

COMP 335 Worksheet

Languages, DFAs and NFAs

1. Prove that if $L_1 \subseteq L_2$, then $L_1^* \subseteq L_2^*$.
2. Let $L = \{w \in \{0, 1\}^* \mid n_0(w) \neq n_1(w)\}$ What is L^* ?
3. What are the strings in the language $L\emptyset$?
4. Under what circumstances is $L^+ = L^* - \{\lambda\}$?
5. Let $\Sigma = \{a, b\}$. Give some examples of strings that are *in* and *not in* the following languages.

will be in Montreal for part of the Fall 2020 semester

 - (a) $\{w \mid \text{for some } u \in \Sigma\Sigma, w = uu^R u\}$
 - (b) $\{w \mid ww = wwww\}$
 - (c) $\{w \mid \text{for some } u, v \in \Sigma^*, uvw = wvu\}$
 - (d) $\{w \mid \text{for some } u \in \Sigma^*, wwww = uu\}$
6. Let $\Sigma = \{a, b\}$. Give DFAs for the following languages over Σ . (Note: A *run* in a string is a substring of length at least two, as long as possible, and consisting entirely of the same symbol. For instance, the string *abbbaab* contains a run of *b*'s of length 3 and a run of *a*'s of length 2.)
 - (a) $L_1 = \{w \mid w \text{ ends with the string } ab\}$
 - (b) $L_2 = \{w \mid w \text{ contains the string } aba\}$
 - (c) $L_3 = \{w \mid w \text{ contains exactly one } a\}$
 - (d) $L_4 = \{a^n b^m \mid n + m \text{ is even}\}$
 - (e) $L_4 = \{a^n b^m \mid n \geq 3, m \text{ is even}\}$
 - (f) $L_5 = \{w \mid |w| \bmod 3 = 1\}$
 - (g) $L_6 = \{w \mid n_a(w) \bmod 3 > 1\}$
 - (h) $L_7 = \{w \mid n_a(w) \bmod 3 > n_b(w) \bmod 3\}$
 - (i) $L_8 = \{w \mid w \text{ contains no runs of length } < 4\}$
 - (j) $L_9 = \{w \mid w \text{ contains exactly two runs of } a\text{'s of length } < 3\}$
 - (k) $L_{10} = \{w \mid w \text{ the second symbol from the end of } w \text{ is a } b\}$
7. Design an NFA with no more than five states for the language $\{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$.
8. Design an NFA with no more than three states for the language $\{ab, abc\}^*$.
9. Design an NFA for the language $\{(ab)^i \mid i \geq 0\} \cup \{(ba)^i \mid i \geq 0\}$.
10. Convert the NFA obtained in the previous question to a DFA.