Gas-Vapor Mixtures

Air is a mixture of nitrogen and oxygen and argon plus traces of some other gases. When water-vapor is not included, we refer to it as **dry air**. If water-vapor is included, we must properly account for it.
Usually, we will consider air and water-vapor (even if water-vapor is at the saturation state) as ideal gases. The error using this assumption will be around 0.2%.

Therefore, from Dalton's law:
The total pressure is the sum of the partial pressure P_a of the dry air and the partial pressure P_v of water-vapor (called vapor pressure):

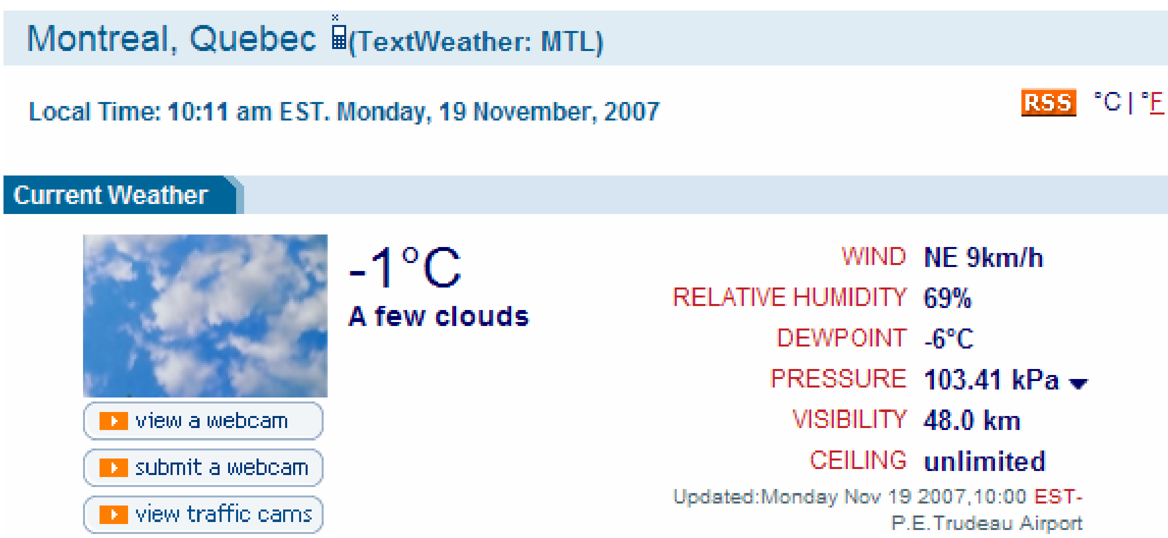
$$P = P_a + P_v$$

Since we assume the water-vapor as an ideal gas, its enthalpy is only dependent on the temperature. Therefore, we will consider the enthalpy of water-vapor as the enthalpy of saturated water-vapor at the same temperature.

$$h_v(T) = h_g(T)$$

This approach is acceptable for situations in which the pressure is relatively low (near atmospheric pressure) and the temperature is below about 60°C.

13.1. Some definitions:



13.1.1. Relative humidity

It is the ratio of the mass of water-vapor m_v to the maximum amount of water-vapor m_g the air can hold at the same temperature.

$$\phi = \frac{m_v}{m_g}$$

using ideal gas law:

$$\phi = \frac{P_v V / R_v T}{P_g V / R_v T} = \frac{P_v}{P_g}$$ with ϕ between 0 and 1.

13.1.2. The humidity ratio ω (specific humidity):

It is the ratio of the mass of water-vapor to the mass of dry air:

$$\omega = \frac{m_v}{m_a}$$

using ideal gas law:

$$\omega = \frac{P_v V / R_v T}{P_a V / R_a T} = \frac{P_v / R_v}{P_a / R_a}$$

but: R_v=0.4615 kJ/kmol K
R_a=0.287 kJ/kmol K

Therefore: $$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{P - P_v}$$

And using the relative humidity:

$$\omega = 0.622 \frac{\phi P_g}{P_a}$$

Example
AIR
25°C, 1 atm
m_a =1 kg
m_v =0.01 kg
$m_{v,\ max}$ =0.02 kg
Specific humidity: ω=0.01 kg H_2O/kg dry air
Relative humidity: 50%.

13.1.3. Dry bulb temperature:

It is the temperature of the air as measured by a conventional thermometer.

13.1.4. Dew-point temperature (T_{dp}):

It is the temperature at which condensation begins if air is cooled at constant pressure.

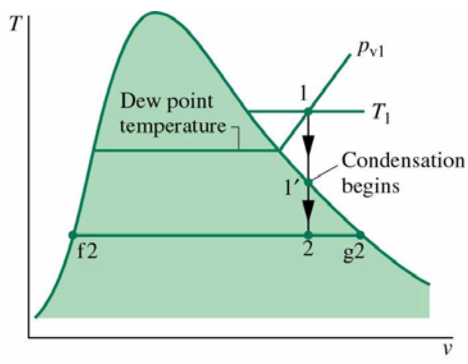


Figure.13.1. Dew-point temperature.

Example

The air at 25°C and 100 kPa in a XXX m^3 room has a relative humidity of 60%. Calculate:

c- The humidity ratio.
d- The dew point.
e- The mass of water-vapor in the air.

13.1.5. Adiabatic saturation and wet-bulb temperature

It is quit difficult to accurately determine directly the relative humidity and the humidity ratio. However, two indirect methods exists:

- Adiabatic saturation process.
- Wet bulb temperature.

13.1.5.a. Adiabatic saturation process

This method uses a relatively long insulated channel. Air with unknown relative humidity (ω_1) enters, moisture is added to the air by the pool of water, and saturated air exits. This process involves no heat transfer because the channel is insulated.

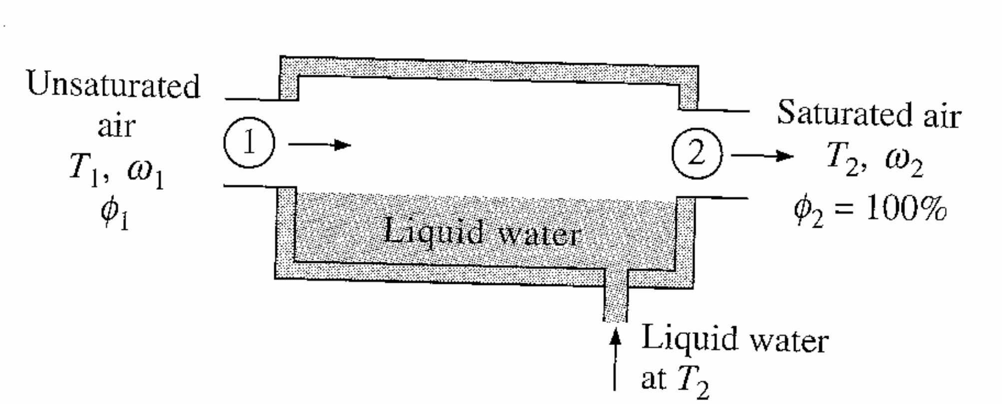


Figure.13.2. Adiabatic saturator.

An energy balance on this control volume, neglecting kinetic and potential energy changes, with Q=W=0, gives:

$$\dot{m}_{v1}h_{v1} + \dot{m}_{a1}h_{a1} + \dot{m}_f h_{f2} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}$$

but

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$$
$$\dot{m}_{v1} + m_f = \dot{m}_{v2}$$

using ω:

$$\dot{m}_a\omega_1 + \dot{m}_f = \dot{m}_a\omega_2$$

substituting with, $h_v \approx h_g$

$$\dot{m}_a \omega_1 h_{g1} + \dot{m}_a h_{a1} + (\omega_2 - \omega_1)\dot{m}_a h_{f2} = \dot{m}_a h_{a2} + \omega_2 \dot{m}_a h_{g2}$$

but at state 2, ϕ_2=100%.

So: $\omega_2 = 0.622 \dfrac{P_{g2}}{P - P_{g2}}$

And, $\omega_1 = \dfrac{\omega_2 h_{fg2} + C_p (T_2 - T_1)}{h_{g1} - h_{f2}}$

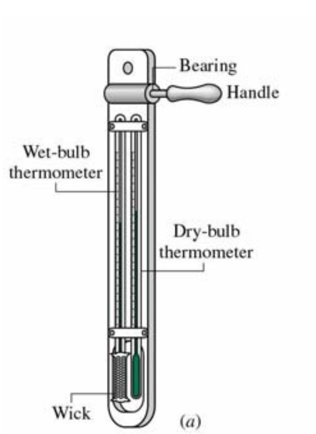
To know ω_2, we have to measure T_1 and T_2 and the total pressure P.

The problem with adiabatic saturation process is that it requires a long channel to achieve saturation conditions at the exit.

Note: usually the amount of dry air in the air-water-vapor mixture remains constant, but the amount of water-vapor changes. Therefore, the enthalpy of atmospheric air is expressed per unit mass of dry air instead of per unit of air-water-air mixture.

13.1.5.b. Wet bulb temperature

A much simpler approach is to wrap the bulb of a thermometer with a cotton wick saturated with water. Then, we can blow air over the wick or swing the thermometer through the air until the temperature reaches a steady state: this is the wet-bulb temperature.
The adiabatic saturation temperature is essentially the same as the wet-bulb temperature if the pressure is approximately atmospheric.



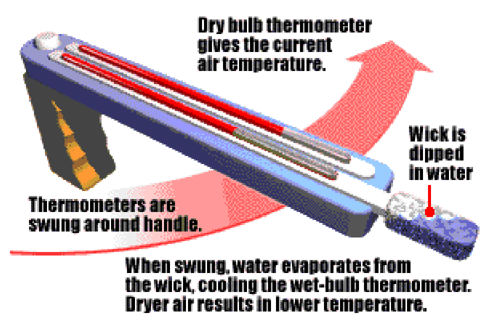


Figure.13.3. dry-bulb and wet bulb temperatures.

Example

The dry and the wet-bulb temperatures of atmospheric air at 1 atm (101.325 kPa) pressure, measured using an adiabatic saturation process are 15°C 25°C respectively. Determine:

a- The specific humidity.
b- The relative humidity.

13.2. Aditional information:

 # Actual method to dermine the relative humidity and the humidity ratio:

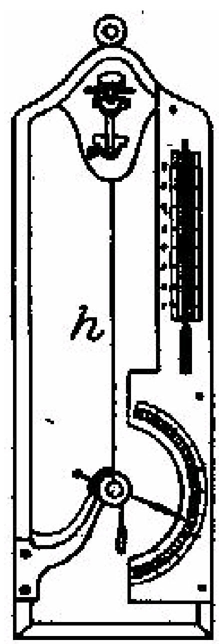
Actually new devices based on the capacitance change of a thin polymer film are used to detemine ω and ϕ.

 # Did you know that hair length increases with humidity?

The range between dry and saturated air can account for a difference in hair length of about three per cent.

In moist air, people with naturally curly hair experience the frizzies as their hair increases in length. Under the same conditions, people with long, straight hair find it going limp.

Hair is such a reliable indicator of good or bad weather, in fact, that it is the primary element of the hair hygrometer, an instrument that was used for years to measure humidity. Invented in 1783, it was used until more sophisticated technology was developed in the 1960s. For its time, it was a very accurate humidity-measuring device, although not in widespread meteorological use today.



 # Humidex factor:

"The humidex is a Canadian innovation, first used in 1965. It was devised by Canadian meteorologists to describe how hot, humid weather feels to the average person."

The humidex is based on the observation that intense heat accompanied by a high vapour content, bring about a physical malaise. In extreme cases, when the combined effects of the temperature and moisture approach the normal temperature of the body (37°C), this malaise becomes dangerous for the human body.

$$\mathbf{H = T + (5 / 9)(P_v - 10)}$$

H is humidex index, T is temperature in °C, P_v is the water vapor pressure in millibar (mbar)}

Range of humidex	Degree of comfort
Less than 29	No discomfort
30 to 39	Some discomfort
40 to 45	Great discomfort; avoid exertion
Above 45	Dangerous
Above 54	Heat stroke imminent

 # Wind chill index:

"The original wind chill formula was derived from experiments conducted in 1939 by Antarctic explorers, Paul Siple and Charles Passel. These hardy scientists measured how long it took for water to freeze in a small plastic cylinder when it was placed outside in the wind. Over the years, the formula was modified somewhat, but remained based on the Antarctic experiments."
This formula was obsolete and in certain circumstances created confusion. Rather than be based on a water cylinder, "the new index is based on a model of how fast a human face loses heat. We chose the face because it is the part of the body most often exposed to severe winter weather, assuming the rest of the body is clothed appropriately for the weather".

The wind chill formula is:

$$\mathbf{R = 13.12 + 0.6215\, T - 11.37\, (V^{0,16}) + 0.3965\, T\, (V^{0,16})}$$

where { R is the wind chill index; T is the air temperature in degrees Celsius (°C); V is the wind speed at 10 metres (standard anemometer height), in kilometres per hour (km/h) }

13.3. The psychrometric chart and air-conditioning processes

13.3.1. The psychrometric chart

All the equations introduced in the upper section are very useful when working at pressures higher than the atmospheric pressure.
For a standard atmospheric pressure, the most conveniant way to determine the various properties associated with a water-vapor mixture is to use a psychrometric chart (Fig.13.4).

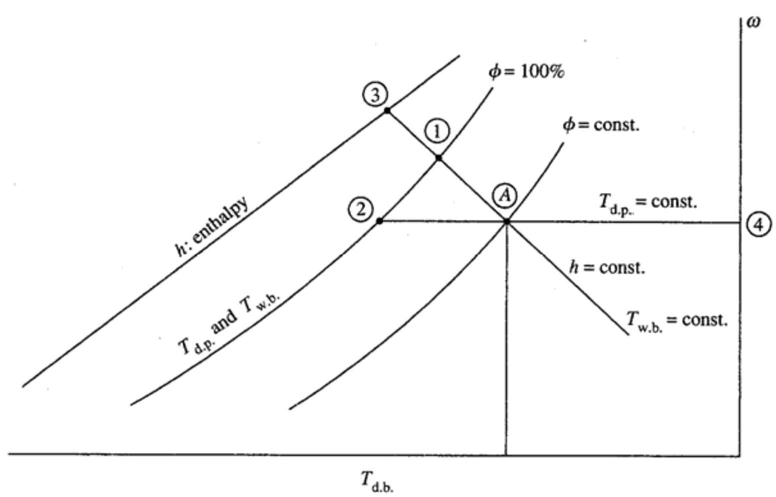


Figure.13.4. Psychrometric chart (principle).

Example

Find the dew-point temperature, the wet bulb temperature the enthalpy and the humidity ratio for the following conditions:

a- dry bulb temperature (T=30°) and a relative humidity of 80%.
b- dry bulb temperature (T=35°) and a relative humidity of 40%.

13.3.2. Air conditioning processes:

Generally, people feel most comfortable when the air is the "comfort zone": the temperature is between 22°C and 27°C and the relative humidity is between 40% and 60% (Fig.13.5) (and usually a wind speed 15 m/min). The area enclosed by the heavy dotted lines represents the ***comfort zone***. There are several situations in which air must be conditioned to put it the comfort zone:

Problem	Solution	Representation
The air is too cold or too hot	Heat is simply added or extracted	A-C and B-C
The air is too cold and the humidity is too low	The air can first be heated, and then moisture added	D-E-C
The temperature is acceptable but the humidity is too high	The air is first cooled, and then moisture is removed. Finally, the air is reheated	F-G; G-H and H-I
The air is too hot and the humidity is low	Moisture is added	J-K
An airstream from the outside is mixed with an airstream form the inside to provide natural cooling or fresh air		I (inside air) + L (outside air) = M (mixed air)

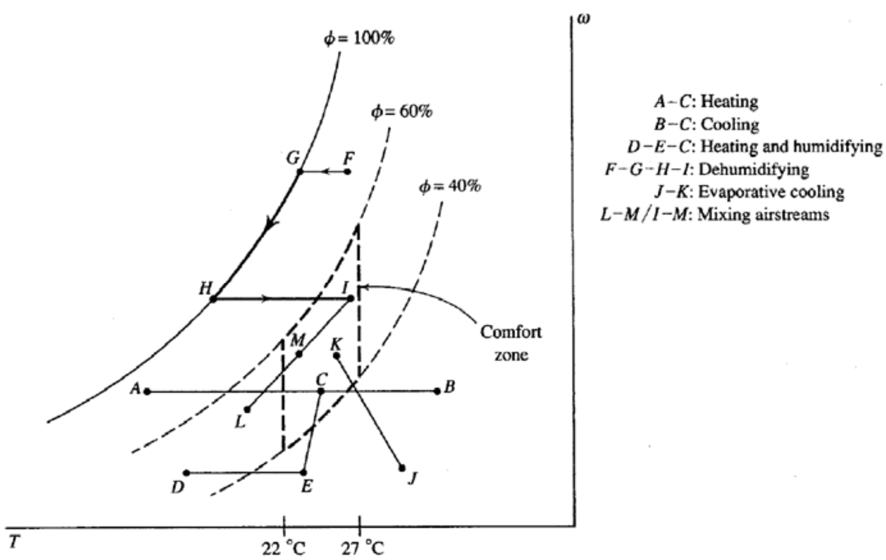


Figure.13.5. The conditioning of air.

13.3.2.1. Air-conditioning processes analysis

Most air-conditioning processes can be modeled as steady-flow processes.

Mass balance for dry air: $\sum m_a\big|_{inlet} = \sum m_a\big|_{outlet}$
Mass balance for water: $\sum m_v\big|_{inlet} = \sum m_v\big|_{outlet}$
Neglecting ΔE_K and ΔE_P, the first law can be written as:

$$\dot{Q}_{in} + \dot{W}_{in} + \sum \dot{m}\ h\Big|_{inlet} = \sum \dot{m}\ h\Big|_{outlet} + \dot{Q}_{out} + \dot{W}_{out}$$

13.3.2.2. Simple heating and cooling (ω=C^t)

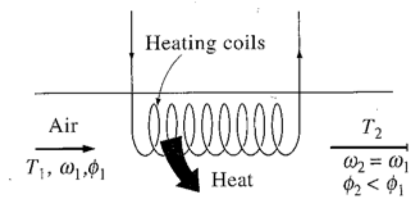


Figure.13.6. Simple heating.

Note: the relative humidity of air decreases during a heating process (ϕ_2<ϕ_1) even if the humidity ratio ω remains constant. This is because the relative humidity is the ratio of moisture content to the moisture capacity of air at the same temperature, and the moisture capacity increases with temperature.

First law (neglecting ΔE_K and ΔE_P and usually the work of the fan is also neglected) can be written under the simple form:

$$\dot{Q} = \dot{m}_a\left(h_2 - h_1\right)$$

13.3.2.3. Heating with humidification:

To overcome the problem of decreasing ϕ with simple heating, air is passed first through a heating section and then through a humidifying section

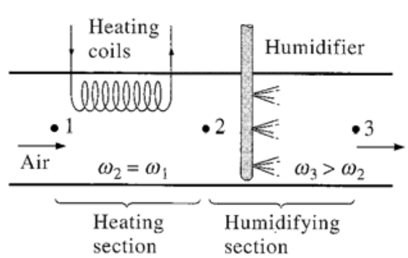


Figure.13.7. Heating and humidification.

If steam is used T_3> T_2
If liquid is used T_3< T_2

13.3.2.4. Cooling with dehumidification

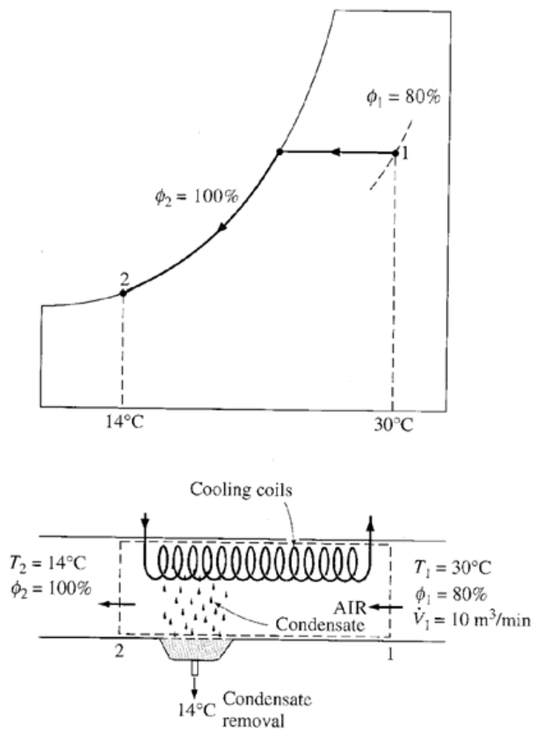


Figure.13.8. Cooling and humidification.

Note that if the T ↓ then ϕ ↑

13.3.2.5. Evaporative cooling

Dry air enters the evaporative cooler where it is sprayed with liquid water. Part of the water evaporates during this process by absorbing heat from the stream. As a result the temperature decreases and humidity increases.

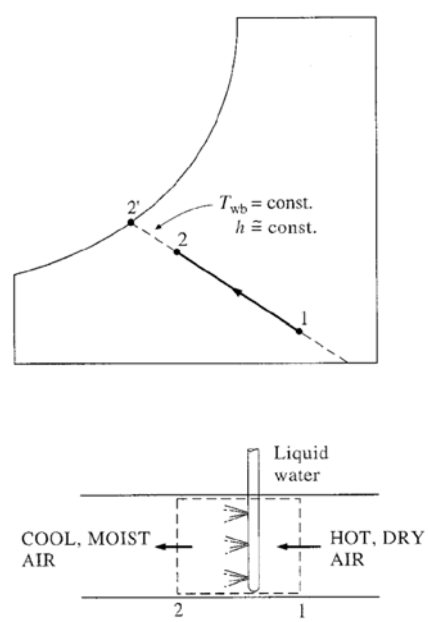


Figure.13.9. Evaporative cooling.

This process is at constant enthalpy and constant wet bulb temperature (constant enthalpy and constant wet bulb temperature lines on the psychrometric chart are almost the same).

13.3.2.6. Adiabatic mixing of airstreams

In large buildings (hospitals, process plants, …), two streams are usually mixed (mixture of conditioned air and new fresh air). The heat transfer with the external medium is small and thus the mixing process can be assumed as adiabatic.

Mass balance for dry air: $\dot{m}_{a,1} + \dot{m}_{a,2} = \dot{m}_{a,3}$

Mass balance for water: $\omega_1 \dot{m}_{a,1} + \omega_2 \dot{m}_{a,2} = \omega_3 \dot{m}_{a,3}$

Neglecting ΔE_K and ΔE_P, the first law can be written as:

$$\dot{m}_{a,1} h_1 + \dot{m}_{a,2} h_2 = \dot{m}_{a,3} h_3$$

Eliminating $\dot{m}_{a,3}$, gives: $\dfrac{\dot{m}_{a,1}}{\dot{m}_{a,2}} = \dfrac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \dfrac{h_2 - h_3}{h_3 - h_1}$

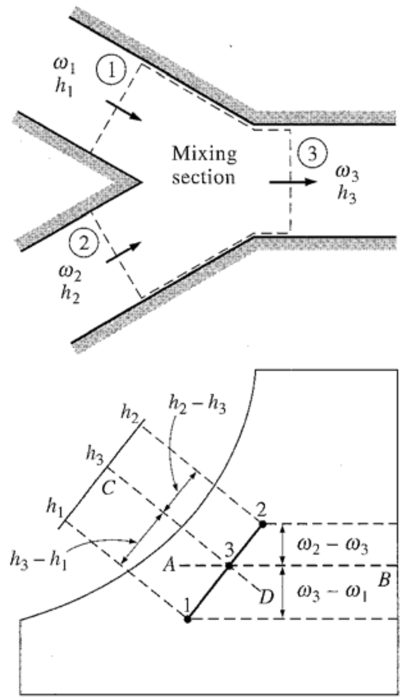


Figure.13.10. adiabatic mixing.

Therefore, when two airstreams at two different states (state 1 and 2) are mixed adiabatically, the state of the mixture (state 3) will lie on the straight line connecting states 1 and 2 on the psychrometric chart, and the ratio of the distances 2-3 and 3-1 is equal to the ratio of mass flow rates at 1 and 2.

Note: if 1 and 2 are close to the saturation line, point 3 may lie to the left of this line. As a consequence, some water will inevitably condense during the mixing process.

Example

Outside cool air at 15°C and 40% relative humidity (airstream 1) is mixed with inside air taken near the ceiling at 32°C and 70% relative humidity (airstream 2). Determine the relative humidity and temperature of the resultant airstream 3 if the outside flow rate is 40 m^3/min and the inside flow rate is 20 m^3/min.

13.3.2.7. Wet cooling towers

In power plants or refrigeration plants, the wasted heat is usually rejected to the sea, lake or river. However, when the water supply is limited, this heat must be rejected to the atmosphere. This is performed using a wet cooling tower.

Air is drawn into the tower from the bottom and leaves through the top. Warm water from the condenser is pumped to the top of the tower and is sprayed into the airstream. The purpose of spraying is to expose a large surface of water to air. As the water droplets fall under the influence of gravity, a small fraction of water (usually a few percent) evaporates and cools the remaining water. The temperature and the moisture content of the air increase during this process. The cooled water is collected at the bottom of the tower and is pumped back to the condenser to pick up additional waste heat (ref. Cengel).

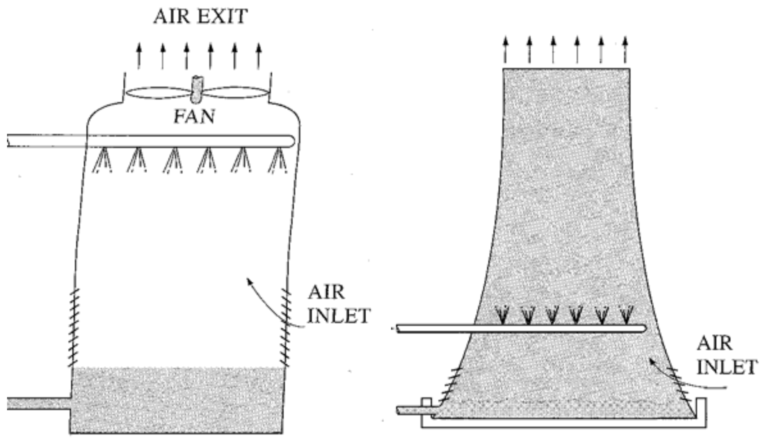


Figure.13.10. Wet cooling towers. (left) An induced-draft couterflow cooling tower. (right) A natural-draft cooling tower.

13.3.3. Additional information

The human body operates like a heat engine, the energy input is food (many thanks to McDO …) and some wasted energy is ejected to the environment. The rate of heat generation for a man depends on the level of activity:

Sleeping	87 W
Resting of doing office work	115 W
Bowling	230 W
Heavy physical effort	440 W

For a woman, the rate of generation is less by about 15% (due to a smaller body size area).

Procedure For Calculating The Properties Of Moist Air

1. Determine what properties you need to calculate. This depends on the problem. Don't forget that the Principles of thermodynamics and the Process descriptions are independent of the method for calculating the Properties.
2. From the Process description, determine the pressure P of the moist air. This is the pressure that would be measured by a pressure gage.
3. From the Process description, determine the temperature T of the moist air. This is the temperature that would be measured by an ordinary thermometer or thermocouple. If the temperature is not known, you must solve the problem by trial and error, so guess a value.
4. Look up the value of the water vapor partial pressure at saturation, P_g, using the Saturated Steam Temperature Table for Water.
5. If the relative humidity, phi, is known from the Process description, write it down and go on to calculating the actual partial pressure of the water vapor in the moist air, P_v, from the equation

$\phi = P_v / P_g$

Then calculate the humidity ratio, omega, from the equation

$\omega = 0.622*P_v / (P - P_v)$

6. Once omega is known, you can calculate the moist air properties from the equations given above.
7. If the relative humidity (phi) is not known, but the humidity ratio (omega) is known and you want to know the relative humidity, you can calculate P_v from

   $P_v = \omega*P / (\omega + 0.622)$

   And then get phi from its definition,

   $\phi = P_v / P_g$

8. If neither ω nor ϕ is known, but the wet bulb temperature T_{wb} is known, then T_{wb} can be used, along with T_{db} (the dry bulb temperature, where $T_{db} = T$), to find omega from the equation given above,

   $\omega = (c_{pa}*(T_{wb} - T_{db}) + \omega_g*h_{fg}) / (h_v(T_{db}) - h_f)$

   Where omega$_g$ is the humidity ratio at saturation at the wet bulb temperature, T_{wb},

   omega$_g$ = $0.622*P_g / (P - P_g)$

   Note that P_g, h_f, and h_{fg} are found from the Saturated Steam Temperature Table at the **wet bulb** temperature (T_{wb}).

9. The relations between ω, ϕ, T_{db}, T_{wb}, are shown graphically on the Psychrometric chart. The enthalpy and volume (per unit mass of dry air) can also be found, although not very accurately, from the chart, but the entropy is not shown at all.