by a fixed input parameter (shrinkage factor). Hsu (2000) presents an uncapacitated, single-item, lot-sizing problem ($LS$) using a deterioration rate factor and considering age-dependent inventory costs. The model is generalized to include back-ordering (Hsu, 2003) and capacities (Waterer, 2007). Using a similar deteriorating coefficient, Chen and Chen (2006) integrate $LS$ and scheduling for a perishable item in maximizing revenue, where demand and production depend on the selling price. Other studies following this approach include Balkhi and Benkherouf (1996), Yang and Cheng (1998), Skouri and Papachristos (2003), and Goyal and Giri (2003). Balkhi (2003) considers lot-sizing problems with time-varying demand of deteriorating items as well as the effect of "learning" in production. Lin et al (2006) present a multi-item production-inventory problem with exponential deterioration rates and constant inventory shrinkage factor. Other studies in this area include Tadj et al (2006), Manna and Chaudhuri (2006), and Belo-Filho et al (2015). Li et al (2014) study dynamic pricing and inventory control policies for perishable products with stochastic disturbance. For reviews of available literature regarding perishable goods in inventory management see Goyal and Giri (2001), Li et al (2010), and Janssen et al (2016). Amorim et al (2011b) and Pahl and Voß (2014) present reviews on production-distribution and supply chain planning for deteriorating items.

The second approach attempts to avoid inventory expiration by limiting the number of periods of production to ensure that products do not reach the end of their shelf-life. Entrup et al (2005) develop MIP

models following this approach to solve production-scheduling problems of yogurt production with shelf-life-dependent selling price. In the same application area, Amorim et al (2011a) propose two multi-objective *LS* and scheduling MIP models for a pure make-to-order system, and for a hybrid make-to-order/make-to-stock scenario. The authors incorporate the maximization of product freshness as the problem objective function. Pahl and Voß (2010) extend this approach without restricting the number of time periods. They allow inventory expiration and penalize it by applying a disposal cost as part of a standard discrete lot-sizing and scheduling problem. Pahl et al (2011) extend this approach to include sequence-dependent setup times and costs.

Other studies relevant to our research in this paper are: Abad (2000) present a constrained non-linear programing model for *LS* problems of perishable goods with exponential decay, partial back-ordering and lost sales. Teunter and Flapper (2003) consider a stochastic Economic Production Quantity model where produced units of a single type product may be non-defective, reworkable-defective, or non-reworkable-defective. Another approach involves traceability in an economic production quantity problem (Wang et al, 2009). Traceability is the ability to trace and follow the product through all stages of production, processing and distribution, which is an important management issue in food industry.

When considering perishability in well-known *economic lot scheduling problems*, most literature is limited to adding shelf-life constraints to the original problems (Amorim et al, 2011b). Soman et al (2004), and Pahl and Voß (2014) present reviews of related literature.

Production time-windows are also used to model perishability constraints (Wolsey, 2005). Chiang et al (2009) study a production-distribution problem applied to the newspaper industry. The authors present a simulation-optimization framework and formulate the problem as an extension of the vehicle routing problem. Chen et al (2009) study a production-scheduling and vehicle routing problem with time-windows and stochastic demands.

The studies presented above consider product perishability and deterioration in production planning and related problems. However, research on raw-material (and/or intermediate products) perishability and deterioration affecting the production of higher-level items is very limited. Cai et al (2008) and Billaut (2011) solve different production scheduling problems with, to certain extent, considerations of raw-material perishability. The former has application in seafood industry, and the latter discusses various aspects in dealing with perishable inventory in operational decisions.

## 3 Lot-sizing with perishable raw material

We consider a production system in which one item (finished product) is to be produced and another item (raw-material), an input of the first, is to be procured from a supplier over a planning horizon with $n$ time periods, $T = \{1, ..., n\}$. This constitutes the simplest version of a two-level product structure. Solving the *two-level lot-sizing problem* (*2LS*) is to determine the production, procurement and inventory plans for the two items to meet the demands of the planning horizon, while minimizing the corresponding costs. In particular, we consider three different types of raw-material deterioration in solving the *2LS* problem: (a) raw-material with fixed shelf-life (*FS*); (b) raw-material with functionality deterioration (*FD*); and (c) raw-material with functionality and volume deterioration (*FVD*).

### 3.1 Fixed Shelf-Life

We first consider the *two-level lot-sizing problem with fixed raw-material shelf-life* (*2LS-FS*). We assume that raw-materials are ordered and received immediately. Associated with each order there are unit batch costs and fixed ordering costs. Received raw-material may be used in production or placed in inventory. However, it can only be kept in stock for a predetermined period of time (shelf-life). If the material reaches the end of its shelf-life and expires, it will be disposed. This causes additional costs depending on when the disposal is made. Raw-material functionality is considered constant during the entire shelf-life period. Production is limited by process capacity and incurs fixed setup costs as well as variable production costs. Demand must be satisfied in every time period. The *2LS-FS* consists of planning the production levels and raw-material ordering for each time period, as well as planning the inventory levels so as to minimize the total production, setup, order-placement, inventory, and raw-material disposal costs.

Applications of *2LS-FS* may arise in the production of plastic films. Depending on their applications, plastic films can be made from a variety of plastic resins and monomers, which are highly reactive and undergo uncontrolled polymerization. However, they are considered fully functional during their shelf-life. The finished product is not considered to be perishable.

In formulating the *2LS-FS* model, we use the following notation:

$d_t$ demand per period $t$
$a$ standard unit production time
$C_t$ available process capacity per period $t$
$p_t$ unit production cost per period $t$
$q_t$ fixed setup cost per period $t$
$h_t$ unit storage cost per period $t$
$b$ fixed raw-material order batch-size
$K_t$ upper ordering limit per period $t$
$\beta$ raw-material shelf-life
$r$ units of raw-material required to produce each finished item
$\rho_t$ fixed cost of placing a raw-material order per period $t$
$\zeta_t$ unit batch cost per period $t$
$\gamma_t$ raw-material unit storage cost per period $t$
$\phi_t$ raw-material unit disposal cost per period $t$

In terms of decision variables, we have the following:

$Q_t$ number of raw-material batches to order in period $t$
$w_{ut}$ amount of raw-material received in period $u$ used for production in period $t$
$e_t$ amount of perished raw-material received in period $t$ to be discarded
$s_t$ finished item stock at the end of period $t$
$y_t$ binary variable equal to 1 if and only if there is positive production in period $t$
$z_t$ binary variable equal to 1 if and only if a raw-material order is placed in period $t$

Variables $w_{ut}$ are defined for $1 \le u \le t \le n$ and $(t-u) < \beta$, given that material received at the beginning of period $u$ can only be used for production during $\beta$ periods of time (including $u$). The last period in which material received at period $u$ can be used for production is given by $\Theta_u = \min\{u+\beta-1,\ n\}$. Let $\Pi_t = \max\{1, t-\beta+1\}$ denote the earliest period in which material can be acquired and still be used in period $t$. We further assume that even though material received during period $u$, $\beta < u \le n$, does not expire during the planning horizon, if not used, will be discarded.

Using these sets of decision variables, the *2LS-FS* can be formulated as:

$$\text{minimize} \sum_{t=1}^{n} \left( \sum_{u=\Pi_t}^{t} p_{ut}^{FS} w_{ut} + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right) \tag{1}$$

$$\text{subject to} \quad bQ_u = \sum_{t=u}^{\Theta_u} w_{ut} + e_u \qquad u \in T \tag{2}$$

$$s_{t-1} + \left(\frac{1}{r}\right) \sum_{u=\Pi_t}^{t} w_{ut} = d_t + s_t \qquad t \in T \tag{3}$$

$$\left(\frac{a}{r}\right) \sum_{u=\Pi_t}^{t} w_{ut} \le C_t y_t \qquad t \in T \tag{4}$$

$$Q_t \le K_t z_t \qquad t \in T \tag{5}$$

$$s_t,\ e_t \ge 0 \qquad t \in T \tag{6}$$

$$w_{ut} \ge 0 \qquad u, t \in T,\ u \le t \tag{7}$$

$$Q_t \ge 0 \text{ and integer} \qquad t \in T \tag{8}$$

$$y_t,\ z_t \in \{0,1\} \quad t \in T \tag{9}$$
$$s_0 = s_0^*, \tag{10}$$

where $s_0^*$ is the number of finished item units available at the beginning of the planing horizon and

$$p_{ut}^{FS} = \begin{cases} \frac{p_t}{r} + \sum\limits_{v=u}^{t-1} \gamma_v & \text{if } u < t, \\ \frac{p_t}{r} & \text{if } u = t. \end{cases}$$

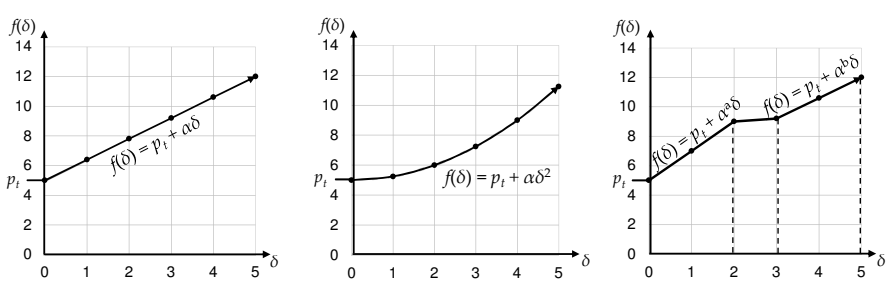
The objective function (1) includes: production and raw-material inventory holding costs, finished item inventory holding and setup costs, raw-material batch and disposal costs, and order-placement costs. Constraints (2) state that the amount of raw-material entering the production system at each period $u$ is equal to the amount used for production at subsequent periods, plus the amount that is discarded if not used before the end of its shelf-life. Constraints (3) represent finished item inventory balance, whereas constraints (4) are capacity limits. Constraints (5) are the upper bounds for the amount of raw-material to order at each time period. Constraints (6) – (7) are non-negativity conditions, and (8) – (9) are integrality and non-negativity conditions. Constraint (10) provides initial finished inventory item units.

*Property 1* When $b = 1$, there exists an optimal solution to *2LS-FS* in which $e_u = 0\ \forall\, u \in T$.

This property states that if it is possible to order raw-material by units, then one should order the exact amount as needed, i.e., $\sum_{u=1}^{n} Q_u = r \sum_{t=1}^{n} d_t$. We focus all our computational study on the case of raw-material batch ordering ($b > 1$).

### 3.2 Functionality deterioration

The second variant is the *two-level lot-sizing problem with raw-material functionality deterioration* (*2LS-FD*). In this case, raw-material functionality decreases as storage time passes. We refer to functionality as the suitability of the material for being used in the production process. Deteriorated materials may cause additional production costs and increased resource consumption in achieving the desired product quality and yields. We represent this effect by considering the unit production cost $p_t$ as an arbitrary non-decreasing function $f(\delta)$ where $\delta = (t - u)$, such that $f(0) = p_t$ and $f(\delta) \leq f(\delta + 1)$, for $0 \leq \delta < \beta$. Figure 1 shows three examples of production cost functions with $\beta = 6$ and $p_t = 5$ for all $t \in T$.




(a) Linear cost function with slope $\alpha$

(b) Cuadratic cost function with a rate of change $\alpha\delta^2$

(c) Piecewise linear cost function slopes $\alpha^a$ for $0 \leq \delta \leq 2$ and $\alpha^b$ for $3 \leq \delta \leq 5$

Fig. 1: Three production cost functions for an example with $\beta = 6$ and $p_t = 5$

Correspondingly, the updated production and raw-material inventory holding costs per period $t$ considering the use of deteriorated raw-material received in $u \leq t$ is given by:

$$p_{ut}^{FD} = \begin{cases} \frac{f(\delta)}{r} + \sum_{v=u}^{t-1} \gamma_v & \text{if } u < t, \\ \frac{p_t}{r} & \text{if } u = t, \end{cases}$$

Note that the parameter $p_{ut}^{FD}$ is not a decision variable but an input of the problem that depends on $u, t$, and $f(\delta)$.

As mentioned, the use of deteriorated but otherwise usable raw-material may require longer production time and consume more resource capacity for setting up the production system. For example, in composite manufacturing processes producing polyimide reinforced fiber composites and other products, slightly hardened resin may still be used in production but it usually requires additional cautions that slower operations.

In this regard, unit production time $a$ is replaced as follows:

$$a_{ut}^{FD} = a + \Delta^{(\delta)} \text{ for } 0 \leq u \leq t \leq n, \quad (t-u) < \beta.$$

where $\Delta^{(\delta)}$ is a non-decreasing function with $\Delta(0) = 0$ and $\Delta(\delta) \leq \Delta(\delta+1)$, for $0 \leq \delta < \beta$.

The *2LS-FD* model can be formulated as follows:

$$\text{minimize} \quad \sum_{t=1}^{n} \left( \sum_{u=\Pi_t}^{t} p_{ut}^{FD} w_{ut} + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right) \tag{11}$$

$$\text{subject to} \quad (2), (3), (5) - (10)$$

$$\left( \frac{a_{ut}^{FD}}{r} \right) \sum_{u=\Pi_t}^{t} w_{ut} \leq C_t y_t \qquad t \in T, \tag{12}$$

A production setup here is the realization of all operations required to reconfigure the production process at the end of period $t$ after producing a batch of products. Thus, constraint (12), ensures that resource capacity in period $t$ to produce all batches and perform reconfigurations using deteriorated material is not exceeded.

### 3.3 Functionality and volume deterioration

We now propose the *two-level lot-sizing problem with raw-material functionality and volume deterioration* (*2LS-FVD*). Here, the perishability nature of the raw-material not only refers to a functionality loss but, in addition, to a progressive volume loss.

We consider that the amount (volume) of available raw-material decreases as a function of the time it has remained in storage. Thereby, $\nu(\delta)$ where $\delta = (t-u)$, denotes the raw-material volume deterioration function, with $\nu(\delta) \leq \nu(\delta+1)$, for $0 \leq \delta < \beta$. A new set of decision variables $c_{ut}$ for $1 \leq u \leq t \leq n$ is introduced to represent the amount of raw-material received in time period $u$ and in storage at the end of $t$. Moreover, the expired raw-material variables $e_t$ now have an additional index to track when the material is received ($u$) and when it is perished/lost ($t$): $e_{ut}$ for $1 \leq u \leq t \leq n$.

Applications of this problem can be found in canning processes such as, canning fruits, vegetables, seafood, and meats, among others. In food processing, considerable amounts of raw-material may be lost throughout the multiple steps of the production process, which may include preliminary preparation, blanching, and filling (Melrose Chemicals Ltd., 2005).

The updated production cost per period $t$ (not including raw-material inventory holding costs) considering the use of deteriorated raw-material received in $u \leq t$ is given by:

$$p_{ut}^{FVD} = \begin{cases} \frac{f(\delta)}{r} & \text{if } u < t, \\ \frac{p_t}{r} & \text{if } u = t, \end{cases}$$

where as before, $f(\delta)$ is the production cost function depending on $\delta = t - u$.

The *2LS-FVD* model can be formulated as follows:

$$\text{minimize} \quad \sum_{t=1}^{n}\left(\sum_{u=\Pi_t}^{t} p_{ut}^{FVD} w_{ut} + h_t s_t + q_t y_t + \zeta_t Q_t + \rho_t z_t\right) + \sum_{u=1}^{n}\sum_{t=u}^{\Theta_u}(\gamma_u c_{ut} + \phi_u e_{ut})$$

$$\text{subject to} \quad (3),\ (5)-(10),\ (12)$$

$$c_{tt} = (bQ_t - w_{tt})(1-\nu(0)) \qquad t \in T \tag{13}$$

$$c_{ut} = (c_{u,t-1} - w_{ut})(1-\nu(\delta)) \qquad u,t \in T,\ 0<\delta<\beta \tag{14}$$

$$e_{tt} = (bQ_t - w_{tt})(\nu(0)) \qquad t \in T \tag{15}$$

$$e_{ut} = (c_{u,t-1} - w_{ut})(\nu(\delta)) \qquad u,t \in T,\ 0<\delta<\beta \tag{16}$$

$$c_{ut},\ e_{ut} \in \mathbb{R}^+ \qquad u,t \in T,\ u \le t, \tag{17}$$

where constraints (13) – (14) represent raw-material inventory levels and constraints (15) – (16) represent raw-material disposal. Note that, if $f(\delta) = p_t$ for $0 \le \delta < \beta$, *2LS-FVD* reduces to a variant with only raw-material volume deterioration. Moreover, if $\nu(\delta) = 0$ for $0 \le \delta < \beta$, *2LS-FVD* reduces to *2LS-FD*.

## 4 Computational experiments and analyses

A computational study was conducted in order to gain in-depth understanding of the considered models and to evaluate our MIP formulations. We tested more than 1,500 randomly generated instances for each of the three problem variants. Appendix A shows how such instances were generated. In Section 4.1, we study the added value of the proposed models in comparison with a sequential approach in which production and raw-material related decisions are made independently. In Section 4.2, we analyze the way that certain key parameters affect the optimal planning decisions. Section 4.3 shows certain computational aspects of the proposed MIP formulations when used with a general purpose solver.

### 4.1 The value of integrating raw-material perishability into classical lot-sizing

To evaluate the added value of integrating raw-material perishability in the classical lot-sizing problems, we first perform a comparative analysis on the optimal solutions obtained with our models that explicitly integrate raw-material perishability, and those of a standard *2LS* model that does not. Details are presented in Appendix B.

As expected, for many problem instances, optimal solutions of the *2LS* model are not feasible for solving their counterpart problems with raw-material perishability, specially for shorter shelf-life instances and for *2LS-FVD*. For the feasible solutions, we compute the optimal deviations as $\%dev = [(SOL_{2LS} - OPT)/OPT] \times 100$, where $SOL_{2LS}$ is the objective function value of the *2LS* solution and $OPT$ the optimal solution value of the model. From this, we observe that $\%dev$ increases when the $\beta$ values increase. In total, 29.8% of the *2LS* solutions were infeasible and the average $\%dev$ is 10.8% with the maximum being 34.2%. Averaging the three problem instances, nearly 24% of the instances showed a $\%dev$ greater than 10%.

We further investigate the instances that are infeasible and those that show greater deviations. In general, infeasibility comes from two different but closely related sources. Firstly, the standard *2LS* model may have solutions where production of finished-items is set to use raw-material stored in inventory for periods longer than its shelf-life, i.e. $w_{ut} > 0$ for $(t-u) \ge \beta$. A second source of infeasibility, specifically for *2LS-FVD*, is that the amount of raw-material ordered in any given period is less than required to cover all production for subsequent periods before a new order is placed, i.e. constraints (13) and (14) are violated.

With a clear understanding that the comparison between the standard *2LS* model and our MIP formulations may not seem fair, we make it in order to quantitatively assess the value of integrating perishability and deterioration into classical lot-sizing problems using our proposed models. To continue this assessment, we propose a sequential approach that adapts in a natural and intuitive fashion the initial standard *2LS* solutions to find feasible and possibly improved solutions for the original problem variants. We then compare how our MIP formulations perform compared to this sequential optimization approach.

Considering that the sources of infeasibility are decisions regarding the size and timing of raw-material orders and the use of such material to meet production requirements, it is natural to adapt the solutions by modifying these decisions in a subsequent phase.

With this in mind, the first step of the sequential approach begins by fixing the production-related decisions obtained in the standard *2LS* solution and use them as exogenous decisions for the following step. The second step applies a rolling-horizon algorithm following the basic idea of the Silver-Meal heuristic (Silver and Meal, 1973) to solve the remaining sub-problem regarding the raw-material related decisions. Thus, the idea is to order enough raw-material to cover the production of one time period $u$ and then the number of periods to cover is increased (in increments of one period $t$) until the average cost per period ($ACP_{ut}$) increases. Details are shown in Appendix C.

Table 1 shows the computational results using the sequential approach to solve each of the original problem variants. The first two columns specify the type of problem variant solved and the $\beta$ values of the

Table 1: Average sequential optimization approach results

| Variant | $\beta$ | %opt | %dev | % > 10 |
|---|---|---|---|---|
| ***2LS-FS*** | 2 | 39.6 | 4.5 | 10.5 |
| | 3 | 52.1 | 4.2 | 20.9 |
| | 4 | 62.5 | 3.9 | 18.8 |
| | | **51.4** | **4.2** | **16.7** |
| ***2LS-FD*** | 2 | 39.6 | 5.0 | 12.5 |
| | 3 | 52.1 | 4.4 | 20.9 |
| | 4 | 52.1 | 4.0 | 18.8 |
| | | **47.9** | **4.5** | **17.4** |
| ***2LS-FVD*** | 2 | 8.3 | 8.7 | 34.4 |
| | 3 | 6.3 | 12.1 | 49.0 |
| | 4 | 4.2 | 14.2 | 63.6 |
| | | **6.3** | **11.7** | **49.0** |
| **Total** | | **35.2** | **6.8** | **27.7** |

instances. The third column shows the percentage of instances for which the sequential approach solution is optimal (%opt). The last two columns show the average deviation (%dev) with respect to the optimal solution (of the non-optimal solutions), and the percentage of instances greater than 10%, respectively.

We can see that in addition to achieving feasibility for all instances, the sequential approach also reached optimal solutions for 51.4%, 47.9%, and 6.3% of the instances for *2LS-FS*, *2LS-FD*, and *2LS-FVD*, respectively. The average deviations (%dev) for the first two problem variants are quite similar, ranging from 3.9% to 5.0%. It is much higher *2LS-FVD*, ranging from 8.7% to 14.2%. Figure 2 clusters the three problem variants and graphically shows the sequential approach deviations (%dev) for each batch-size value $b$. The dotted lines represent the average %dev for each shelf-life $\beta$ value. It shows a clear trend that the average deviation increases as the batch-size value ($b$) increases.

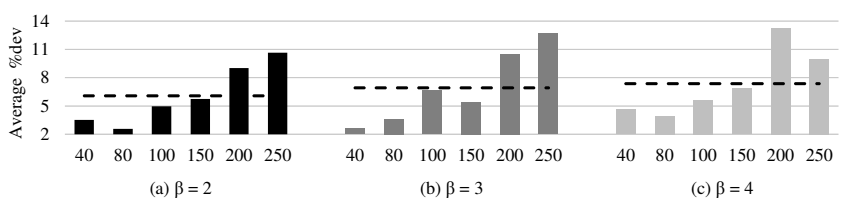



Fig. 2: Average %dev by shelf-life ($\beta$) and batch size ($b$)

Since the sequential approach focuses on the modification of raw-material related decisions, it is relevant to see how its deviation changes with respect to the total costs corresponding to these decisions (%RM) from the total value of the optimal solution. Figure 3 shows the scatter plots for *2LS-FD* and *2LS-FVD* within the range $25 \leq$ %RM $\leq 75$. Figure 3 shows that the deviation values are somewhat scattered. However, we note that for instances with %RM above 65%, the sequential approach found near optimal solutions for

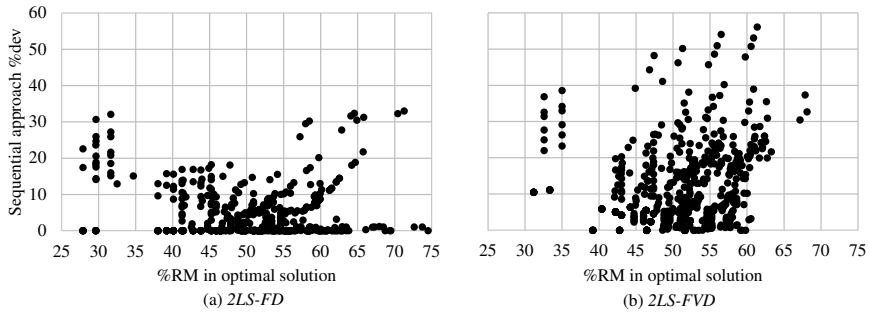



Fig. 3: Average sequential algorithm %dev vs. raw-material costs percentage %RM in optimal solution

*2LS-FD*. This is not the case for *2LS-FVD*, where higher %RM represent also higher deviations. Similarly, when %RM is below 40%, some instances are optimally solved for *2LS-FD*, whereas the *2LS-FVD* variant shows higher deviations.

The highest deviations for the *2LS-FS* and *2LS-FD* (%dev $> 20$) were found on instances with $b \geq 100$ in which the sequential approach resulted in solutions with a lower raw-material ordering frequency than the one observed in the optimal solution of the problem. These instances have %RM lower than 35 and greater than $> 55$. By resulting in a lower raw-material ordering frequency, the fixed order-placement are evidently reduced. However, this reduction is not sufficient compared to the substantial increase in raw-material inventory holding and disposal costs.

On the other hand, the highest deviations for the 2LS-FVD (%dev $> 30$) were found on instances with $n \geq 80$. In these cases, the discrepancy between the sequential and the integrated solution is that the former makes raw-material orders that are much higher than those required for production. This increases the general raw-material related costs, including inventory and disposal.

## 4.2 Key parameters for optimal planning

The shelf-life parameter $\beta$ limits the number of periods that the raw-material can remain in storage and be used for production. Along with $\beta$, functions $f(\delta)$ and $\nu(\delta)$ that model the loss of material functionality and volume, respectively, constitute the core features of the studied problems. In addition, raw-material order batch-size is another parameter that requires detailed analysis to see how it affects optimal solutions. For this analysis, we solved a set of instances with $n = 7$, $r = 3$, $L = 0$, and various $\beta$ and $b$ values, keeping all other parameters unchanged. Figure 4 shows a comparison of the corresponding raw-material disposal costs $\left(\sum_{u=1}^{n} \sum_{t=u}^{\Theta_u} (\gamma_u c_{ut} + \phi_u e_{ut})\right)$ in the optimal solution for each problem variant.

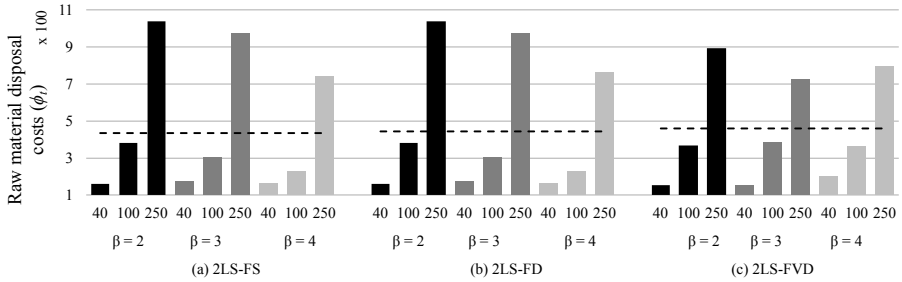



Fig. 4: Disposal costs by shelf-life ($\beta$) and batch size ($b$) values

The most important variations in the structure of optimal solutions largely arise from the relation between the order batch-size $b$, the bill of material $r$, and the finished-item demand $d_t$ levels. This relation affects the flexibility to manage raw-material inventories and the possibility to avoid disposing units. The greater

flexibility is found on instances with ordering batch-size $b = 1$ (see Property 3.1). Results shown in this sections are for instances with $b > 1$. As observed in Figure 4, for every problem instance, raw-material disposal increases consistently as the batch-size increases.

For the same $\beta$ and $b$ values, Figure 5 shows the changes in the corresponding order-placement costs ($\sum_{t=1}^{n} \rho_t z_t$) in the optimal solution. These costs represent the frequency in which raw-material orders are placed.

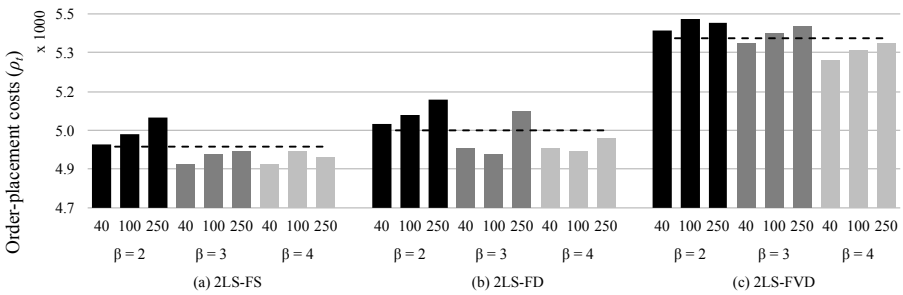



Fig. 5: Order-placement costs by shelf-life ($\beta$) and batch size ($b$) values

As observed, longer shelf-lives result in lower average order-placement costs. This can be partially attributed to the fact that shorter shelf-lives represents fast functionality and/or volume deterioration, which consequently results in a higher setup frequency. Although not shown in the figure, this fact also results in an increase in the average finished-item inventory holding costs, since production tends to take place in earlier periods to avoid raw-material disposal.

A clear observation from Figure 5 is that the *2LS-FVD* problem variant incurs the highest order-placement costs. This is somewhat expected due to the progressive raw-material volume loss in each period, resulting in the need to place orders more frequently.

### 4.3 Computational performance of MIP formulation

For the following computational study, we have implemented our MIP formulation on a set of randomly generated capacitated instances for each problem variant (see Appendix A). All computational experiments were implemented and executed using the Callable Library of IBM CPLEX 12.6.2 on an Intel(R) Xeon(R) CPU E3-1270 v3 processor with 3.50GHz and 24GB of RAM memory and Microsoft Windows 7 Enterprise operating system. A maximum time limit of two hours was used in all experiments.

Table 2 presents the computational performance of the test instances for each problem variant. The first three columns show the different variant, $n$ and $\beta$ values for which results are presented. We show the average performance of 1,620 instances for each variant (i.e. 135 for each $\beta$ value and 540 for each $n$).

The second couple of columns refer to the number of branch and bound nodes explored by CPLEX showing the minimum value found and the average. The next couple of columns refer to the linear programming relaxation gaps (LP Gaps %). And the last two columns refer to the CPU time (in seconds), showing the average and the maximum value observed. Wherever "limit" is registered it means that at least one of the solutions was not solved to optimality within the two hours.

A first observation is that the increase in the shelf-life values corresponds to an increment of the number of branch and bound nodes explored in most of the tested instances. This is partially due to the fact that a lower shelf-life restricts the solution space decreasing the number of possibilities to decide upon regarding the usage of raw-material for production in later periods. Having a longer shelf-life value increases the solution space to explore.

Generally, LP Gaps range from 6.51% to 26.81%. The lowest LP Gaps ($< 10\%$) were observed in *2LS-FS* and *2LS-FD* instances with lower fixed costs of placing raw-material orders $\rho_t$. Whereas the highest LP Gaps ($> 20\%$) were mostly observed in *2LS-FVD* instances with $b \geq 100$ values. In terms of computational times, the average CPU time ranges between 6.77 and $1,577$ seconds. The highest values ($> 60$ minutes)

Table 2: Computational performance of MIP formulations

| Variant | $n$ | $\beta$ | Nodes (x1,000) | | LP Gap (%) | | Time (s) | | Solved |
|---|---|---|---|---|---|---|---|---|---|
| | | | min | avg | min | avg | avg | max | |
| *2LS-FS* | 18 | 2 | 1.69 | 158.66 | 7.37 | 13.56 | 7.30 | 66.94 | 135/135 |
| | | 4 | 1.59 | 740.71 | 7.31 | 13.96 | 35.70 | 1684.83 | 135/135 |
| | | 6 | 1.12 | 1,480.00 | 6.94 | 14.15 | 73.64 | 2131.91 | 135/135 |
| | | 8 | 1.97 | 3,529.23 | 7.87 | 14.21 | 192.37 | limit | 134/135 |
| | | **Total** | **1.12** | **1,477.15** | **6.94** | **13.97** | **77.25** | **limit** | **539/540** |
| | 20 | 2 | 2.47 | 663.44 | 6.51 | 13.59 | 30.28 | 1137.07 | 135/135 |
| | | 4 | 5.80 | 1,191.96 | 7.95 | 13.95 | 59.54 | 1077.24 | 135/135 |
| | | 6 | 6.11 | 4,180.69 | 7.25 | 14.30 | 226.90 | limit | 134/135 |
| | | 8 | 5.43 | 9,106.74 | 8.08 | 14.25 | 604.65 | limit | 130/135 |
| | | **Total** | **2.47** | **3,785.71** | **6.51** | **14.02** | **230.34** | **limit** | **534/540** |
| | 22 | 2 | 4.76 | 1,741.72 | 7.56 | 13.45 | 87.56 | 1685.92 | 135/135 |
| | | 4 | 6.55 | 5,565.82 | 7.23 | 13.95 | 332.51 | limit | 133/135 |
| | | 6 | 15.93 | 8,269.32 | 7.03 | 13.97 | 480.59 | limit | 133/135 |
| | | 8 | 21.21 | 16,195.62 | 8.40 | 14.16 | 1193.36 | limit | 120/135 |
| | | **Total** | **4.76** | **7,943.12** | **7.03** | **13.88** | **523.50** | **limit** | **521/540** |
| | **Variant Total** | | **1.12** | **4,401.99** | **6.51** | **13.96** | **277.03** | **limit** | **1,594/1,620** |
| *2LS-FD* | 18 | 2 | 1.28 | 139.98 | 7.80 | 14.08 | 6.77 | 60.29 | 135/135 |
| | | 4 | 2.94 | 661.71 | 7.32 | 14.37 | 36.11 | 1916.44 | 135/135 |
| | | 6 | 2.15 | 783.55 | 7.23 | 14.46 | 51.86 | 918.77 | 135/135 |
| | | 8 | 4.67 | 1,840.65 | 8.00 | 14.46 | 140.16 | limit | 134/135 |
| | | **Total** | **1.28** | **856.47** | **7.23** | **14.34** | **58.73** | **limit** | **539/540** |
| | 20 | 2 | 0.92 | 622.76 | 7.00 | 14.08 | 28.87 | 1054.34 | 135/135 |
| | | 4 | 6.30 | 1,032.04 | 7.96 | 14.38 | 60.36 | 902.31 | 135/135 |
| | | 6 | 9.43 | 3,239.19 | 7.46 | 14.64 | 216.48 | 2881.92 | 135/135 |
| | | 8 | 6.18 | 5,500.59 | 8.33 | 14.51 | 452.89 | limit | 134/135 |
| | | **Total** | **0.92** | **2,598.65** | **7.00** | **14.40** | **189.65** | **limit** | **539/540** |
| | 22 | 2 | 3.66 | 1,715.70 | 8.16 | 13.95 | 88.01 | 1573.18 | 135/135 |
| | | 4 | 7.19 | 4,317.31 | 7.96 | 14.34 | 291.21 | limit | 134/135 |
| | | 6 | 6.30 | 6,087.04 | 7.50 | 14.30 | 491.10 | limit | 132/135 |
| | | 8 | 15.29 | 11,610.36 | 8.49 | 14.41 | 1135.40 | limit | 125/135 |
| | | **Total** | **3.66** | **5,932.60** | **7.50** | **14.25** | **501.43** | **limit** | **526/540** |
| | **Variant Total** | | **0.92** | **3,129.24** | **7.00** | **14.33** | **249.94** | **limit** | **1,604/1,620** |
| *2LS-FVD* | 18 | 2 | 1.54 | 232.54 | 10.80 | 17.85 | 13.87 | 335.50 | 135/135 |
| | | 4 | 3.52 | 1,107.24 | 11.04 | 18.20 | 114.48 | limit | 134/135 |
| | | 6 | 3.84 | 1,093.42 | 11.90 | 18.30 | 133.27 | 2090.75 | 135/135 |
| | | 8 | 3.02 | 1,015.08 | 11.97 | 18.40 | 142.19 | 2510.95 | 135/135 |
| | | **Total** | **1.54** | **862.07** | **10.80** | **18.19** | **100.95** | **limit** | **539/540** |
| | 20 | 2 | 1.99 | 947.69 | 11.38 | 17.95 | 56.41 | 1132.17 | 135/135 |
| | | 4 | 10.68 | 2,358.18 | 11.84 | 18.22 | 242.20 | 6684.31 | 135/135 |
| | | 6 | 4.44 | 5,677.68 | 11.75 | 18.54 | 765.87 | limit | 131/135 |
| | | 8 | 7.79 | 4,146.80 | 11.97 | 18.37 | 674.60 | limit | 130/135 |
| | | **Total** | **1.99** | **3,282.59** | **11.38** | **18.27** | **434.77** | **limit** | **531/540** |
| | 22 | 2 | 5.71 | 3,215.44 | 11.92 | 17.82 | 213.35 | 6195.72 | 135/135 |
| | | 4 | 9.38 | 6,323.53 | 11.61 | 18.19 | 708.45 | limit | 131/135 |
| | | 6 | 18.28 | 5,933.21 | 11.57 | 18.33 | 847.36 | limit | 132/135 |
| | | 8 | 26.13 | 8,659.96 | 12.08 | 18.40 | 1577.27 | limit | 119/135 |
| | | **Total** | **5.71** | **6,033.03** | **11.57** | **18.18** | **836.61** | **limit** | **517/540** |
| | **Variant Total** | | **1.54** | **3,392.56** | **10.80** | **18.21** | **457.44** | **limit** | **1,587/1,620** |

were mostly observed in instances with $\beta \geq 4$. All instances that were not solved within the two hour limit have $\beta \geq 6$.

## 5 Conclusions

We studied how raw-material perishability considerations can be integrated into classical lot-sizing problems. We introduced three variants of the *two-level lot-sizing problem* with different types of raw-material perishability: (a) *fixed shelf-life*, (b) *functionality deterioration*, and (c) *functionality and volume deterioration*. We proposed MIP formulations for each of these variants. We analyzed the impact of perishability on production processes regarding: manufacturing, inventory, and disposal costs.

From the study presented in Section 4.1 we can infer that there is a significant added value for using our proposed MIP formulations to integrate these considerations into classical lot-sizing models. Clearly, the use of a standard two-level lot-sizing model within a sequential approach is insufficient to solve the problems discussed in this study.

We are currently investigating the integration of other relevant factors to make our models more robust, such as multi-raw-material items, different product structures, capacity restrictions, time-dependent batch sizes, and other raw-material inventory-related assumptions. Finally, considering the extensive computational times to solve a portion of medium to large size problem instances, we are also working in the development of solution algorithms for efficiently solving certain variants of these problems.

## 6 Acknowledgments

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## A Description of test instances

All problem instances used to perform the computational experiments in Section 4 were randomly generated as follows. Three parameters were set a priori as a basis for comparison: planning horizon ($n$), shelf-life ($\beta$), and batch-size ($b$). In particular, we tested instances with $n \in \{18, 20, 22\}$, $\beta \in \{2, 4, 6, 8\}$, and $b \in \{50, 100, 150, 200, 250\}$. Another parameter set a priori is $r = 3$ for all considered instances. The remaining parameters were randomly generated using continuous or discrete uniform distributions, depending on the parameter, as follows:

- *Demand:* $d_t \sim U\,[150, 300]$
- *Unit production time:* $a \sim U\,[2.5, 3.5]$
- *Unit production cost:* $p_t \sim U\,[10.0, 13.0]$
- *Fixed setup cost:* $q_t \sim U\,[380.0, 420.0]$
- *Unit storage cost:* $h_t \sim U\,[5.0, 7.0]$
- *Unit raw-material cost:* $\zeta_t \sim U\,[1.0, 3.0]$
- *Upper ordering limit:* $K_t \sim U\,[K_{coef} \times 4.5, K_{coef} \times 4.75]$,

where the $K_t$ values were generated considering the product of the average demand and the bill of material $r$, divided by the batch size $b$. With the purpose of integrating variability in a controlled manner, for the remaining parameters, three different value levels were generated each for 1/3 of the instances.

- *Fixed order-placement cost:* A *high level* using $\rho_t \sim U\,[500, 550]$, a *medium level* using $\rho_t \sim U\,[250, 300]$, and a *low level* using $\rho_t \sim U\,[150, 200]$.
- *Raw-material unit storage cost:* A *high level* using $\gamma_t \sim U\,[8.0, 12.0]$, a *medium level* using $\gamma_t \sim U\,[3.0, 7.0]$, and a *low level* using $\gamma_t \sim U\,[1.0, 2.0]$.

Finally, the capacitated instances used the following:

- *Available process capacity:* The $C_t$ values were generated considering the upper demand limit 300 as follows: a *high level* using $C_t \sim U\,[300 \times 4.5, 300 \times 4.75]$, a *medium level* using $C_t \sim U\,[300 \times 4.25, 300 \times 4.5]$, and a *low level* using $C_t \sim U\,[300 \times 4.0, 300 \times 4.25]$.

## B A standard two-level lot-sizing model

As described in Section 3, the *standard two-level lot-sizing 2LS* problem considers a production system in which one item (finished product) is to be produced and another item (raw-material), an input of the first, is to be procured from a supplier over a planning horizon with $n$ time periods, $T = \{1, ..., n\}$. Solving the *2LS* problem is to determine the production, procurement and inventory plans for the two items to meet the demands of the planning horizon, while minimizing the corresponding costs.

To evaluate the added value of integrating raw-material perishability into classical lot-sizing problems, we initially performed a comparative analysis on the optimal solutions obtained with our MIP formulations and those of the *2LS* model.

For each of our three problem variants, we evaluate the solutions of the *2LS*. If they are feasible for the counterpart problems with raw-material deterioration, these solutions are compared with the optimal solutions of the proposed MIP formulations. Table 3 presents these results for a set of instances with $n = 7$ where the only varying parameters are $\beta = \{2, 3, 4\}$ and $b = \{40, 80, 100, 150, 200, 250\}$. The same instances are used for the computational experiments presented in Section 4.1.

The first two columns in Table 3 specify the type of problem variant solved and the $\beta$ values of the instances. The third column shows the percentage of instances for which the standard *2LS* solution is infeasible (%inf) when adapted to solve its counterpart problem variant. The next column shows the average deviation (%dev) of the feasible solutions from the optimal solution of the actual problem considering raw-material perishability. The deviations are computed as $\%dev = [(SOL_{2LS} - OPT)/OPT] \times 100$, where $SOL_{2LS}$ is the objective function value of the feasible solution and $OPT$ the optimal solution value. Finally, the last two columns show the percentage of instances with %dev greater than 10% and the maximum %dev observed, respectively.

Table 3: Average *standard 2LS* solution deviations

| Variant | $\beta$ | %inf | %dev | % > 10 | max |
|---|---|---|---|---|---|
| ***2LS-FS*** | 2 | 33.3 | 8.8 | 12.6 | 15.5 |
| | 3 | 16.6 | 10.2 | 20.3 | 18.1 |
| | 4 | 0.0 | 12.2 | 21.8 | 23.6 |
| | | **16.6** | **10.4** | **18.8** | **23.6** |
| ***2LS-FD*** | 2 | 33.3 | 8.9 | 12.6 | 15.5 |
| | 3 | 16.6 | 10.5 | 20.3 | 18.1 |
| | 4 | 0.0 | 12.3 | 21.8 | 23.6 |
| | | **16.6** | **10.6** | **18.8** | **23.6** |
| ***2LS-FVD*** | 2 | 56.2 | 9.2 | 33.8 | 26.3 |
| | 3 | 56.2 | 11.9 | 42.9 | 30.6 |
| | 4 | 56.2 | 13.0 | 47.5 | 34.2 |
| | | **56.2** | **11.4** | **41.4** | **34.2** |
| **Total** | | **29.8** | **10.8** | **23.9** | **34.2** |

## C A sequential approach

The goal of the sequential approach is to adapt the initial *2LS* solutions to find feasible and possibly improved solutions for the considered problem variants. We begin by defining the terms:

$\hat{x}_t$ production decisions to fix obtained from standard *2LS* solution for $1 \leq t \leq n$, where $\hat{x}_t = \sum_{u=1}^{t} w_{ut}$.

$\hat{X}_{ut}$ fixed cumulative production to cover from period $u$ to $t$.

$\bar{Q}_{ut}$ order quantity (raw-material batches) in period $u$ to cover fixed production up to $t$.

$\bar{z}_{ut}$ binary raw-material for order placement variable.

$\bar{w}_{ut}$ variables used to modified the original $w_{ut}$ variables within the heuristic to avoid violation of the $(t-u) < \beta$ condition.

$ACP_{ut}$ average cost per period for an order placed in period $u$ to cover fixed production requirements up to $t$, where:

$$ACP_{ut} = \frac{\rho_u \bar{z}_u + \zeta_u \bar{Q}_{ut} + \sum_{t=u}^{\Theta_u} p_{ut}^i \bar{w}_{ut} + \phi_u \left( b\bar{Q}_{ut} - \sum_{t=u}^{\Theta_u} \bar{w}_{ut} \right)}{(t-u)+1},$$

where $i = FS$ for the *2LS-FS* variant and $i = FD$ for the *2LS-FD* variant. We note that original Silver-Meal heuristic cannot be directly applied to any our problem variants and thus, an extended version is developed. For the *2LS-FS* and *2LS-FD* variants, the procedure used in the second step of the sequential approach are summarized in Algorithm 1.

**Algorithm 1** Sequential approach for *2LS-FS* and *2LS-FD*

1: Solve standard *2LS* to obtain $\hat{y}_t$, $w_{ut}$ for $u, t \in T$, $u \leq t$
2: $u \leftarrow 1$
3: **while** $t < n$ **do**
4: $\quad t \leftarrow u$
5: $\quad \hat{X}_{ut} \leftarrow \dfrac{\sum_{t=u}^{n} w_{ut}}{r}$
6: $\quad$ **if** $\hat{X}_{ut} = 0$ **then**
7: $\quad\quad \bar{Q}_{ut} \leftarrow 0$
8: $\quad\quad \bar{z}_u \leftarrow 0$
9: $\quad$ **else**
10: $\quad\quad \bar{Q}_{ut} \leftarrow \max\left\{ \left\lceil \dfrac{r\hat{X}_{ut}}{b} \right\rceil, L \right\}$
11: $\quad\quad \bar{z}_u \leftarrow 1$
12: $\quad$ **end if**
13: $\quad \bar{w}_{ut} \leftarrow r\hat{x}_t$
14: $\quad$ Evaluate $ACP_{ut}$
15: $\quad$ **if** $ACP_{ut} > ACP_{u,t-1}$ or $t + 1 = \Theta_u$ **then**
16: $\quad\quad$ **go to** step 20
17: $\quad$ **else**
18: $\quad\quad t \leftarrow (t + 1)$ and **go to** step 5
19: $\quad$ **end if**
20: $\quad z_u \leftarrow \bar{z}_{u,t-1}$, $Q_u \leftarrow \bar{Q}_{u,t-1}$, and $w_{u,t-1} \leftarrow \bar{w}_{u,t-1}$
21: $\quad u \leftarrow (u + 1)$ and **go to** step 4
22: **end while**
23: **return** $z_u, Q_u, w_{ut}$ for $u, t \in T$, $0 \leq (t - u) < \beta$

For the *2LS-FVD* variant, $ACP_{ut}$ is computed as follows:

$$ACP_{ut} = \frac{\rho_u \bar{z}_u + \zeta_u \bar{Q}_{ut} + \sum_{t=u}^{\Theta_u} p_{ut}^{FVD} w_{ut} + \sum_{t=u}^{\Theta_u} (\gamma_u c_{ut} + \phi_u e_{ut})}{(t - u) + 1},$$

and the steps are shown in Algorithm 2.
Where $\bar{w}_{ut} = r\hat{x}_t$; $\bar{c}_{ut} = (\bar{c}_{u,t-1} - r\hat{x}_t)(1 - \nu(\delta))$ and $\bar{c}_{uu} = r\hat{x}_{u+1}$, and $\bar{e}_{ut} = (\bar{c}_{u,t-1} - r\hat{x}_t)(\nu(\delta))$ and $\bar{e}_{uu} = \left(b\bar{Q}_{ut} - \hat{x}_t\right)(\nu(\delta))$.

**Algorithm 2** Sequential approach for *2LS-FVD*

1: Solve standard *2LS*, **return** $\hat{y}_t$, $w_{ut}$ for $u, t \in T$, $u \leq t$
2: $u \leftarrow 1$
3: **while** $t < n$ **do**
4: $\quad t \leftarrow u$
5: $\quad$ **if** u = t **then**
6: $\quad\quad$ **if** $\hat{x}_u = 0$ **then**
7: $\quad\quad\quad \bar{Q}_{ut} \leftarrow 0$
8: $\quad\quad\quad \bar{z}_u \leftarrow 0$
9: $\quad\quad$ **else**
10: $\quad\quad\quad \bar{Q}_{ut} \leftarrow \max\left\{\left\lceil \frac{r\hat{x}_u}{b} \right\rceil, L\right\}$
11: $\quad\quad\quad \bar{z}_u \leftarrow 1$
12: $\quad\quad$ **end if**
13: $\quad$ **else**
14: $\quad\quad$ **if** $\hat{x}_u = 0$ **then**
15: $\quad\quad\quad \bar{Q}_{ut} \leftarrow 0$
16: $\quad\quad\quad \bar{z}_u \leftarrow 0$
17: $\quad\quad$ **else**
18: $\quad\quad\quad \bar{Q}_{ut} \leftarrow \max\left\{\left\lceil \frac{\left[\frac{\bar{c}_{ut}}{(1-\nu(0))} + r\hat{x}_u\right]}{b} \right\rceil, L\right\}$
19: $\quad\quad\quad \bar{z}_u \leftarrow 1$
20: $\quad\quad$ **end if**
21: $\quad$ **end if**
22: $\quad$ Evaluate $\bar{w}_{ut}$, $\bar{c}_{ut}$, and $\bar{e}_{ut}$ for all $u \leq t \leq \Theta_u$
23: $\quad$ Evaluate $ACP_{ut}$
24: $\quad$ **if** $ACP_{ut} > ACP_{u,t-1}$ or $t + 1 = \Theta_u$ **then**
25: $\quad\quad$ **go to** step 29
26: $\quad$ **else**
27: $\quad\quad t \leftarrow (t + 1)$ and **go to** step 18
28: $\quad$ **end if**
29: $\quad z_u \leftarrow \bar{z}_{u,t-1}$, $Q_u \leftarrow \bar{Q}_{u,t-1}$ for $u \leq t \leq \Theta_u$
30: $\quad w_{u,t-1} \leftarrow \bar{w}_{u,t-1}$, $c_{u,t-1} \leftarrow \bar{c}_{u,t-1}$, and $e_{u,t-1} \leftarrow \bar{e}_{u,t-1}$ for $u \leq t \leq \Theta_u$
31: $\quad u \leftarrow (u + 1)$ and **go to** step 4
32: **end while**
33: **return** $z_u, Q_u, w_{u,t}, c_{ut}, e_{ut}$ for $u, t \in T$, $0 \leq (t - u) < \beta$