COMP 335 Worksheet
Properties of regular languages

1. Let $\Sigma = \{a, b\}$. If L is a regular language over Σ, prove that the following languages are regular.

   (a) $Prefix(L) = \{u \mid uv \in L\}$

   (b) $Reverse(L) = \{w \mid w^R \in L\}$

   (c) $even(L) = \{w_2 w_4 \ldots w_n \mid w_1 w_2 w_3 \ldots w_m \in L \text{ where } m \in \{n, n+1\}\}$

   (d) $min(L) = \{w \in L \mid \text{there is no } u \in L, v \in \Sigma^+ \text{ such that } w = uv\}$

2. Which of the following languages is regular? Prove your answer.

   (a) $\{a^{n^2} \mid n \geq 1\}$

   (b) $\{a^n \mid n \text{ is prime}\}$

   (c) $\{a^{2^k} \mid k \geq 1\}$

   (d) $\{a^n \mid n \text{ is not a perfect square}\}$

   (e) $\{a^n b^l \mid n/l \in Z\}$

   (f) $\{a^n b^n \mid n \geq 1\} \cup \{a^n b^m \mid n \geq 1, m \geq 1\}$

   (g) $\{a^n b^n \mid n \geq 1\} \cup \{a^n b^{n+2} \mid n \geq 1\}$

   (h) $\{a^n b^\ell c^k \mid n > 5, \ell > 3, k \leq \ell\}$

   (i) $\{a^n b^\ell \mid n \leq \ell \leq 2n\}$

   (j) $\{w_1 c w_2 \mid w_1, w_2 \in \{a, b\}^\star, w_1 \neq w_2\}$

   (k) $\{w \in (a+b)^* \mid n_a(w) = 2n_b(w)\}$

   (l) $\{w w^R v \mid v, w \in (a+b)^+\}$

3. True or false? Explain.

   (a) If L is regular, then any subset of L is regular.

   (b) If L is regular, then any superset of L is regular.

   (c) If L_1 and L_2 are regular, then $\{w \mid w \in L_1, w^R \in L_2\}$ is regular.

   (d) Suppose L_1 is finite and $L_1 \cup L_2$ is regular. Then L_2 is regular.