

# COMP 335 Worksheet

## Push-Down Automata

1. Let  $\Sigma = \{a, b\}$ . Find non-deterministic push-down automata for the following languages:

- (a)  $\{a^n b^{2n} \mid n \geq 0\}$
- (b)  $\{wcw^R \mid w \in (a+b)^*\}$
- (c)  $\{ww^R \mid w \in (a+b)^*\}$
- (d)  $\{a^n b^m c^{n+m} \mid n, m \geq 1\}$
- (e)  $\{a^n b^{n+m} c^m \mid n, m \geq 1\}$
- (f)  $\{a^n b^m \mid n \neq m\}$
- (g)  $\{a^n b^m \mid n \neq 2m\}$
- (h)  $\{a^n b^m \mid n \leq m \leq 3n\}$
- (i)  $\{w \in (a+b)^* \mid n_a(w) \neq n_b(w)\}$
- (j)  $\{w \in (a+b+c)^* \mid n_a(w) + n_b(w) = n_c(w)\}$
- (k)  $\{w_1 c w_2 \mid w_1, w_2 \in (a+b)^*, w_1 \neq w_2^R\}$
- (l)  $\{uvvw^R \mid |u| = |w| = 3, u, v, w \in (a+b)^*\}$
- (m)  $\{a^n b^m c^k \mid k = |n - m|\}$

2. Using the CFG-to-NPDA construction, construct an npda that accepts the language generated by the grammar below:

$$\begin{aligned} S &\rightarrow aABBB \mid aAA \\ A &\rightarrow aBB \mid a \\ B &\rightarrow bBB \mid A \end{aligned}$$

3. Given an NPDA  $M = \{Q, \Sigma, \Gamma, \delta, q_0, z, F\}$ , let  $N(M) = \{w \in \Sigma^* : (q_0, w, z) \vdash^* (p, \lambda, \lambda)\}$  be defined as the language  $M$  accepts *by empty stack*. Show that for every npda  $M'$ , there exists an npda  $M$  such that  $L(M') = N(M)$ .

4. For each of the following languages, say whether or not it is a deterministic cfl:

- (a)  $\{a^n b^n : n \geq 1\}$
- (b)  $\{a^n b^{2n} : n \geq 0\}$
- (c)  $\{a^n b^m : n = m \text{ or } n = 2m\}$
- (d)  $\{ww^R : w \in (a+b)^*\}$