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Two-Level Lot-Sizing with Raw-Material Perishability and Deterioration

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In many industries and production systems it is common to face significant rates of product deterioration, referring to physical exhaustion, loss of functionality, and even obsolescence. This deterioration property, known as *perishability*, prevents such products from being stored and used for unlimited periods of time. In this paper, we present production planning problems that incorporate raw-material perishability and deterioration and analyze how these considerations enforce specific constraints on a set of fundamental decisions, especially in the case of multi-level product structures. In particular, we study four variants of the two-level lot-sizing problem incorporating different types of raw-material perishability: (a) *fixed shelf-life*, (b) *functionality deterioration*, (c) *volume deterioration*, and (d) *functionality-volume deterioration*. We propose mixed integer programming formulations for each of these variants. We perform computational experiments and carry out sensitivity analyses from two different perspectives. We show the computational performance of the proposed formulations and analyze the added value of incorporating perishability considerations into standard production planning problems. We finally study the impact of key parameters in the structure of optimal solutions.

Keywords: perishability; shelf-life; deterioration; production planning; two-level lot-sizing; mixed-integer programming;

1. Introduction

A common assumption in most of the production planning literature is that finished and intermediate products involved in the production process have unlimited *lifespans*, meaning they can be stored and used indefinitely. However, in practice, most items deteriorate over time, referring not only to physical exhaustion or loss of functionality, but also to obsolescence. Often, the rate of deterioration is low or can be ignored and there is little need for considering it in the planning process. Nonetheless, in many types of industries it is common to deal with items that are subject to significant rates of deterioration. These items are referred to as *perishable products*.

Although there are multiple definitions of perishability depending on the type of product or system, the concept basically relates to items that cannot be stored infinitely without deterioration or devaluation (Billaut 2011). Clear cases of this type of products can be found in the food or pharmaceutical industries (Farahani, Grunow, and Gunther 2012; Vila-Parrish, Ivy, and King 2008). For instance, in the yogurt industry perishability is found at different stages of the production process: from the highly perishable raw-material (milk) that enters the dairy factories to the finished products, which are all stamped with a best-before-date (Entrup et al. 2005).

As mentioned by Amorim et al. (2011), perishability and deterioration enforce specific constraints on a set of crucial production planning decisions, specially in the case of multi-level production structures where two or more items are produced and at least one item is required as an input

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(raw-material, component) of another. These intermediate products are often inventoried, allowing one to produce and consume them at different moments and rates in time (Pochet 2001). When dealing with perishability, most of the data associated with inventories has to be extended to trace the age and usability status of items with specific time-stamps. Besides the amount of inventory kept in stock we also need to know when the material has been acquired and to what level it has deteriorated, as well as the impact that such deterioration may have in the production process. Moreover, production planning decisions determine the size and timing of production lots or batches and therefore the frequency of setups. Meanwhile, setups affect lead times of items waiting in line to be processed which consequently increases deterioration. To reach acceptable quality levels and/or production yields a deteriorated material may consume more resources that would otherwise be available for production and therefore it may also have an effect on waiting times.

If a perishable item reaches the end of its *shelf-life* and becomes unsuitable for use, it may have to be discarded. Thus, besides the obvious waste of valuable resources and the negative impact it may have on the quality of the finished products, this aspect causes additional costs, as disposed material may need to be transported to a certain disposal site and incur also a treatment cost.

To the best of our knowledge, raw-material (and/or intermediate products) perishability and the way it affects the production of higher level items in the product structure have not been studied extensively. The main contribution of this paper is to study how raw-material perishability can be incorporated into classical lot-sizing problems and its impact in production processes regarding: manufacturing, inventory, disposal costs, and quality of the finished products. The initial motivation for this study comes from the area of composite manufacturing and related industries, in the production of fiber reinforced polymer composites and other applications. These industries use fibers, such as glass or graphite, impregnated with polyimide monomeric reactants, and other material that are subject to limited shelf-life, very sensitive to premature aging, and affect the production process in several ways (see Alston and Gahn 2000, and Alston and Scheiman 1999, for reports on this type of applications). However, once these materials are used for production they become stable and no longer deteriorate. We study four variants of a two-level lot-sizing problem involving different types of perishability. We present mixed integer programming (MIP) formulations to model these problems. We perform a series of computational experiments to evaluate the proposed models and formulations when used with a general purpose solver. We also analyze the impact of key parameters in the structure of optimal solutions.

The paper is organized as follows: In Section 2 we review different characteristics considered when dealing with product perishability and deterioration. Afterwards, we review the most relevant modeling approaches for integrating perishability in production planning and related problems. In Section 3 we present the four considered problem variants: *fixed shelf-life, functionality deterioration, volume deterioration, and functionality-volume deterioration.* In Section 4 we show computational results for all the problem variants, as well as the analyses carried out on them. Finally, Section 5 brings final conclusions and future research directions.

2. Characteristics of perishability and modeling approaches

A general definition of perishability presented by Wee (1993) describes it as the decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of an item that results in decreasing usefulness from the original one. As mentioned by Pahl and Voß (2014), most authors working in this field use deterioration, perishability, and depreciation interchangeably. In general, all perishable goods are items with a fixed maximum lifetime, usually referred to as *shelf-life*. Shelf-life is the maximum length of time during which a product is considered of satisfactory quality and can be stored under specified (or expected) conditions, remaining suitable for use, consumption or for its intended functions. Shelf-life is usually considered from the moment the product is produced or acquired.

2.1 Characteristics and classification of perishability

We can distinguish three main viewpoints to characterize and classify product perishability: (a) the utility or functionality of the product, (b) the physical state of the product, and (c) the mathematical modeling point of view of perishability.

When the interest is in the utility of the products (Raafat 1991; Pahl and Voß 2010), and based on the value of inventory as a function of time, perishability is classified in: (1) constant-utility: items undergo decay but face no considerable decrease in value (prescription drugs), (2) decreasingutility: items lose functional value throughout their shelf-life (milk, fruits and vegetables), and (3) increasing-utility: items increase in value (wines, cheese, antiques).

Similarly, but with an interest in how the customer perceives the functional value of items (Ferguson and Koenigsberg 2007), we can make two distinctions: (1) items whose functionality deteriorates over time (fresh produce), and (2) items whose functionality does not degrade, but the utility perceived by the customers does (fashionable clothing and high-technology products).

The emphasis on the physical state of the product is found in early inventory control papers. Ghare and Schrader (1963) characterize perishability according to the type of deterioration: (1) direct spoilage (fresh food), (2) physical depletion (gasoline and alcohol), and (3) decay and obsolescence (newspapers). With a similar interest, Lin, Kroll, and Lin (2006) take into account age-related characteristics and distinguish between: (1) age-dependent deterioration (milk, fruits), and (2) age-independent deterioration (volatile liquids, radioactive and other chemicals).

The third perspective refers the treatment of perishability from a purely modeling viewpoint. That is, the interest is not in the origin of the perishable nature of the products, but only in aspects necessary for the mathematical formulation of the problems. In this regard, Nahmias (1982) divides perishable products as follows: (1) with fixed shelf-life: cases where the shelf-life is known *a priori*, and (2) with random shelf-life: cases where the product shelf-life is a random variable with a specified probability distribution.

One of the most complete classification of perishability is the one proposed by Amorim et al. (2011), considering three classifying dimensions: (1) physical product deterioration: reflects if the item is suffering physical modifications or not, (2) authority limits: represents the external regulations or other conventions that influence perishability, and (3) customer value: reflects the customer willingness to pay for a certain good.

2.2 Modeling approaches to perishability

We can distinguish two different approaches in which perishability and deterioration have been taken into account in the fields of inventory control, scheduling, and production and distribution planning. The first approach assumes a loss of a portion of inventory, determined by a fixed input parameter (shrinkage factor). In this regard, Hsu (2000) presents an uncapacitated, single-item, lotsizing problem (LS) using a deterioration rate factor and considering age-dependent inventory costs. The model is later generalized to include back-ordering (Hsu 2003) and capacities (Waterer 2007). Using a similar deteriorating coefficient, Chen and Chen (2006) integrate LS and scheduling for a perishable item in a problem maximizing revenue, where demand and production depend on the selling price. Other studies within this first approach but for Economic Production Quantity (EPQ) or inventory control and replenishment type settings, normally considering infinite time horizons for a single-item, are the following: Balkhi and Benkherouf (1996) and Yang and Cheng (1998), who assume a continuous and constant deterioration rate as a time-dependent function, as well as a constant demand rate; Skouri and Papachristos (2003), who later extends Yang and Cheng (1998)'s work for the case of time-dependent back-ordering; and Goyal and Giri (2003), who based their study on Balkhi and Benkherouf (1996)'s approach considering time-varying demand. Similarly, Balkhi (2003) also considers time-varying demand in addition to the effect in deterioration of the so-called "learning phenomenon" (introduced by Wright 1936), which implies an improvement in the production rate and/or a cost reduction through a repetitive learning process. Moreover, Alamri and Balkhi (2007) study the effects of learning-and-forgetting: the system is subject to learning in the production stage and to forgetting while production ceased, so that the optimal quantity is dependent on the instantaneous production rate. Lin, Kroll, and Lin (2006) present a multiitem production-inventory problem with exponential deterioration rates and a constant inventory shrinkage factor. Other studies in this area of production-inventory and production-distribution systems are the ones by Tadj, Bounkhel, and Benhadid (2006), Manna and Chaudhuri (2006), and Belo-Filho, Amorim, and Almada-Lobo (2015). Li, Zhang, and Tang (2014) study dynamic pricing and inventory control policies for perishable products with stochastic disturbance.

For reviews of available literature regarding perishable goods in the context of inventory management see Goyal and Giri (2001) and Li, Lan, and Mawhinney (2010). Amorim et al. (2011) and Pahl and Voß (2014) present reviews on perishability in production-distribution and supply chain planning.

The second approach avoids inventory expiration by limiting the number of periods of demand that can be produced and thus, ensures that products do not reach the end of their shelf-life. Entrup et al. (2005) develop MIP models that integrate this type of approach into production-scheduling problems for an industrial case study of yogurt production with shelf-life-dependent selling price. Their objective is to maximize the contribution margin and they use a block planning approach, where a block is formed from all product variants based on a same recipe. In the same application area, Amorim, Antunes, and Almada-Lobo (2011) propose two multi-objective LS and scheduling MIP models for a pure make-to-order environment, and for a hybrid make-to-order/make-to-stock scenario. The authors use the same block planning approach as Entrup et al. (2005), and incorporate the maximization of product freshness as a problem objective. Pahl and Voß (2010) extend this approach without restricting the considered time periods. They allow inventory expiration to then penalize it by applying a disposal cost and it is integrated into well known discrete lot-sizing and scheduling problems. Pahl, Voß, and Woodruff (2011) later extend this approach to consider sequence-dependent setup times and costs.

Other approaches that are relevant to our study are: Abad (2000), who present a constrained non-linear program for a *LS* for a perishable good with exponential decay, partial back-ordering and lost sales and Teunter and Flapper (2003), who consider a stochastic EPQ model where produced units of a single product may be non-defective, reworkable-defective, or non-reworkable-defective. In industries such as the food or pharmaceutical, perishable reworkable-defective products are quite common. Another approach involves the integration of traceability for an EPQ problem (Wang, Li, and O'Brien 2009). Traceability is the ability to trace and follow the product through all stages of production, processing and distribution, which is a highly important operations management function in the food industry.

When integrating perishability into the well-known *economic lot scheduling problem*, most of the literature is limited to the addition of a shelf-life constraint (Amorim et al. 2011). Soman, van Donk, and Gaalman (2004) present a review of available literature, which is later complemented by Pahl and Voß (2014).

Finally, another approach used to model product perishability constraints is the use of the socalled production time-windows, in which demand has to be satisfied from products manufactured within a given time interval (Wolsey 2005). Chiang et al. (2009) study a production-distribution problem applied to the newspaper industry, where daily product shelf-life is typically only a few hours. The authors present a simulation-optimization framework and formulate the problem as an extension of the vehicle routing problem with time-windows. For the case of highly perishable food products, Chen, Hsueh, and Chang (2009) study a production-scheduling and vehicle routing problem with time-windows under stochastic demands. Here, the revenue of the supplier depends on the value of the finished items, which starts to decay once they are produced, as well as on the time-windows fulfillment set by the retailers.

The studies presented above constitute the state of the art for the treatment of perishability and deterioration in production planning and other related problems. However, there is an aspect that, to the best of our knowledge, has not been considered in this field and is the way in which raw-

material (and/or intermediate products) perishability and deterioration may affect the production of higher-level items in the product structure for multi-level systems. The work of Cai et al. (2008) and Billaut (2011), although it does not refers to the problems we study, as they are focused on scheduling settings, is still relevant in the sense that they study raw-material perishability. The former refers to a specific application in the seafood industry, and the latter discusses various different aspect to consider when dealing with perishable inventory in operational decisions.

3. Lot-sizing with perishable raw material

We next consider a production system in which one item (finished product) is to be produced and another item (raw-material), required as an input of the first, is to be procured from a supplier. This constitutes the simplest version of a two-level production structure. The *two-level lot-sizing problem* (2LS) consists of finding the production, procurement, and inventory plans for the two items over a discretized planning horizon (divided into *n* time periods, where $T = \{1, ..., n\}$) so as to meet finished item demand in every period, while minimizing the corresponding costs. As mentioned above, the core aspect of the problems under study is the perishable condition of the raw-material. In particular, we study four different variants of the 2LS problem: (a) raw-material subject to fixed shelf-life (FS); (b) raw-material subject to functionality deterioration rate (FD); (c) rawmaterial subject to volume deterioration rate (VD); and (c) raw-material subject to functionality and volume deterioration rate (FVD).

3.1 Fixed Shelf-Life

The first considered variant is the two-level lot-sizing problem with fixed raw-material shelf-life (2LS-FS). Raw-material orders are acquired in batches under the immediate receipt assumption (no ordering lead-time). Associated with each order there are unit batch costs and fixed orderplacement costs. Once the raw-material is received, it is used to satisfy finished item production requirements or can be inventoried. However, on-hand raw-material can only be kept in stock and used for a predetermined period of time (shelf-life). If the material reaches the end of its shelf-life and expires, it will have to be disposed. This causes additional costs that vary depending on when the disposal is made. Raw-material functionality is considered constant during the entire shelf-life period. Finished-item production is subject to process capacity, as well as to per-unit basic production costs and fixed setup costs. Production requirements are derived from the forecast demand and must be satisfied in every period (back-ordering not allowed). The 2LS-FS consists of planning the production levels and raw-material orders batches for each time-period, as well as planning the inventory levels so as to minimize the total production, setup, order-placement, inventory, and raw-material disposal costs.

Applications of 2LS-FS may arise in the production of plastic films. A plastic film is a thin continuous polymeric material used to separate areas or volumes, to act as barriers, or as printable surfaces (Hawkins 2002). Depending on the properties of the desired application, plastic films can be made from a variety of plastic resins and monomers, which are highly reactive and undergo uncontrolled polymerization. As raw-material for plastic films, these resins and monomers are considered fully functional during their shelf-life and, once they are used in production, they become stable. Furthermore, even though the finished product may present sings of deterioration during the storage and handling period, they have long enough shelf-lives not to be considered perishable.

In order to formulate the 2LS-FS we propose the following notation:

- d_t forecast demand
- *a* constant production rate
- B_t amount of capacity C_t consumed per setup
- C_t available machine (or process) capacity
- m_t upper bound in production quantity
- p_t basic unit production cost per period t
- q_t fixed setup cost per period t
- h_t unit storage cost per period t
- b fixed order batch-size
- K_t variable upper ordering limit for $t \in T$

- Llower ordering limit (if a raw-material order is placed in any period t, at most K_t and at least L batches have to be ordered)
- raw-material shelf-life (maximum number of β periods material can be stored)
- fixed order-placement cost per period t (cost ρ_t of placing a raw-material order)
- bill of material (amount of raw-material units rrequired to produce each finished item)

In terms of decision variables, we have the following:

- Q_t : raw-material order-size (number of batches to order in period t)
- w_{ut} : amount of raw-material, received at period $u \in T$, used to satisfy production requirements in period t, for $1 \le u \le t \le n$, as long as $(t-u) < \beta$

- ζ_t unit batch cost per period t
- raw-material unit storage cost per period t γ_t
- raw-material unit disposal cost per period t ϕ_t
- last period in which raw-material (received at Θ_t period t) can be used for production ($\Theta_t =$ $\min\{\theta_t, n\}, \text{ where } \theta_t = t + \beta - 1)$
- Π_t earliest period in which material can be acquired and still be used for production in period t ($\Pi_t = \max\{1, \pi_t\}$, where $\pi_t = t - \beta + 1$)
- e_t : amount of expired (perished) raw-material, received at period t, to be discarded
- finished item stock at the end of period t s_t :
- y_t : binary setup variable
- z_t : binary order-placement variable (equal to 1 if a raw-material order is placed in period t, and 0 otherwise)

Note that the condition $(t-u) < \beta$ for w_{ut} implies that material received at the beginning of period $u \in T$ can only be used for production during β periods of time (including period u). Hence, the last period in which material received at period t can be used for production is given by $\Theta_u = \min\{\theta_t, n\}$, where $\theta_t = t + \beta - 1$. Similarly, let $\Pi_t = \max\{1, \pi_t\}$, where $\pi_t = t - \beta + 1$ denote the earliest period in which material can be acquired and still be used for production in period t. We further assume that, even though material received during period $u : \beta < u \leq n$ does not expire during the planning horizon n, if it is not used, it is also discarded.

Using the above decision variables, the 2LS-FS can be formulated as follows:

minimize
$$\sum_{t=1}^{n} \left[\mathcal{P}(w,t) + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right] + \sum_{t=1}^{n-1} \mathcal{Y}(w,t)$$
 (1)

subject to
$$bQ_u = \sum_{t=u}^{\Theta_u} w_{ut} + e_u \qquad u \in T$$
 (2)

$$s_{t-1} + \left(\frac{1}{r}\right) \sum_{u=\Pi_t}^t w_{ut} = d_t + s_t \qquad t \in T$$
(3)

$$\sum_{u=\Pi_t}^t w_{ut} \le rM_t y_t \qquad t \in T \tag{4}$$

$$\left(\frac{a}{r}\right)\sum_{u=\Pi_t}^t w_{ut} \le \left(C_t - B_t\right) y_t \qquad t \in T$$
(5)

$$Q_t \le K_t z_t \qquad \qquad t \in T \tag{6}$$

$$Q_t \ge L z_t \qquad \qquad t \in T \tag{7}$$

$$Q_u, s_t, w_{ut}, e_u \in \mathbb{N}^+ \qquad u, t \in T : u \le t$$
(8)

$$y_t, z_t \in \{0, 1\}$$
 $u, t \in T : u \le t$ (9)

$$s_0 = s_0^*.$$
 (10)

Here, the objective function (1) is conformed by: production cost $\mathcal{P}(w,t) = \frac{p_t}{r} (\sum_{u=\Pi_t}^t w_{ut})$, finished item inventory holding costs, setup costs, raw-material batch costs, raw-material inventory holding costs, where $\mathcal{Y}(w,t) = \gamma_t (\sum_{u=\Pi_t^{(2)}}^t \sum_{\tau=t+1}^{\Theta_{tu}^{(2)}} w_{u\tau})$, for $1 \leq t \leq n-1$, is the raw-material inventory holding cost function ($\Theta_{tu}^{(2)} = \min\{\theta_t, \theta_u, n\}$, and $\Pi_t^{(2)} = \max\{1, \pi_t + 1\}$), lost/perished raw-material disposal costs, and order-placement costs.

Constraints (2) state that the amount of raw-material entering the production system at each period u is equal to the amount of this material used to meet production requirements at subsequent periods $(\sum_{t=u}^{\Theta_u} w_{ut})$ plus the amount that is discarded because it is not used before its shelf-life. Constraints (3) represent finished item inventory balance, whereas constraints (4) are the production upper bound and setup enforcement constraints, where $M_t = \min\{m_t, \sum_{r=t}^n d_r\}$. Constraints (5) represent machine (or process) capacity C_t consumption, ensuring that there is enough capacity in period t to produce all batches of finished items and perform the setup operations. Constraints (6) and (7) are the variable upper and lower bound constraints for the amount of raw-material to order at each time-period. Constraints (8) and (9) are integrality constraints. Constraints (10) represent the assumption that initial finished item stock is fixed and known.

Property 1. When b = 1, there exists an optimal solution to 2LS-FS in which $e_u = 0 \,\forall u \in T$.

This property states that when it is possible to order any number of raw-material items, then it will never be optimal to dispose products, and thus, to incur in disposal costs, because it is always feasible to order exactly what is needed, i.e., $\sum_{u=1}^{n} Q_u = r \sum_{t=1}^{n} d_t$.

3.2 Functionality deterioration

The second variant is the two-level lot-sizing problem with raw-material functionality deterioration rate (2LS-FD). Here, in addition to having on-hand raw-material subject to a fixed shelf-life, its functionality decreases progressively as storage time passes. Thus, we introduce a raw-material functionality deterioration rate parameter, denoted by $\alpha_{ut} \in [0, 1]$, for $1 \leq u \leq t \leq n$, representing the fraction of decrease in the functionality level of material received in period u, to be used for production in period t. It is assumed that $\alpha_{ut} \geq \alpha_{jt}$ for $1 \leq u \leq j \leq t \leq n$, meaning that the longer an item is carried in stock, the greater its level of deterioration. It is also assumed that $\alpha_{tt} = 0 \ \forall t \in T$. Moreover, the shelf-life β parameter, although it remains fixed, it is determined by the number of periods it takes for α_{ut} to reach 1. Progressive decrease in material functionality has a direct impact on production. This impact may be, for instance, on the quality of the finished items produced with deteriorated material, or an increase in the production cost in order to reach the same desired quality or production yield, using material that is not fully functional. Either way, we represent such impact by an increase in the production cost as: $p_t^+ = p_t + \sum_{u=1}^{t-1} \alpha_{ut} w_{ut} p_t$, where p_t^+ is the production cost considering deteriorated raw-material. Figure 1 graphically shows an instance with a planning horizon $T = \{1, 2, 3\}$ with basic production cost p_3 for period t = 3. The following are the functionality deterioration rates for material used in production at period $t = 3: \alpha_{13} = 0.4, \ \alpha_{23} = 0.2.$



Production cost for t = 3: $p_3(1 + \alpha_{13}w_{13} + \alpha_{23}w_{23})$

Figure 1.: Functionality deterioration rates and production cost for period t = 3 when $\beta = 3$.

Furthermore, process capacity C_t consumption may also be affected by material deterioration. As expected, the use of any amount of material with, e.g., functionality level of 50% ($\alpha_{ut} = 0.5$) for production will consume a higher amount of capacity C_t for setup than any material with $\alpha_{ut} < 0.5$. This feature may arise in applications in which machines or processes must be reconfigured in order to be able to use deteriorated material and still achieve the same level of finished item quality or yield, as in the previously mentioned composite manufacturing and related industries, when producing polyimide reinforced fiber composites and other products. For this reason, the amount of capacity consumed per setup is now denoted by B_{ut} , to incorporate the variation depending on when the material is received (u), and when it is being used for production (t). Thus, for $1 \leq u \leq j \leq t \leq n$, $B_{ut} \geq B_{jt}$, meaning that the higher the deterioration of the material used for production, the more capacity C_t it requires for setup. To model this, we define an additional set of binary variables Z_{ut} which are equal to 1 if there is production in period t using raw-material units received in period u, and 0 otherwise.

The 2LS-FD can be formulated as follows:

minimize
$$\sum_{t=1}^{n} \left[\mathcal{P}^{+}(w,t) + h_{t}s_{t} + q_{t}y_{t} + \zeta_{t}Q_{t} + \phi_{t}e_{t} + \rho_{t}z_{t} \right] + \sum_{t=1}^{n-1} \mathcal{Y}(w,t)$$
(11)

subject to
$$(2) - (4), (6) - (11)$$

 $w_{ut} \le r M_t Z_{ut}$
 $u, t \in T : 0 \le (t - u) < \beta$ (12)

$$\left(\frac{a}{r}\right)\sum_{u=\Pi_t}^t w_{ut} + \Gamma \sum_{u=\Pi_t}^{t-1} \Delta_{ut} Z_{ut} \le (C_t - B_t) y_t \quad t \in T$$
(13)

$$Z_{ut} \in \{0, 1\} \qquad u, t \in T : u \le t, \tag{14}$$

where $\Gamma \geq 0$ is the coefficient for production cost and capacity consumption depending on the functionality deterioration.

A setup here is considered to be the realization of all the operations required to reconfigure the production process at the end of a batch in period t, in order to reach the required level of finished item quality using deteriorated raw-material. Thus, constraints (12) - (13), ensure that there is enough capacity in period t to produce all batches and perform the reconfiguration operations when deteriorated material is used for production. Production cost in the objective function (11) is represented by the following function: $\mathcal{P}^+(w,t) = \frac{p_t}{r} (\sum_{u=\Pi_t}^t w_{ut} + \Gamma \sum_{u=\Pi_t}^{t-1} \alpha_{ut} w_{ut})$. We note that when $\Gamma = 0$, the 2LS-FD reduces to the 2LS-FS.

3.3 Volume deterioration

We next present the two-level lot-sizing problem with raw-material volume deterioration rate $(2LS \cdot VD)$. Here, the perishability nature of the raw-material refers to a progressive volume loss. In this sense, we have a volume deterioration rate, denoted by $\delta_{ut} \in [0,1]$, for $1 \leq u \leq t \leq n$, representing the fraction of the material acquired in period u, which is lost at the end of period t. We assume that $\delta_{ut} \geq \delta_{jt}$ for $1 \leq u \leq j \leq t \leq n$, meaning that the longer the material is carried in stock, the faster it deteriorates and, consequently, the higher the proportion of lost material. Here the shelf-life β is determined by the number of periods it takes for δ_{ut} to reach 1. A new set of inventory variables c_{ut} for $1 \leq u \leq t \leq n$ is introduced as the raw-material stock at the end of t and received at u.

Applications of this problem can be found in production processes arising in the canning industry, such as: canning fruits, vegetables, seafood, and meats, among others. The primary objective of food processing is the preservation of highly perishable goods in a stable form that can be stored and shipped to distant markets. However, it is normal to face considerable levels of raw-material loss throughout the multiple steps of the production process, which includes preliminary preparation, blanching, and filling (Melrose Chemicals Ltd., 2005).

The 2LS-VD problem can be formulated as follows:

minimize
$$\sum_{t \in T} \left[\frac{p_t}{r} \left(\sum_{u = \Pi_t}^t w_{ut} \right) + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right] + \sum_{t = 1}^{n-1} \mathcal{Y}(w, t)$$

subject to (3) - (11)

$$c_{tt} = (bQ_t - w_{tt})(1 - \delta_{tt}) \qquad t \in T$$
(15)

$$c_{ut} = (c_{u,t-1} - w_{ut}) (1 - \delta_{ut}) \qquad u, t \in T : 0 < (t-u) < \beta$$
(16)

$$e_{tt} = (bQ_t - w_{tt})(\delta_{tt}) \qquad t \in T$$
(17)

$$e_{ut} = c_{u,t-1} - w_{ut} \qquad u, t \in T : 0 < (t-u) < \beta$$
(18)

$$c_{ut} \in \mathbb{N} \qquad \qquad u, t \in T : u \le t, \tag{19}$$

where constraints (15) - (16) represent raw-material inventory levels and constraints (17) - (18) represent raw-material disposal.

3.4 Functionality and volume deterioration

We now propose the two-level lot-sizing problem with raw-material functionality and volume deterioration (2LS-FVD), which generalizes all the previous problem variants. Here, the perishability nature of the raw-material is biphasic, referring not only to a functionality loss but, in addition, to a progressive volume loss.

Several industries and processes, including some of the cases previously mentioned, under certain conditions, can also constitute scenarios where this problem finds applications. For instance, in many beverage and food related industries, the perishable nature of the materials may have multiple sources and consequences. In some cases, material waste may be caused by packaging, handling and/or storing conditions, resulting in material loss due to evaporation, damage, or spillage. In addition, the intrinsic perishable characteristics of the material may also be a source of deterioration, resulting in a decrease of freshness or usefulness. In this way, in order to minimize raw-material waste from several possible sources, these industries need an integrated approach to optimize the conditions associated with loss of physical quantities of material, as well as those related to functionality deterioration of such material.

The 2LS-FVD can be formulates as follows:

minimize
$$\sum_{t \in T} \left[\mathcal{P}^+(w,t) + h_t s_t + q_t y_t + \zeta_t Q_t + \phi_t e_t + \rho_t z_t \right] + \sum_{t=1}^{n-1} \mathcal{Y}(w,t)$$
subject to (3) - (4), (6) - (11), (13) - (15), (17) - (21).

Note that, when $\Gamma = 0$, the 2LS-FVD reduces to the 2LS-VD. Moreover, if $\delta_{ut} = 0$ for $1 \le u \le t \le n$, the 2LS-FVD reduces to the 2LS-FD.

4. Computational experiments and analyses

In this section we present the results and analyses of extensive computational experimentation performed to gain a thorough understanding of the considered problems and their solutions, and to evaluate our MIP formulations when solved with a general purpose solver. In Section 4.1, we first show the complexity of the problems in terms of computational performance. In addition, we analyze the added value of the proposed formulations in contrast with two algorithms developed from the solution obtained with a traditional lot-sizing model. In Section 4.2, we then present an analysis of the way in which key parameters impact the structure of optimal solutions and therefore, the production planning decisions.

4.1 Computational performance of MIP formulations

In order to elucidate the added value of the proposed models, we perform the following analysis. For each of the four 2LS variants (FS, FD, VD, and FVD), we start by evaluating the feasibility of the optimal solution obtained by solving the standard 2LS (i.e., assuming no raw-material perishability or deterioration). Subsequently, based on this initial solution, we implement two different algorithms to find feasible and possibly improved solutions for the original problem variants. We then compare the deviations of the solutions obtained with these algorithms with respect to the actual optimal solution obtained with our MIP formulations. This deviation is computed as $\% dev = (SOL_i - OPT)/OPT \times 100$, where SOL_i denotes the solution value obtained with algorithm *i* and OPT the optimal solution value.

Fixing-orders (f.o.): The first algorithm fixes the ordering decisions Q_t obtained in the initial standard 2LS solution and optimizes the subproblem associated with the production decisions.

Fixing-production (f.p.): Contrary to the f.o. algorithm, the second approach fixes the production decisions obtained in the 2LS solution and optimizes the subproblem associated with the ordering and w_{ut} decisions.

Using a randomly generated instance with n = 6, $\beta = 2$, and b = 15 as an example, Figure 2 shows the raw-material inventory levels for the different solutions when solving it as a 2LS-FD.



Figure 2.: A comparison of solutions for the 2LS-FD

As shown in Figure 2(a), the standard 2LS gives a solution with raw-material orders at periods 1 and 4. There is a strictly positive production at each of the six time periods, which corresponds to the following lot sizes: $X_1 = 28$, $X_2 = 16$, $X_3 = 27$, $X_4 = 18$, $X_5 = 11$, and $X_6 = 10$, with no finished item inventory at any period (X_t denotes the finished item lot size at period t). However, the standard 2LS solution is infeasible for the 2LS-FDR variant, since it uses $w_{13} = 81$, $w_{46} = 18$, and $w_{16} = 12$ units of raw-material for production that are in fact lost/perished and have to be disposed, given that $\beta = 2$. Based on this initial solution, the *f.o.* and *f.p.* algorithms are used to find feasible solutions for the 2LS-FDR.

As shown in Figure 2(b), the *f.o.* approach results in a modification of the w_{ut} decisions, which are the cause of infeasibility. Production now occurs only in $t \in \{1, 2, 4, 5\}$, which corresponds to

the following lot sizes: $X_1 = 28$, $X_2 = 47$, $X_4 = 14$, $X_5 = 21$; and to the following finished item inventory levels: $s_2 = 31$, $s_3 = 4$, and $s_5 = 10$. This results in an objective value with a 8.4% deviation from the optimal solution of the problem, which is shown in Figure 2(d).

On the other hand, Figure 2(c) shows the solution with the *f.p.* approach, resulting in a modification of the orders, now received at $t \in \{1, 3, 5\}$. In addition, some w_{ut} decisions also changed, resulting in a total $e_1 + e_5 = 15$ raw-material units lost/perished and disposed, but in a reduced objective value with a 2.3% deviation.

Finally, the optimal solution for the 2LS-FD instance obtained with our MIP formulation, as shown in Figure 2(d), gives a solution with orders at $t \in \{1, 3, 4\}$; and strictly positive production at $t \in \{1, 2, 3, 4, 5\}$, which corresponds to the following lot sizes: $X_1 = 28$, $X_2 = 17$, $X_3 = 26$, $X_4 = 18$, and $X_5 = 21$.

Considering the same instance, the procedure is used for the 2LS-FVD variant. Figure 3(a) shows the same initial 2LS solution. Figures 3(b) and 3(c) show the solutions obtained by implementing the *f.o.* and *f.p.* algorithms, respectively, which result in solution values with 11.6% and 2.9% deviation.



Figure 3.: Solutions for the 6x2x15x3x5 2LS-FVD problem instance

We have implemented this same procedure for a set of 136 problem instances, which are divided into four main different groups: (1) Uncapacitated 2LS-FS and 2LS-VD (-U), (2) 2LS-FS and 2LS-VD with Constant Capacities (-CC), (3) Uncapacitated 2LS-FD and 2LS-FVD (-U), and (4) 2LS-FD and 2LS-FVD with Constant Capacities (-CC). The average computational results for each of the four group instances are presented in Tables 1 to 4.

Each of these sets consists of three subsets of eight instances with $n \in \{6, 10, 16\}$ and two subsets of five instances with $n \in \{24, 30\}$. Moreover, within each subset, we have evaluated instances with $\beta \in \{2, 3, 4, 6, 8, 9, 12\}$, as well as different values for the b, r, and L. For each of the resulting tables, the first column represents the instance size depending on the number of periods in the planning horizon. The second and third columns show, respectively, the amount of infeasible solutions obtained when considering the instance to be a standard 2LS, and the averaged deviation from the optimal solution of the feasible ones. The fourth and fifth columns show the average deviation from the optimal solution of the problem when implementing the f.o. and f.p. algorithms, respectively. The next five columns are all regarding the optimal solution of the corresponding problem variant when solved with our proposed MIP formulations, in the following order: number of instances that were solved to optimality, averaged optimality gap for the instances that were not solved optimally, number of branch and bound nodes explored, CPU time (in seconds), and linear programming gap (LP Gap %). Furthermore, in the right-hand half of each table, the first four columns show the number of infeasible solutions and the deviation from the optimal solution of the problem when implementing the f.o and f.p. algorithms. Finally, the last five columns of each table refer to the optimal solution of the corresponding problem variant, in the same manner as in the left-hand half of the table.

All computational experiments were implemented using the Callable Library of IBM CPLEX 12.6.2 on an Intel(R) Xeon(R) CPU E3-1270 v3 processor with 3.50GHz and 24GB of RAM memory under Microsoft Windows 7 Enterprise operative system. The instances that CPLEX was not able to solve to optimality were due to memory limitations.

	2LS-FS-U										2LS-VD-U								
	2LS	adpt.	f.o.	f.p.	Opti	mal	Node	Time	LPGap	J	.0.	J	f.p.	Optii	nal	Node	Time	LPGap	
n	inf.	% dev	%dev	%dev	count	(%)	count	(sec)	(%)	inf.	% dev	inf.	% dev	count	(%)	count	(sec)	(%)	
6	6/8	0.0	1.3	2.3	8/8	-	156	0.07	11.3	5/8	8.2	5/8	2.4	8/8	-	1,693	0.06	17.9	
10	4/8	0.0	1.3	0.3	8/8	-	1,041	0.16	14.6	6/8	8.7	6/8	1.5	8/8	-	28,606	0.69	19.4	
16	3/8	0.0	1.4	0.4	8/8	-	19,679	0.49	15.1	6/8	15.1	7/8	3.1	7/8	7.0	32,432,807	2,919.60	19.8	
24	3/5	0.0	0.2	0.1	5/5	-	17,117,360	570.10	16.6	3/5	8.6	4/5	3.2	3/5	9.5	108,047,800	10,519.70	23.2	
30	3/5	0.0	0.5	0.1	3/5	0.9	288,043,986	29,970.60	17.5	2/5	12.6	4/5	3.3	1/5	9.9	$116,\!859,\!144$	14,775.70	24.1	

Table 1.: Results for 2LS-FS-U and 2LS-VD-U problem instances

Table 2.: Results for <i>2LS-FS-CC</i> and <i>2LS-V</i>	VD-CC	problem	instances
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	2LS-FS-CC									2LS-VD-CC								
	2LS	adpt.	f.o.	<i>f.p.</i>	Opti	mal	Node	Time	LPGap	j	.0.	J	f.p.	Opti	mal	Node	Time	LPGap
n	inf.	% dev	%dev	%dev	count	(%)	count	(sec)	(%)	inf.	% dev	inf.	% dev	count	(%)	count	(sec)	(%)
6	6/8	0.1	1.3	2.1	8/8	0.0	176	0.03	13.4	7/8	3.3	4/8	1.8	7/8	-	796	0.08	16.5
10	5/8	0.1	1.0	0.3	8/8	0.0	1,944	0.08	13.8	8/8	-	6/8	1.4	8/8	0.0	40,884	1.05	22.3
16	6/8	0.1	0.8	0.3	8/8	0.0	22,503	0.56	12.8	8/8	-	7/8	2.7	7/8	9.9	109,998,863	7,459.40	24.3
24	2/5	0.0	0.1	0.1	5/5	0.0	16,206,814	736.96	12.3	5/5	-	4/5	2.1	1/5	6.0	$615,\!170,\!135$	24,211.30	16.3
30	4/5	0.0	0.5	0.1	5/5	0.0	$91,\!452,\!139$	4,744.00	12.8	4/5	7.1	4/5	1.9	1/5	9.5	247,782,238	$21,\!377.10$	16.5

Observing the results given in Tables 1 and 2, we note that, for the most basic variant (2LS-FS) the standard 2LS solution is often infeasible. However, when feasible, its deviation from the optimal solution is rather small. Moreover, the *f.o.* and *f.p.* algorithms always reach feasible solutions for the 2LS-FS, with average deviations ranging from 0.5% to 1.4%, and from 0.1% to 2.3%, respectively. On the contrary, both algorithms do not reach feasibility for every instance when solved as a 2LS-VD or as a 2LS-FVD. For the majority of the instances, specially for those with longer planning horizons, the *f.p.* algorithm seems to have a better performance. This is somehow expected as the standard 2LS ignores the disposal and the added production costs when using deteriorated raw-material, and focuses on the setup and ordering costs.

Table 3.: Results for 2LS-FD-U and 2LS-FVD-U problem instances

	2LS-FD-U										2LS-FVD-U								
	2LS adpt. f.		f.o.	f.p.	Optimal		Node	Time	LPGap	f.o.		<i>f.p.</i>		Opti	nal	Node	Time	LPGap	
n	inf.	% dev	%dev	%dev	count	(%)	count	(sec)	(%)	inf.	% dev	inf.	% dev	count	(%)	count	(sec)	(%)	
6	6/8	3.4	4.1	1.7	8/8	0.0	137	0.03	12.5	5/8	8.2	5/8	2.5	8/8	0.0	1,609	0.04	17.8	
10	4/8	2.4	2.9	0.8	8/8	0.0	1,121	0.05	15.7	6/8	8.7	6/8	1.5	8/8	0.0	14,278	0.37	19.3	
16	6/8	2.8	4.0	0.4	8/8	0.0	51,607	1.33	16.7	6/8	15.3	7/8	3.4	8/8	0.0	$16,\!681,\!906$	1,009.02	19.7	
24	4/5	1.3	1.9	0.5	5/5	0.0	50,411,588	2,870.64	18.2	3/5	8.6	4/5	3.5	4/5	7.9	52,271,838	4,998.75	22.6	
30	5/5	-	3.1	0.7	1/5	4.7	$118,\!332,\!468$	$14,\!557.80$	19.4	2/5	12.6	4/5	3.5	2/5	6.6	$118,\!686,\!343$	17,733.13	22.2	

Table 4.: Results for 2LS-FD-CC and 2LS-FVD-CC problem instances

[2LS-FD-CC									2LS-FVD-CC								
	2LS adpt. f.o.		f.o.	<i>f.p.</i>	Optimal		Node	Time	LPGap	f.o.		<i>f.p.</i>		Opti	mal	Node	Time	LPGap
n	inf.	$\% \mathrm{dev}$	%dev	%dev	count	(%)	count	(sec)	(%)	inf.	% dev	inf.	% dev	count	(%)	count	(sec)	(%)
6	6/8	1.0	3.0	1.2	8/8	0.0	154	0.07	15.8	7/8	3.3	4/8	1.9	7/8	-	498	0.05	17.3
10	5/8	3.1	2.2	0.8	8/8	0.0	1,245	0.04	15.9	8/8	-	6/8	1.5	8/8	0.0	36,052	0.94	24.1
16	6/8	1.2	3.8	0.3	8/8	0.0	81,886	2.15	15.0	8/8	-	7/8	3.3	7/8	8.9	$38,\!152,\!752$	2,947.40	18.8
24	2/5	1.8	1.5	0.4	5/5	0.0	30,761,622	$1,\!435.32$	14.3	5/5	-	4/5	2.2	5/5	0.0	52,728,983	4,287.57	16.3
30	4/5	81.7	12.3	1.1	3/5	2.3	93,440,018	$9,\!472.11$	15.4	4/5	7.0	3/5	1.4	2/5	6.4	93,201,304	10,062.55	16.4

Table 5 presents the computational results for instances that showed higher complexity in terms of computational time and number of branch and bound nodes explored (i.e. with n = 24 and n = 30). The instances are now organized in terms of the shelf-life β values. We can observe how there is no clear trend in these results. For instance, if we focus on the 2LS-FS-CC version, we see that there is an increase in the number of nodes and time as β also increases. However, if we focus on the 2LS-FD and 2LS-FVD versions, we see a decrease in the number of nodes and time as β increases.

Shefl-life	Opt.	Node	Time	LPGap	Opt.	Node	Time	LPGap			
β	count	count	(sec)	(%)	count	count	(sec)	(%)			
	2LS-F	S-U			2LS-VD-U						
4	3/4	129,803,247	12,493.84	17.2	3/4	128,076,280	11,542.77	20.1			
8	4/4	182, 113, 641	18,598.78	16.7	2/4	144,693,916	19,181.14	22.4			
12	2/4	139,069,590	14,166.60	17.5	4/4	16,726,967	1,790.60	33.4			
	2LS-F	S-CC			2LS-V	D-CC					
4	4/4	40,845,010	1,676.29	11.8	2/4	204,447,510	18,324.86	15.4			
8	4/4	$47,\!595,\!288$	2,600.39	12.4	4/4	$172,\!436,\!936$	21,807.73	18.4			
12	4/4 92,266,789		5,147.90	14.4	4/4	403,612,040	93,867.90	14.2			
	2LS-F	D-U			2LS-FVD-U						
4	2/4	99,491,544	10,068.21	19.0	3/4	112,949,084	8,844.56	19.9			
8	2/4	76,495,389	8,403.63	18.4	2/4	98,495,104	14,365.35	21.6			
12	4/4	69,886,275	6,627.45	19.3	4/4	4,507,079	409.89	28.8			
	2LS-F	D-CC			2LS-FVD-CC						
4	3/4	94,335,621	8,206.19	13.8	4/4	48,684,928	2,893.02	15.1			
8	3/4	51,330,883	5,114.20	14.9	2/4	114,731,022	13,367.82	16.6			
12	4/4	$19,\!171,\!093$	1,061.86	16.8	2/4	$246,\!697,\!075$	$21,\!601.15$	18.1			

Table 5.: Results for all problem variants in terms of the shelf-life β

4.2 Analysis on the impact of input parameters

We now analyze the way in which different key parameters constrain and impact the planning decisions and the structure of optimal solutions. In addition to the particular parameters we have integrated into traditional lot-sizing to model perishability, such as shelf-life β , and functionality α_{ut} and volume deterioration δ_{ut} rates, other parameters of traditional use in combination with the above, markedly affect structure of optimal solutions. One aspect that has greater influence in this regard is the relation between the ordering batch-size b and the bill of material r. This relation determines the flexibility to manage raw-material inventories and, depending on β , to avoid disposing units and incurring in higher finished item inventory costs. The greater flexibility is found on problem instances with ordering batch-size b = 1 (see Property 1). Figure 4 shows the changes in the optimal solutions for a single problem instance with a planning horizon of n = 6 periods, and for various β and b values. In particular, Figure 4(a) shows a comparison of the optimal solutions for the 2LS-FD and 2LS-FVD values for nine different values of β , keeping all other parameters unchanged.



Figure 4.: A comparison of solutions for different values of β and b

As it can be observed, when solving the instance as a 2LS-FD, the optimal solution and the β values have an inversely proportional relation. This is due to the fact that a shorter shelf-life requires production to be done faster, which consequently leads to higher finished item inventory levels and to an important increase of inventory holding costs. On the other hand, a longer shelf-life allows for more flexibility to balance raw-material and finished item inventory costs, reducing the total solution value. In contrast, when solving the same instance as a 2LS-FVD, the optimal solution values are clearly higher, but present no significant variation with respect to the β values. This is due to a significant increase in raw-material disposal, given its continuous and incremental volume loss, as well as to larger and more frequent raw-material orders. Figure 4(b) shows the optimal solutions for the same instance with an additional variation on the ordering batch-size $b \in \{20, 30, 40, 50\}$. Although the effect that b has on the optimal solutions is linked to the bill of material r, we can observe that, for the case of the 2LS-FD, its increment also corresponds to an increment in the value of the objective function value.

5. Conclusions

In this paper, we studied how raw-material perishability considerations can be integrated into classical lot-sizing problems. We introduced four variants of the *two-level lot-sizing problem* with different types of raw-material perishability: (a) *fixed shelf-life*, (b) *functionality deterioration*, (c) *volume deterioration*, and (c) *functionality-volume deterioration*. We proposed MIP formulations for each of these variants. We pointed out the impact that perishability has in the production process of finished items regarding: manufacturing, inventory, disposal costs, capacity consumption, and quality. To the best of our knowledge, these aspects have not been previously studied in the literature.

Additionally, we studied two solution algorithms using a standard lot-sizing model as a basis, and performed computational experimentation and analyses to demonstrate the usefulness of our proposed models. The number of instances for which these algorithms failed to reach feasible solutions, and the solution deviations for problems with volume deterioration, demonstrate the advantages of the use of our models.

Considering the memory limitations that CPLEX had and the long computational times to solve an important portion of medium to large size problem instances, we are currently developing solution algorithms for efficiently solving particular variants of these problems.

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