

COMP 335 Worksheet: Context-free grammars

1. Let $\Sigma = \{a, b\}$. Find context-free grammars for the following languages:

- (a) $\{a^n b^m \mid n \leq m + 3\}$
- (b) $\{a^n b^m \mid n \neq 2m\}$
- (c) $\{a^n b^m \mid 2n \leq m \leq 3n\}$
- (d) $\{w \in (a + b)^* \mid n_a(w) \neq n_b(w)\}$
- (e) $\{w \in (a + b + c)^* \mid n_a(w) + n_b(w) = n_c(w)\}$
- (f) $\{a^n b^m c^{n+m} \mid n, m \geq 0\}$
- (g) $\{a^n b^{n+2m} c^m \mid n, m \geq 0\}$
- (h) $\{w_1 c w_2 \mid w_1, w_2 \in (a + b)^*, w_1 \neq w_2^R\}$
- (i) $\{u v w v^R \mid |u| = |w| = 3, u, v, w \in (a + b)^*\}$
- (j) $\{a^n b^m c^k \mid k = |n - m|\}$

2. Show a derivation tree for the string $aabbbb$ with the grammar:

$$\begin{aligned} S &\rightarrow AB \mid \lambda \\ A &\rightarrow aB \\ B &\rightarrow Sb \end{aligned}$$

3. Suppose $\lambda \notin L$ and we have CFG for L that has no λ -productions and no unit productions. Let $w \in L$ with $|w| = n$. What is the maximum length of a derivation for w ?

4. Show that the following grammars are ambiguous:

- (a) $\begin{aligned} S &\rightarrow AB \mid aaB \\ A &\rightarrow a \mid Aa \\ B &\rightarrow b \end{aligned}$
- (b) $S \rightarrow ASbS \mid bSaS \mid \lambda$