## Student Name:

## Student ID:

## Question 1 \{3 marks\}

A circular specimen of MgO is loaded using a three-point bending mode. Compute the minimum possible radius of the specimen, given that it can be loaded with 425 N without fracture, the flexural strength is 105 MPa , and the separation between load points is 50 mm .

## Solution:

We are asked to calculate the maximum radius of a circular specimen of MgO that is loaded using three point bending. Solving for $R$ from Equation 12.7b

$$
R=\left[\frac{F_{f} L}{\sigma_{f s} \pi}\right]^{1 / 3}
$$

which, when substituting the parameters stipulated in the problem statement, yields

$$
\begin{aligned}
& R=\left[\frac{(425 \mathrm{~N})\left(50 \times 10^{-3} \mathrm{~m}\right)}{\left(105 \times 10^{6} \mathrm{~N} / \mathrm{m}^{2}\right)(\pi)}\right]^{1 / 3} \\
= & 4.0 \times 10^{-3} \mathrm{~m}=4.0 \mathrm{~mm}(0.16 \mathrm{in} .)
\end{aligned}
$$

Question 2 \{ 7 marks $\}$
The density and associated percent crystallinity for two polytetrafluoroethylene materials are as follows:

| $\rho\left(\mathrm{g} / \mathrm{cm}^{3}\right)$ | crystallinity $(\%)$ |
| :---: | :---: |
| 2.144 | 51.3 |
| 2.215 | 74.2 |

(a) Compute the densities of totally crystalline and totally amorphous polytetrafluoroethylene.
(b) Determine the percent crystallinity of a specimen having a density of $2.26 \mathrm{~g} / \mathrm{cm}^{3}$.

## Solution

$$
C=\frac{\rho_{c}\left(\rho_{s}-\rho_{a}\right)}{\rho_{s}\left(\rho_{c}-\rho_{a}\right)}
$$

Rearrangement of this expression leads to

$$
\rho_{c}\left(C \rho_{s}-\rho_{s}\right)+\rho_{c} \rho_{a}-C \rho_{s} \rho_{a}=0
$$

For the two conditions given, we can generate two equations and solve them simultaneously to find the densities of totally crystalline and totally amorphous polytetrafluoroethylene, that is

$$
\begin{aligned}
& \rho_{c}\left(C_{1} \rho_{s 1}-\rho_{s 1}\right)+\rho_{c} \rho_{a}-C_{1} \rho_{s 1} \rho_{a}=0 \\
& \rho_{c}\left(C_{2} \rho_{s 2}-\rho_{s 2}\right)+\rho_{c} \rho_{a}-C_{2} \rho_{s 2} \rho_{a}=0
\end{aligned}
$$

Therefore, the density of the totally amorphous material is

$$
\rho_{a}=\frac{\rho_{s 1} \rho_{s 2}\left(C_{1}-C_{2}\right)}{C_{1} \rho_{s 1}-C_{2} \rho_{s 2}}=\frac{\left(2.144 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(2.215 \mathrm{~g} / \mathrm{cm}^{3}\right)(0.513-0.742)}{(0.513)\left(2.144 \mathrm{~g} / \mathrm{cm}^{3}\right)-(0.742)\left(2.215 \mathrm{~g} / \mathrm{cm}^{3}\right)}=2.000 \mathrm{~g} / \mathrm{cm}^{3}
$$

And that of the totally crystalline material is

$$
\begin{gathered}
\rho_{c}=\frac{\rho_{s 1} \rho_{s 2}\left(C_{2}-C_{1}\right)}{\rho_{s 2}\left(C_{2}-1\right)-\rho_{s 1}\left(C_{1}-1\right)} \\
=\frac{\left(2.144 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(2.215 \mathrm{~g} / \mathrm{cm}^{3}\right)(0.742-0.513)}{\left(2.215 \mathrm{~g} / \mathrm{cm}^{3}\right)(0.742-1)-\left(2.144 \mathrm{~g} / \mathrm{cm}^{3}\right)(0.513-1)}=2.301 \mathrm{~g} / \mathrm{cm}^{3}
\end{gathered}
$$

(b) Now we are to determine the $\%$ crystallinity for $7 \mathrm{~s}=2.26 \mathrm{~g} / \mathrm{cm} 3$. So, using Equation 14.8

$$
\begin{gathered}
\% \text { crystallinity }=\frac{\rho_{c}\left(\rho_{s}-\rho_{a}\right)}{\rho_{s}\left(\rho_{c}-\rho_{a}\right)} \times 100 \\
=\frac{\left(2.301 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(2.260 \mathrm{~g} / \mathrm{cm}^{3}-2.000 \mathrm{~g} / \mathrm{cm}^{3}\right)}{\left(2.260 \mathrm{~g} / \mathrm{cm}^{3}\right)\left(2.301 \mathrm{~g} / \mathrm{cm}^{3}-2.000 \mathrm{~g} / \mathrm{cm}^{3}\right)} \times 100 \\
=87.9 \%
\end{gathered}
$$

