

# COMP 335 Worksheet

## Languages, DFAs and NFAs

1. Prove that if  $L_1 \subseteq L_2$ , then  $L_1^* \subseteq L_2^*$ .
2. Let  $L = \{w \in \{0,1\}^* \mid n_0(w) \neq n_1(w)\}$  What is  $L^*$ ?
3. What are the strings in the language  $L\emptyset$ ?
4. Under what circumstances is  $L^+ = L^* - \{\lambda\}$ ?
5. Let  $\Sigma = \{a,b\}$ . Give some examples of strings that are *in* and *not in* the following languages.
 

will be in Montreal for part of the Fall 2020 semester

  - (a)  $\{w \mid \text{for some } u \in \Sigma\Sigma, w = uu^Ru\}$
  - (b)  $\{w \mid ww = www\}$
  - (c)  $\{w \mid \text{for some } u, v \in \Sigma^*, uvw = wvu\}$
  - (d)  $\{w \mid \text{for some } u \in \Sigma^*, www = uu\}$
6. Let  $\Sigma = \{a,b\}$ . Give DFAs for the following languages over  $\Sigma$ . (Note: A *run* in a string is a substring of length at least two, as long as possible, and consisting entirely of the same symbol. For instance, the string *abbbaab* contains a run of *b*'s of length 3 and a run of *a*'s of length 2. )
  - (a)  $L_1 = \{w \mid w \text{ ends with the string } ab\}$
  - (b)  $L_2 = \{w \mid w \text{ contains the string } aba\}$
  - (c)  $L_3 = \{w \mid w \text{ contains exactly one } a\}$
  - (d)  $L_4 = \{a^n b^m \mid n + m \text{ is even}\}$
  - (e)  $L_4 = \{a^n b^m \mid n \geq 3, m \text{ is even}\}$
  - (f)  $L_5 = \{w \mid |w| \bmod 3 = 1\}$
  - (g)  $L_6 = \{w \mid n_a(w) \bmod 3 > 1\}$
  - (h)  $L_7 = \{w \mid n_a(w) \bmod 3 > n_b(w) \bmod 3\}$
  - (i)  $L_8 = \{w \mid w \text{ contains no runs of length } < 4\}$
  - (j)  $L_9 = \{w \mid w \text{ contains exactly two runs of } a\text{'s of length } < 3\}$
  - (k)  $L_{10} = \{w \mid w \text{ the second symbol from the end of } w \text{ is a } b\}$
7. Design an NFA with no more than five states for the language  $\{abab^n \mid n \geq 0\} \cup \{aba^n \mid n \geq 0\}$ .
8. Design an NFA with no more than three states for the language  $\{ab, abc\}^*$ .
9. Design an NFA for the language  $\{(ab)^i \mid i \geq 0\} \cup \{(ba)^i \mid i \geq 0\}$ .
10. Convert the NFA obtained in the previous question to a DFA.