

## COMP 335 Worksheet: Context-free grammars

1. Let  $\Sigma = \{a, b\}$ . Find context-free grammars for the following languages:

- (a)  $\{a^n b^m \mid n \leq m + 3\}$
- (b)  $\{a^n b^m \mid n \neq 2m\}$
- (c)  $\{a^n b^m \mid 2n \leq m \leq 3n\}$
- (d)  $\{w \in (a + b)^* \mid n_a(w) \neq n_b(w)\}$
- (e)  $\{w \in (a + b + c)^* \mid n_a(w) + n_b(w) = n_c(w)\}$
- (f)  $\{a^n b^m c^{n+m} \mid n, m \geq 0\}$
- (g)  $\{a^n b^{n+2m} c^m \mid n, m \geq 0\}$
- (h)  $\{w_1 c w_2 \mid w_1, w_2 \in (a + b)^*, w_1 \neq w_2^R\}$
- (i)  $\{u v w v^R \mid |u| = |w| = 3, u, v, w \in (a + b)^*\}$
- (j)  $\{a^n b^m c^k \mid k = |n - m|\}$

2. Show a derivation tree for the string  $aabbbb$  with the grammar:

$$\begin{aligned} S &\rightarrow AB \mid \lambda \\ A &\rightarrow aB \\ B &\rightarrow Sb \end{aligned}$$

3. Suppose  $\lambda \notin L$  and we have CFG for  $L$  that has no  $\lambda$ -productions and no unit productions. Let  $w \in L$  with  $|w| = n$ . What is the maximum length of a derivation for  $w$ ?

4. Show that the following grammars are ambiguous:

- (a)  $S \rightarrow AB \mid aaB$   
 $A \rightarrow a \mid Aa$   
 $B \rightarrow b$
- (b)  $S \rightarrow ASbS \mid bSaS \mid \lambda$