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1: Intro, Strategic-form Games

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In this lecture, we introduce the notion of strategic interactions between decision makers. We consider one-shot games, where a finite number of players act simultaneously. Each player makes a single decision, everyone observes the outcome, and then game over.

What is strategic decision making?

> FATHER: Son, I caught you reading a teen girly magazine the other day? Why were you doing that?
>
> SON: I was reading an article called “Best places to meet men.” I just wanted to know where I’m supposed to be!

Example 0.1 (Student’s dilemma)**.** Two students in the class. Grades are relative.

	study	don’t study
study	B+, B+	A, C
don’t study	C, A	B, B

Example 0.2 (Generalized beauty contest[1])**.** Each student writes his or her name on the card and a number between 0 and 100; collect the cards and average the numbers on them. The student whose choice is closest to half of the average wins CAD 1.

1 Examples

Example 1.1 (Three voters, three candidates.)**.** The voter preferences and weights are:

Voter 1 (48%)	Voter 2 (47%)	Voter 3 (5%)
Donald	Hillary	Gary
Gary	Gary	Hillary
Hillary	Donald	Donald

Example 1.2 (Hotelling (1929))**.** Why are fuel station, pharmacies, restaurants, etc., all located next to each other? Why do election candidates have similar views? Consider an election with two candidates. Suppose that we parametrize their campaign opinions as a point in an interval $[0, 1]$: 0 means leftist, 1 means rightist. Each voter also has an opinion in the same interval. Suppose that voters vote for the candidate with the closest opinion and that votes are split equally if there are ties. Suppose that the voter opinions are uniformly distributed, what is the best strategy? Could Hillary have won[2]?

[1] `http://www.sscnet.ucla.edu/polisci/faculty/chwe/austen/dixit2005.pdf`

[2] `http://www.economist.com/news/united-states/21702805-anger-and-fickleness-voters-are-forcing-chang`

Example 1.3 (Advertising[3]). Two companies compete to provide internet service. Each currently makes 5 million each year. At the start of the year, they each have to decide whether to run an ad campaign or not. An ad campaign costs 2 million. If one company advertises, but the other does not, then the advertising company captures 3 million in revenu from the competitor.

	B advertises	B doesn't advertise
A advertises	3, 3	6, 2
A doesn't advertise	2, 6	5, 5

Example 1.4 (Pub duopoly). There are two pubs in town, serving two groups of customers: locals and tourists. Tourists visit the two pubs evenly and purchase 3000 pints of beer in each pub. Locals visit the pub with the cheapest beer, or the two pubs evenly if the price is the same. They account for 4000 pints. Should each pub charge \$2, \$4, or \$5 per pint?

	\$2	\$4	\$5
\$2	10, 10	14, 12	14, 15
\$4	12, 14	20, 20	28, 15
\$5	15, 14	15, 28	25, 25

Example 1.5 (Poker). There are a number of common player types:

- Calling station
- Nit
- Tight aggressive
- Loose aggressive

What should your strategy be?

Example 1.6 (Peer evaluation). Final project peer evaluation (among members of a group).

2 Strategic form

A game in strategic or normal form is composed of

1. A finite set of players $\{1, 2, \ldots, I\}$,
2. A set of strategy or action spaces $\{S_1, \ldots, S_I\}$, where S_i is the set of possible strategies of player i,
3. A set of payoff or utility functions $\{u_1, \ldots, u_I\}$, where $u_i : S_1 \times \ldots \times S_I \to \mathbb{R}$ is the utility function of player i.

Examples of strategy spaces:

[3] http://www.maa.org/sites/default/files/pdf/ebooks/GTE_sample.pdf

10.3

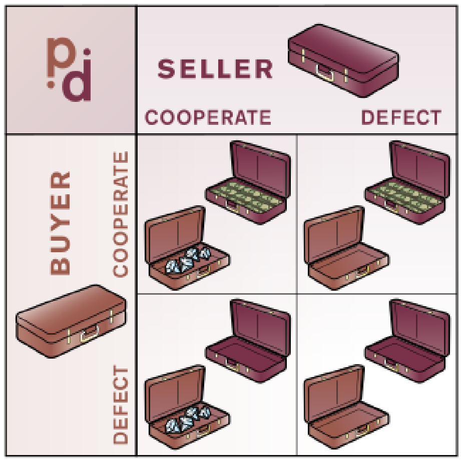

Figure 1: From Wikipedia.

- Advertising game: $\{\text{advertise}, \text{don't advertise}\}$,
- Competition game (Pub duopoly): prices $\{0.01;\ 0.01;\ 0.02;\ \ldots\}$,
- Hotelling model: set of real numbers $[0, 1]$.

An outcome or strategy profile is a vector $(s_1, s_2, \ldots, s_I) \in S_1 \times \ldots \times S_I$. A payoff function maps a strategy profile to a real number. Each player i chooses s_i and tries to maximize the value $u_i(s_i, s_{-i})$—we use the notation $-i$ to denote the set of players except i.

Remark 1 (Preferences). Do we really need real-valued functions $u_1, \ldots, u_I$ to define games? Preference relations are also OK (cf. Student's dilemma).

Example 2.1 (Uncomparable outcomes)**.** Two students have two exams the next day, but only time to study for one.

	Calculus	Linear Algebra
Calculus	(A,C), (A,C)	(A,C), (C,A)
Linear Algebra	(C,A), (A,C)	(C,A), (C,A)

What are other examples of uncomparable outcomes?

Definition 2.1 (Zero-sum)**.** A game is said to be zero-sum if for every fixed strategy profile $(s_1, \ldots, s_I)$, we have $\sum_{i=1}^{I} u_i(s_i, s_{-i}) = 0$.

3 Dominance

Example 3.1 (Prisoner's dilemma (Golden Balls))**.** Two people meet and exchange closed suitcases, with the understanding that one of them contains money, and the other contains items (drugs, diamonds, etc.).

	cooperate	defect
cooperate	1, 1	0, 2
defect	2, 0	0, 0

Example 3.2 (Iterated strict dominance)**.** Consider

	L	M	R
U	4, 3	5, 1	6, 2
M	2, 1	8, 4	3, 6
D	3, 0	9, 6	2, 8

dominated by R

	L	R
U	4, 3	6, 2
M	2, 1	3, 6
D	3, 0	2, 8

Player 1's M and D actions are likewise dominated.

	L	R
U	4, 3	6, 2

Finally, the only non-dominated outcome is (U,L).

Remark 2. Use iterated dominance to solve the Pub duopoly example.

Definition 3.1 (Dominance)**.** Consider a fixed player i, if there exist strategies $s_i \in S_i$ and $s_i' \in S_i$ such that

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \quad \text{for all } s_{-i}, \tag{1}$$

then we say that the strategy s_i is strictly dominated by s_i' for player i. The strategy s_i is weakly dominated if

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}) \quad \text{for some } s_{-i} \tag{2}$$

and

$$u_i(s_i', s_{-i}) \geq u_i(s_i, s_{-i}) \quad \text{for all } s_{-i} \tag{3}$$

Example 3.3 (Second-price auction)**.** An auctioneer has one item for sale. There are I potential buyers. Each buyer i has a (private or publicly known) valuation v_i for the item. All buyers submit bids $s_1, \dots, s_I$ at the same time. The buyer with highest bid receives the item and pays the second highest bid. The other buyers do not pay anything. Ties are resolved at random.

For each buyer i, bidding the valuation $s_i = v_i$ weakly dominates all other bids:

- If bidding higher than valuation ($s_i > v_i$),
  - if he or she loses, $s_i \leq \max_{j \neq i} s_j$, then the utility is 0, which is not better than bidding v_i,

- if he or she wins and $v_i \geq \max_{j \neq i} s_j$, and the utility is $v_i - \max_{j \neq i} s_j$, which is not better than bidding v_i,
- if he or she wins and $v_i < \max_{j \neq i} s_j < s_i$, then the utility is negative,

- if bidding lower than valuation ($s_i < v_i$),
  - if wins and $s_i \geq \max_{j \neq i} s_j$, utility is not better than bidding v_i,
  - if loses and $v_i \leq \max_{j \neq i} s_j$, then he or she loses anyways by bidding v_i,
  - if loses and $s_i < \max_{j \neq i} s_j < v_i$, then he or she loses should have bid v_i!

Remark 3 (Generalized beauty contest). In the generalized beauty contest example, every strategy above 50 is strictly dominated by the strategy 50. In turn, if every player knows that every player knows that 50 dominates strategies above 50, and every player acts optimally, then the average cannot exceed 50, and 25 dominates strategies above 25. In the limit, 0 is the only remaining strategy by iterated dominance.

4 Reading material

- Chapter 1 of Fudenberg and Tirole.
- Chapter 12 of Microeconomics Theory (Mas-Colell, Whinston, Green).