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8: Competition

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In this lecture, we consider the effect of competition in supply chains. Competition arises from the strategic interactions between multiple decision makers with individual objectives. The tools and notions that we use come from game theory. As in previous lectures though, we are interested in optimal quantities of goods, but we also introduce problems where the decision is an optimal price.

These notes are based on the book Microeconomics Theory (Mas-Colell, Whinston, Green).

Figure 1: Monopoly Man by Vectorius, Bertrand, Cournot, Stackelberg (from Wikipedia).

1 Monopoly

Consider a firm that is the only produce of a good (e.g., iPhones, petroleum). For simplicity, we only consider divisible goods in this lecture, so that production quantities and demands are real numbers. Let p denote the price per unit of this good. Let $x : \mathbb{R}_+ \to \mathbb{R}_+$ be a strictly decreasing function, such that $x(p)$ models the demand as a function of the price p. Moreover, suppose that there exists $\bar{p}$ such that $x(p) = 0$ for all $p \geq \bar{p}$. Let $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$ denote a strictly increasing production cost function, such that $\gamma(q)$ is the cost for producing q units. We assume that p and γ are non-negative, continuous, and twice differentiable. Let $\pi = x^{-1}$ (the inverse of the demand function). The value $\pi(q)$ models the price at which all production is entirely bought in the market when the produced quantity is q. Observe that π is also strictly decreasing since x is strictly decreasing.

The pricing problem, given x and γ, is to find the price that maximizes profit:

$$\max_{p \in \mathbb{R}_+} \quad p \cdot x(p) - \gamma(x(p)).$$

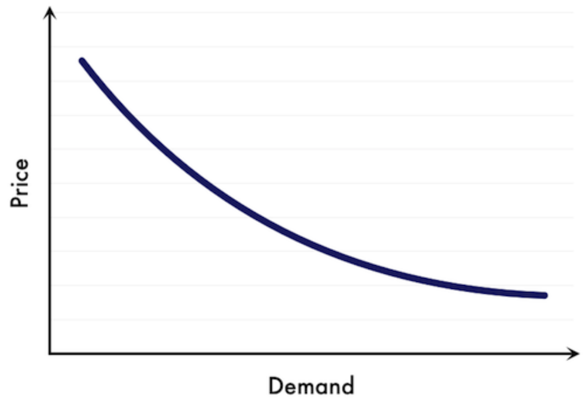

Figure 2: Market-clearing price function (inverse demand function) from `http://floriankugler.com/`.

An equivalent problem is to find the production quantity q that maximizes profit:

$$\max_{q \in \mathbb{R}_+} \quad \pi(q) \cdot q - \gamma(q).$$

By taking derivatives, we find that the optimal production quantity q^*[1] satisfies:

$$\pi'(q^*)q^* + \pi(q^*) = \gamma'(q^*).$$

In other words, the marginal revenue is equal to the marginal cost. Since $\pi'(q) < 0$ (strictly decreasing assumption) for all values of q, we have that $\pi(q^*) > \gamma'(q^*)$—the price under monopoly exceeds marginal cost. This is in contrast to the competitive market equilibrium (e.g., the perfect competition case with a large number of competitors), where $\pi(Q) = \gamma'(Q)$.

In the next sections, we consider the case of multiple firms producing the same good (e.g., Android phones). These cases are called "oligopolies[2]" in general, but we will focus on the special case of duopolies with two firms. Example of oligopolies in supply chains include: OPEC (twelve oil-producing countries), and the mobile phone market in Canada.

2 Bertrand duopoly

Consider two firms that produce the same good, and simultaneously set their asking prices for this good. We assume that they can not change their prices afterwards—relaxing this assumption requires different notions and analyses, which is done in the game theory and economics literature. This model was advanced in 1883 by Joseph Louis Francois Bertrand. The demand function x satisfies the same assumptions as the previous section.

[1] In this talk, we talk about produced goods, but the results hold as well for goods ordered from suppliers (e.g., by a newsboy).

[2] From the greek ὀλίγος (few) and πωλεῖν (to sell).

Firm 1 and firm 2 simultaneously anounce their prices p_1 and p_2. Let $x_j(p_j, p_k)$ denote the demand (order size) for Firm j—$x_1(p_1, p_2)$ sales for Firm 1 and $x_2(p_2, p_1)$ sales for Firm 2. The competition between firm 1 and firm 2 results in the following outcome (demands, order sizes):

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ x(p_j)/2 & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k, \end{cases}$$

for $j = 1, 2$ and $k \neq j$.

We assume that the two firms have the same cost function, which is linear in the production quantity: $\gamma(z) = c \cdot z$. The objective of Firm j is to maximize its profit:

$$p_j \cdot x_j(p_j, p_k) - c \cdot x_j(p_j, p_k).$$

2.1 Equilibrium prices

The notion of equilibrium in the interactions between two decision makers (Firm 1 and Firm 2) is defined as follows. First, let A denote the set of possible decisions of each decision maker; A is the set $\mathbb{R}_+$ of possible prices for Bertrand duopoly. Let $U_j : A \times A \to \mathbb{R}$ for $j = 1, 2$ denote the profit function of Firm j.

Definition 2.1. A *Nash equilibrium* is a pair of decisions $(p_1^*, p_2^*) \in A \times A$ such that

$$U_1(p_1^*, p_2^*) \geq U_1(p_1, p_2^*), \quad \text{for all } p_1 \in A,$$
$$U_2(p_1^*, p_2^*) \geq U_2(p_1^*, p_2), \quad \text{for all } p_2 \in A.$$

In other words, when Firm decides p_2^*, Firm 1 is best off deciding p_1^* and vice versa.

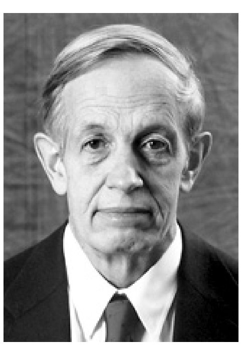

Figure 3: John Forbes Nash. From http://www.nobelprize.org/

How can we find a Nash equilibrium? In the Bertrand duopoly case, we will do so by guessing and verifying whether our guess satisfies the defining properties of a Nash equilibrim. Consider the pair of prices $(p_1^*, p_2^*) = (c, c)$, i.e., each firm sells the goods for zero profit at the production cost of the good. Lowering prices results in loses, or negative profit, as opposed to zero profit. Neither firm can gain by raising its price because its sales would drop to zero, resulting in the same profit of zero. Therefore, $(p_1^*, p_2^*) = (c, c)$ is an equilibrium pair of prices.

The equilibrium for the Bertrand duopoly is unique and more desirable than the monopoly outcome, but it does not reflect reality. Can we find an equilibrium that is more realistic, but still more desirable than the monopoly outcome?

3 Cournot duopoly

Antoine Augustin Cournot proposed the following model in 1838. Consider two firms who simultaneously compete on their production quantities $q_1 \in \mathbb{R}_+$ and $q_2 \in \mathbb{R}_+$ of the same good instead of prices. Let π denote the inverse demand function as in the monopoly case. The price of the good resulting from this competition is

$$\pi(q_1 + q_2),$$

which clears the market by definition of π.

Remark 1. An example of Cournot duopoly is farmers picking up perishable goods from their fields each morning and bringing them to the market. They sell all them items at once in the manner of a Dutch auction: starting with a high price, reducing it until the total of their goods equals the demand.

As in the case of Bertrand duopoly, we assume that the production cost per unit is a constant c; but we also assume that $p(0) > c$. The function π and constant c are given as input to the decision maker; the profit of each Firm j is

$$\pi(q_j + q_k) \cdot q_j - cq_j.$$

3.1 Equilibrium production quantities

Observe that, in contrast to single-decision-maker problems, the optimal decision of each firm is a function of the other firm's decision:

$$q_1^{\text{opt}}(q_2) = \arg\max_{q_1} \quad \pi(q_1 + q_2) \cdot q_1 - cq_1,$$
$$q_2^{\text{opt}}(q_1) = \arg\max_{q_2} \quad \pi(q_1 + q_2) \cdot q_2 - cq_2.$$

These optimal decision functions are called "best-response functions" and are illustrated in Figure 3.1 for the special case of a linear π function.

In the case of Cournot duopoly, an equilibrium—if it exists—is a pair of production quantities that we denote (q_1^*, q_2^*). One approach to find an equilibrim by finding the intersection of the best-response functions by visual inspection. An alternative, analytical approach is to set the first derivative ($\frac{\mathrm{d}}{\mathrm{d}q_1}$ and $\frac{\mathrm{d}}{\mathrm{d}q_2}$, respectively) of each firm's profit equal to zero to find the optimal decisions, which is similar to the EOQ solution. We obtain the following necessary conditions for the optimal production quantities for both firms[3]:

$$\pi'(q_1^* + q_2^*)q_1^* + \pi(q_1^* + q_2^*) = c,$$
$$\pi'(q_1^* + q_2^*)q_2^* + \pi(q_1^* + q_2^*) = c.$$

[3]Homework: check that $q_1 = 0$ is not optimal for Firm 1, nor is $q_2 = 0$ for Firm 2.

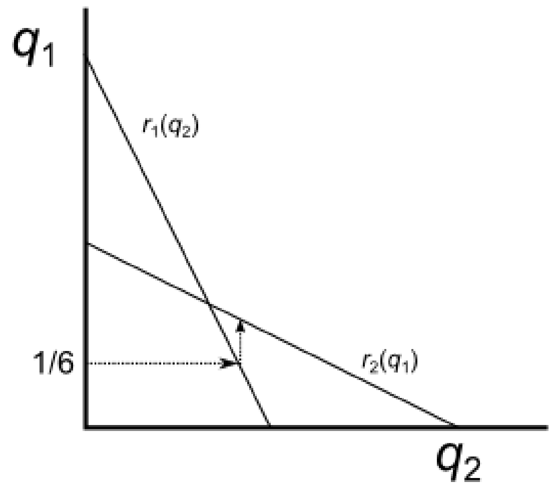

Figure 4: Optimal quantity $q_1^{\text{opt}} = r_1$ as a function of q_2 and vice versa. From `http://mindyourdecisions.com/`.

We can solve this system of two equations in two unknowns by rewriting it as

$$\pi'(2q_1^*)q_1^* + \pi(2q_1^*) = c,$$
$$q_1^* = q_2^*,$$

where we used the fact that $\pi'(q) < 0$ (strictly decreasing assumption) for all $q \geq 0$.

How does $q_1^* + q_2^*$ compare with the solution q^* from the monopoly market?

4 Stackelberg duopoly

In the previous sections, we looked at competition among decision makers who make decisions simultaneously. We now look at sequential decisions. Heinrich von Stackelberg proposed the following model in 1934. Firm 1 is the leader and picks its production quantity q_1 first. The follower, Firm 2, then observes q_1 and picks its production quantity q_2 accordingly. Other that the distinction of the order of actions, we use the same assumptions as Cournot duopoly.

To find a Nash equilibrium in the Stackelberg duopoly, we find the best-response function for each firm by backward induction because the firms make decision sequentially, one after the other.

1. The follower's optimal decision (after observing q_1) is:

$$q_2^{\text{opt}}(q_1) = \arg\max_{q_2} \quad \pi(q_1 + q_2)q_2 - cq_2.$$

   This is a best-response function, as in Cournot duopoly. It can be found by setting the first derivative $(\frac{\mathrm{d}}{\mathrm{d}q_2})$ to zero.

2. The leader's optimal decision (knowing that the follower will respond to its decision q_1 according to the best-response function q_2^{opt}) is:

$$q_1^\star = \arg\max_{q_1} \quad \pi(q_1 + q_2^{\text{opt}}(q_1))q_1 - cq_1.$$

This can also be found by setting the first derivative ($\frac{\mathrm{d}}{\mathrm{d}q_1}$) to zero.

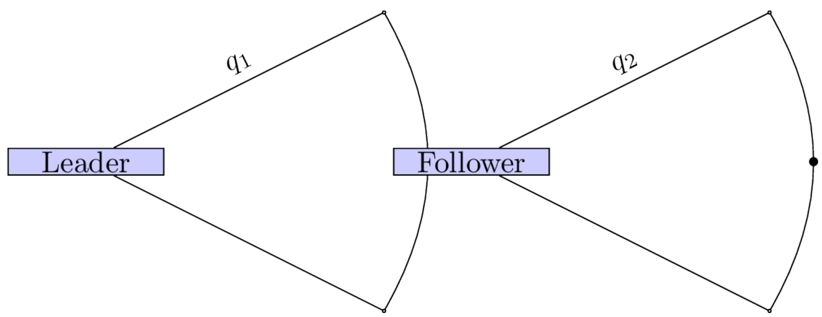

Figure 5: Stackelberg setting. From `http://vknight.org/`.

How does $(q_1^\star, q_2^{\text{opt}}(q_1^\star))$ in Stackleberg duopoly compare with the Cournot case?

5 Reading material

- Chapter 12 of Microeconomics Theory (Mas-Colell, Whinston, Green).