INSE6290 Jia Yuan Yu

Assignment 2

Concordia November 18, 2016

Assignment answers should be submitted on Moodle in PDF format.

1 Risk pooling

Pick values for h and p; for instance, $h = 0.2$ and $p = 0.5$. Suppose that the demand random variables $D_1, \ldots, D_N$ are independent. Compare S to Σ (cf. notes for Lecture 6 for the definitions) for the following settings:

1. (10 points) Let $N = 5$, and let the demand distributions $F_1, \ldots, F_N$ all be normal with mean 10 and variance 2.
2. (1 point) Let $N = 10$, and let the demand distributions $F_1, \ldots, F_N$ all be normal with mean 10 and variance 2.
3. (1 point) Let $N = 20$, and let the demand distributions $F_1, \ldots, F_N$ all be normal with mean 10 and variance 2.
4. (8 points) How does the ratio S/Σ change with N?
5. (10 points) Let $N = 10$, and let the demand distributions $F_1, \ldots, F_N$ all be normal with mean 10 and variance 1.
6. (1 point) Let $N = 10$, and let the demand distributions $F_1, \ldots, F_N$ all be normal with mean 10 and variance 2.
7. (1 point) Let $N = 10$, and let the demand distributions $F_1, \ldots, F_N$ all be normal with mean 10 and variance 3.
8. (8 points) How does the ratio S/Σ change with the variance?
9. (10 points) What is the take-away lesson from this exercise?

2 Linear Programming

This problem comes from neos-guide.org.

The diet problem was one of the first optimization problems studied in the 1930s and 1940s. The problem was motivated by the Army's desire to minimize the cost of feeding GIs in the field while still providing a healthy diet. One of the early researchers to study the problem was George Stigler, who made an educated guess of an optimal solution using a heuristic method. His guess for the cost of an optimal diet was \$39.93

per year (1939 prices). In the fall of 1947, Jack Laderman of the Mathematical Tables Project of the National Bureau of Standards used the newly developed simplex method to solve Stigler's model. As the first "large scale" computation in optimization, the linear program consisted of nine equations in 77 unknowns. It took nine clerks using hand-operated desk calculators 120 man days to solve for the optimal solution of $39.69. Stigler's guess was off by only $0.24 per year!

Given a set of foods, along with the nutrient information for each food and the cost per serving of each food, the objective of the diet problem is to select the number of servings of each food to purchase (and consume) so as to minimize the cost of the food while meeting the specified nutritional requirements. Typically, the nutritional requirements are expressed as a minimum and a maximum allowable level for each nutritional component. Other constraints such a minimum and/or maximum number of servings may be included to improve the quality of the menu.

Consider the following simple example (from The Diet Problem: A WWW-based Interactive Case Study in Linear Programming). Suppose there are three foods available, corn, milk, and bread, and there are restrictions on the number of calories (between 2000 and 2250) and the amount of Vitamin A (between 5000 and 50000). The first table lists, for each food, the cost per serving, the amount of Vitamin A per serving, and the number of calories per serving.

Food	Cost per serving	Vitamin A	Calories
Corn	$0.18	107	72
Milk	$0.23	500	121
Wheat Bread	$0.05	0	65

Table 1: Diet problem

Suppose that the maximum number of servings is 7. What is the optimal diet (30 points)?

3 Dynamic Programming

Consider Figure 3, where the numbers on the edges denote the travel time from one node to another. What is the quickest path from Harrisonburg to Virginia Beach (10 points)?

How about Blacksburg to Newport News (10 points)?

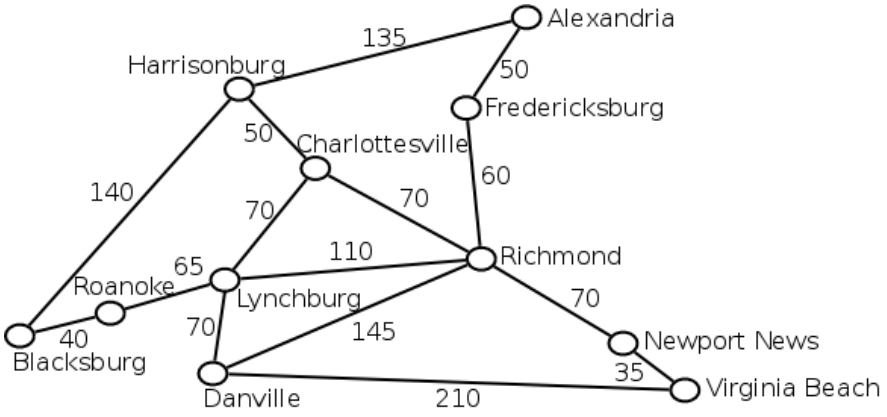

Figure 1: From `http://cs.umw.edu/~finlayson/class/spring15/cpsc230/notes/20-graphs.html`