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12: Refinements	
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In this lecture, we see a refinement for static games in strategic form.

Example 0.1 (Pirate game)**.** Make groups of four players. Order players from P1 to P4. At turn i, player i proposes a distribution of coins within the group. Players vote to accept or reject. In case of tie, the proposer gets an additional vote. If accepted, the game ends, if rejected, player i is eliminated (thrown overboard).

Repeat the pirate game with coalitions: players can negotiate contracts before play.

1 Trembling-hand Perfect Equilibrium

What happens when players may make mistakes? Consider the following verions of Prisoner's dilemma:

	cooperate	defect
cooperate	1, 1	0, 0
defect	0, 0	0, 0

Is (defect, defect) a reasonable outcome in the presence of mistakes?

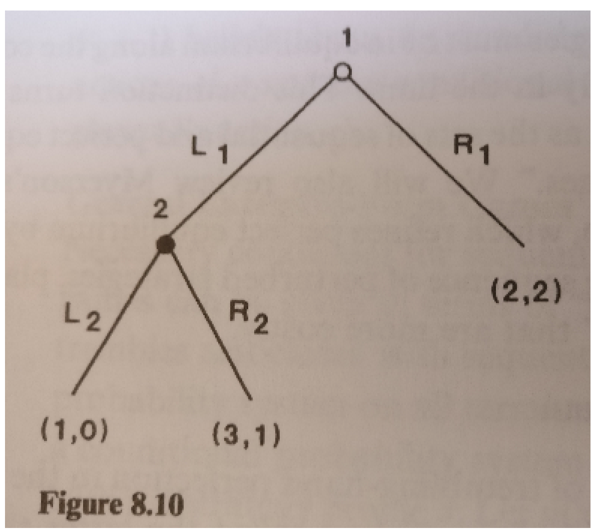

Figure 1: $(R1, L2)$ is a NE, but not great (from Fudenberg Tirole).

Definition 1.1 (Trembling-hand perfection)**.** For every player i, and a vector $\varepsilon \in \mathbb{R}_{>0}^{\sum_j |S_j|}$, let

$$Z_i(\varepsilon) = \{z \in \Delta(S_i) : z_k \geq \varepsilon_{i,k} \text{ for } k = 1, \ldots, |S_i|\}$$

denote the set of mixed strategies bounded from below by ε. An ε-constrained equilibrium of a strategic form game is a totally mixed strategy profile σ^ε such that for every player i, there exists g such that:

$$u_i(\sigma_i^\varepsilon, \sigma_{-i}^\varepsilon) \geq u_i(z, \sigma_{-i}^\varepsilon), \quad \text{for all } z \in Z_i(\varepsilon).$$

A profile σ^* is a Trembling-hand Perfect Equilibrium (THPE) if there exists a sequence $\{\varepsilon^t\}_{t=1,2,\ldots}$ and a sequence of ε^t-constrained equilibria $\{\sigma^{\varepsilon_t}\}$ such that

$$\sigma^{\varepsilon_t} \to \sigma^* \quad \text{and } \varepsilon^t \to 0 \quad \text{as } t \to \infty.$$

1.1 Example

Consider the following strategic form game[1]:

	L	R
U	1, 1	2, 0
D	0, 2	2, 2

There are two NE: (U, L) and (D, R).

Suppose that the row player plays $(1-\varepsilon, \varepsilon)$, then the column player's payoff is:

$$u_2((1-\varepsilon,\varepsilon), L) = 1(1-\varepsilon) + 2\varepsilon = 1+\varepsilon,$$
$$u_2((1-\varepsilon,\varepsilon), R) = 0(1-\varepsilon) + 2\varepsilon = 2\varepsilon.$$

By symmetry, when the column player plays $(1-\varepsilon, \varepsilon)$, the row player's payoff is:

$$u_1(U, (1-\varepsilon,\varepsilon)) = 1(1-\varepsilon) + 2\varepsilon = 1+\varepsilon,$$
$$u_1(D, (1-\varepsilon,\varepsilon)) = 0(1-\varepsilon) + 2\varepsilon = 2\varepsilon.$$

Hence, we have a sequence of ε-constrained NE $((1-\varepsilon_t, \varepsilon_t), (1-\varepsilon_t, \varepsilon_t))$, which converges to the THNE (U, L).

Next, suppose that the row player plays $(\varepsilon, 1-\varepsilon)$, then the column player's payoff is:

$$u_2((1-\varepsilon,\varepsilon), L) = 1\varepsilon + 2(1-\varepsilon) = 2-\varepsilon,$$
$$u_2((1-\varepsilon,\varepsilon), R) = 0\varepsilon + 2(1-\varepsilon) = 2-2\varepsilon,$$

so that the best response for the column is to play $(1-\varepsilon, \varepsilon)$ in a ε-constrained NE for a small value of ε.

2 Reading material

- Chapter 8 of Fudenberg and Tirole.

Some topics for self-study:

- Coalitional games (with side payments)
- Stochastic games
- Computing Nash equilibria

[1] https://en.wikipedia.org/wiki/Trembling_hand_perfect_equilibrium