MECH 221
Multiple Choice
True/False Answer Sheet

Student Name: SOLUTIONS
(Last name, First name)

2017 Dec

Student ID # ☐☐☐☐☐☐☐☐

1. For questions (1-50), fill in, or put a distinct **X** in the box containing the letter of the "**best**" answer to that question, E.g. If you want answer (b) fill in the box containing (b) as indicated.
2. The numbers refer to the question numbers on the exam paper.
3. Fill in only one box per question. A line with two or more responses will be treated as blank.

1	a	▨	c	d	e

Q	Multiple choice				
1	[✓ a]	b	c	d	e
2	a	b	[✓ c]	d	e
3	a	b	[✓ c]	d	e
4	a	[✓ b]	c	d	e
5	a	b	c	[✓ d]	e
6	[✓ a]	b	c	d	e
7	a	b	[✓ c]	d	e
8	a	[✓ b]	c	d	e
9	a	b	c	[✓ d]	e
10	a	[✓ b]	c	d	e
11	a	b	c	[✓ d]	e
12	a	b	c	[✓ d]	e
13	a	b	[✓ c]	d	e
14	a	b	c	d	[✓ e]
15	a	b	c	[✓ d]	e
16	a	[✓ b]	c	d	e
17	[✓ a]	b	c	d	e
18	a	[✓ b]	c	d	e
19	a	b	c	[✓ d]	e
20	a	b	c	d	[✓ e]
21	a	[✓ b]	c	d	e
22	a	b	c	[✓ d]	e
23	a	b	c	d	[✓ e]
24	a	[✓ b]	c	d	e
25	a	[✓ b]	c	d	e
26	a	b	c	[✓ d]	e
27	a	b	c	d	[✓ e]
28	[✓ a]	b	c	d	e
29	[✓ a]	b	c	d	e
30	a	b	[✓ c]	d	e

Q	True or False	
31	[✓ T]	F
32	T	[✓ F]
33	[✓ T]	F
34	[✓ T]	F
35	[✓ T]	F
36	T	[✓ F]
37	T	[✓ F]
38	[✓ T]	F
39	[✓ T]	F
40	T	[✓ F]
41	[✓ T]	F
42	T	[✓ F]
43	T	[✓ F]
44	[✓ T]	F
45	[✓ T]	F
46	T	[✓ F]
47	T	[✓ F]
48	T	[✓ F]
49	[✓ T]	F
50	[✓ T]	F

60 + 20

SOLUTIONS

Concordia University
Engineering and Computer Science

Student Name..
(Last name, First name)

2017

60 + 20 + 35

Student ID # ☐☐☐☐☐☐☐☐

DEPARTMENT OF MECHANICAL & INDUSTRIAL ENGINEERING

COURSE Materials Science		**NUMBER** MECH 221	**SECTION** T, X & M
EXAMINATION FINAL	**DATE** December 14th	**TIME** 9.00 am – 12.00 pm	**# OF PAGES** 15

PROFESSORS
M. Pugh (X), M. Medraj (T), S. Joshi (Y), N. Sharifi (M)

MATERIALS ALLOWED: Pens, pencils, erasers, rulers, French-English dictionaries.

CALCULATORS ALLOWED: YES – Faculty Approved

SPECIAL INSTRUCTIONS:

1. Attempt all questions.

2. For the multiple choice and True/False questions, select the **best** answer from those given, and shade in the box with the appropriate letter for that question number on the special answer sheet provided. Make sure you put your name and student number on the answer sheet as well as on this Exam Booklet. Fill in only one box per question. A line with two or more responses will be treated as blank.

   E.g. If you select answer (b), fill in the box containing [b] as indicated.

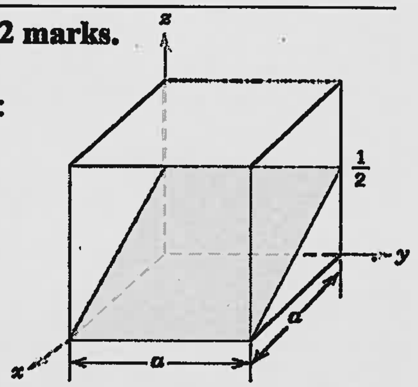
   1 a ▨ c d e

   E.g. If you think the statement is False, fill in the box containing [F] as indicated.

   21 T ☒

3. Answer Questions 51- 52 in the space provided below each question.
4. Each multiple choice question is worth 2 marks. Each true or false question is worth 1 mark.
5. The marks for Questions 51- 52 are indicated at the beginning of the question.
6. The total mark is 115.
7. You may use the blank sides of the pages for rough working.
8. The Equation sheet and the MC answer sheet can be detached carefully from the exam paper.

For Questions 1- 30, select the best answer. Each correct answer is worth 2 marks.



1. The Miller indices for the plane shown in the adjacent cubic unit cell are:
   - [✓ a)] (1 0 2)
   - b) (1 1 ½)
   - c) (1 ∞ ½)
   - d) (1 1 0)
   - e) (1 0 ½)

x y z
1 ∞ 1/2
1 0 2
(1 0 2)

2. The number of vacancies per cubic meter at 1000°C for a metal that has an energy for vacancy formation of 1.22 eV/atom, a density of 6.25 g/cm³, and an atomic weight of 37.4 g/mol is:
   - a) 7.25×10^{16} m^{-3}
   - b) 1.49×10^{22} m^{-3}
   - [✓ c)] 1.49×10^{24} m^{-3}
   - d) 2.57×10^{24} m^{-3}
   - e) 1.49×10^{18} m^{-3}

$n_v = n e^{-Q/RT}$

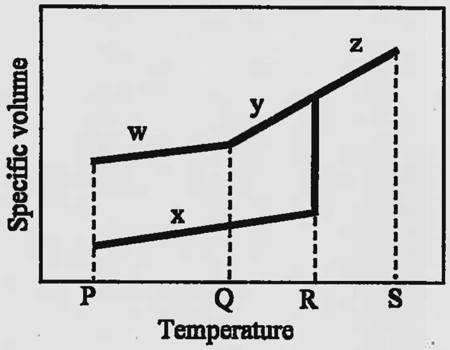
$n = 1.04 \times 10^{29}$

$n_v = 1.04 \times 10^{29} \left(e^{-1.22/1273 \times 8.62 \times 10^{-5}} \right)$

$n_v = 1.55 \times 10^{24}$

3. The electron band structure for materials determines their electrical behavior (such as electrical conductivity). Which of the following statements is **FALSE**?
   - a) Metals, such as copper, have many available empty energy states adjacent to the filled states.
   - b) In some metals, such as magnesium, there is an overlap of the empty band with the filled band.
   - [✓ c)] In a polymer, there is a mixture of overlapping energy bands and small band gaps.
   - d) Semi-conductors, such as silicon have relatively narrow band gaps between the empty conduction band and the filled valence band.
   - e) Insulators such as ceramics, have a large energy band gap between the empty conduction band and the filled valence band.

4. Which of the following best describes the material behaviour exhibited at the designated regions in the adjacent graph of Specific Volume versus Temperature ?
   - a) (w) crystalline, (x) glassy, (y) supercooled liquid, (z) liquid.
   - [✓ b)] (w) glassy, (x) crystalline, (y) supercooled liquid, (z) liquid.
   - c) (w) glassy, (x) single crystal, (y) liquid, (z) gas.
   - d) (w) linear, (x) crosslinked, (y) rubbery, (z) liquid.
   - e) (w) crystalline, (x) leathery, (y) rubbery, (z) liquid.



5. In the figure above:
   - a) P refers to the Glass Transition temperature, T_g, and S is the Melting temperature, T_m.
   - b) P refers to the Glass Transition temperature, T_g, and R is the Melting temperature, T_m.
   - c) Q refers to the Glass Transition temperature, T_g, and S is the Melting temperature, T_m.
   - [✓ d)] Q refers to the Glass Transition temperature, T_g, and R is the Melting temperature, T_m.
   - e) R refers to the Glass Transition temperature, T_g, and S is the Melting temperature, T_m.

6. A sample of polymer has a density of 2.144 g.cm^{-3}. The densities of the completely amorphous structure and the completely crystalline structure of this polymer are respectively: 2.000 g.cm^{-3} and 2.301 g.cm^{-3}. What is the percentage crystallinity of the sample?
   - [✓ a)] 51.3%
   - b) 48.7%
   - c) 2.1%

$$\% \, xst = \frac{\rho_c(\rho_s - \rho_a)}{\rho_s(\rho_c - \rho_a)} = \frac{2.301(2.144 - 2)}{2.144(2.301 - 2)}$$

$$= 0.513$$

Question continues on next page

- d) 21%
- e) 79%

7. Ductile failure of metals at room temperature is:
   - a) when the % elongation is greater than 0%.
   - b) when the grains separate from each other along the grain boundaries.
   - [✓ c)] when there is a cup-cone type fracture which starts by the opening of cracks around impurity particles inside the neck.
   - d) the kind of failure in which the elastic region extends right up to final fracture.
   - e) promoted by increasing the rate of loading.

8. Wood has a Young's modulus of 1×10^{10} Nm^{-2} when tested parallel to the grain (up and down the tree) and a value of 7×10^8 Nm^{-2} perpendicular to the grain (across the tree). A wooden beam loaded in compression will therefore:
   - a) compress more if the loading is along the grain than if it is perpendicular to the grain.
   - [✓ b)] compress less if the loading is along the grain than if it is perpendicular to the grain.
   - c) compress the same amount whether loaded parallel to or perpendicular to the grain.
   - d) most likely fracture since wood is almost always loaded in tension.
   - e) compress the same amount if loaded parallel or perpendicular to the grain but failure will occur at a lower stress if the load is perpendicular to the grain.

9. On a particularly heavy traffic day, enough traffic stops on a bridge to increase the tensile stress in each of the support cables to 550 MPa (which is above the yield strength). If the total strain in the cable under load is 0.006, what is the permanent strain in the cable? The Young's modulus of the cable is 200 GPa.
   - a) 0
   - b) 0.0016
   - c) 0.0027
   - [✓ d)] 0.0033
   - e) 0.006

$0.006 = \varepsilon_p + \varepsilon_E$

$\varepsilon_E = \frac{\sigma}{E} = \frac{550 \times 10^6}{200 \times 10^9} = 0.00275$

$\varepsilon_p = 0.006 - 0.00275 = 0.00325$

10. A ceramic part fails in tension at 50 MPa. Microscopy reveals an external flaw (0.095 mm long) in the side of the failed part that caused the failure. What is the K_{IC} of the material, assuming the geometric shape parameter, Y, is equal to 1:
    - a) 0.611 $MPa.m^{1/2}$
    - [✓ b)] 0.864 $MPa.m^{1/2}$
    - c) 1.222 $MPa.m^{1/2}$
    - d) 27.3 $MPa.m^{1/2}$
    - e) 38.63 $MPa.m^{1/2}$

$K_{IC} = Y\sigma\sqrt{\pi a} = 1 \times 50 \sqrt{\pi \times 0.095 \times 10^{-3}}$

$= 0.864 \ MPa\,m^{1/2}$

11. In a binary eutectic phase system between metal A and metal B, the eutectic mixture of solid phases that forms when a liquid of the eutectic composition is slowly cooled below the eutectic temperature tends to form a lamellar structure of alternating "layers" of α-phase (A-rich) and β-phase (B-rich). With regard to this lamellar mixture, select the best answer below:
    - a) the layer structure forms because the α-phase solidifies first and then the liquid layer on top solidifies as β-phase and then this keeps repeating as the temperature drops below the liquidus temperature.
    - b) this structure has the lowest amount of free energy compared to equi-axed grains of the two-phases.
    - c) The crystal structures of the two phases which have different compositions can only grow in 2 dimensions thus sheets have to form as the liquid solidifies.

Question continues on next page

[✓ d)] The two solid phases have very different compositions and in order for these solids to form from one liquid with an intermediate composition, lamellar structures are favoured as the diffusion distances are shorter.

e) The α-phase forms first above the eutectic temperature and then the β-phase forms as a micro-constituent in the α + Liquid region and then the final mixture solidifies below the eutectic temperature.

12. A specimen of steel has a rectangular cross section 20 mm wide and 40 mm thick, an elastic modulus of 207 GPa, and a Poisson's ratio of 0.30. If this specimen is pulled in tension with a force of 60 kN, what is the change in width if the deformation is totally elastic?

$\varepsilon_x = -\nu \varepsilon_z$

$\varepsilon_x = \frac{\Delta w}{w_o}$ $\quad \varepsilon_z = \sigma/E = F/Ewb$

$$\Delta l = \frac{(-0.3)(60{,}000)(20\times10^{-3})}{207\times10^{9}(20\times10^{-3})(40\times10^{-3})} \qquad \Delta l = \frac{-\nu F L}{E b w}$$

a) Increase in width of 3.62 μm
b) Decrease in width of 7.24 μm
c) Increase in width of 7.24 μm
[✓ d)] Decrease in width of 2.18 μm
e) Decrease in width of 4.35 μm

13. When a semi-crystalline polymer has been "drawn":

a) The crystalline regions decrystallize and become amorphous thus weakening the polymer
b) The entropy has been increased and the chains are more entangled thus the polymer is stronger
[✓ c)] The polymer chains become aligned with the drawing direction increasing the secondary bonding and increasing the polymer strength and stiffness
d) The intermolecular spaces have been filled with water thus increasing the density of the polymer
e) The crystalline regions have been oriented perpendicular to the drawing direction increasing the resistance to fibre splitting.

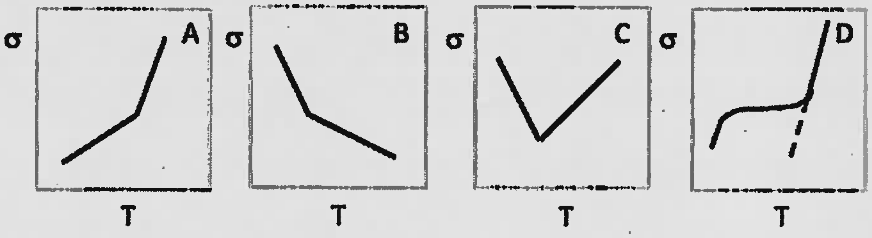
14. A rod of a hypothetical metal 50 cm long elongates 0.40 mm on heating from 50°C to 151°C. The value of the linear coefficient of thermal expansion for this metal is:

$\Delta l = L\alpha\Delta T$, $\alpha = \frac{\Delta l}{\Delta T L} = \frac{0.4\times10^{-3}}{101\times 0.5}$

$= 7.92\times10^{-6}$

a) 1.24×10^{-6} °C^{-1}
b) 1.60×10^{-6} °C^{-1}
c) 5.30×10^{-6} °C^{-1}
d) 8.01×10^{-5} °C^{-1}
[✓ e)] 7.92×10^{-6} °C^{-1}

15. The electrical conductivity (σ) of an n-type extrinsic semiconductor varies with temperature (T). Which of the *σ vs T* diagrams in the adjacent figure is most similar to the expected behaviour?



a) A
b) B
c) C
[✓ d)] D
e) none of the figures is representative of the real behaviour.

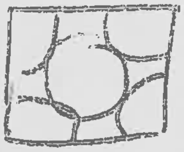
16. Two hypothetical metals, A and B, have identical densities; however A has an FCC crystal structure whereas B is BCC. If the lattice parameters of A and B are 0.3 and 0.4 nm respectively, which is the heavier of the two atoms? (i.e. Which has the highest atomic weight?)

a) A
[✓ b)] B

Question continues on next page

c) Both have the same weight.
d) Not enough information has been given.
e) None of the above.

17. The lattice parameter of FCC copper is 0.361nm. The atomic density in atoms per square nm on the face of the unit cell is therefore:
   - [✓ a)] 15.35 atoms/nm^2
   - b) 23.02 atoms/nm^2
   - c) 2 atoms/nm^2
   - d) 38.4 atoms/nm^2
   - e) 7.67 atoms/nm^2



$2 \text{ atoms}/(0.361 \times 10^{-9})^2$

18. 10% tin can be added to copper in the molten state and cooled to give a substitutional solid solution known as bronze. This means that the crystal structure of the solid solution :
   - a) consists of unit cells of FCC copper interspersed with unit cells of FCC tin.
   - [✓ b)] is basically that of FCC copper with some of the copper atoms replaced by tin atoms.
   - c) consists of grains of pure copper which make up 85% of the weight of the material. The remaining 15% occurs as grains of pure tin.
   - d) is that of tin with the copper atoms squeezed in the gaps.
   - e) is that of copper with the tin atoms squeezed into the spaces between the copper atoms.

19. Single crystal aluminum oxide (Al_2O_3) is a colourless, transparent material known as "Sapphire". If this material is doped with a small amount of chromium oxide (0.5 to 2 at% Cr_2O_3) it gives the material a deep red colour and is known as "Ruby".
   - a) This occurs because the sapphire is a glass and when it is doped it becomes a crystalline material.
   - b) An electron energy band gap is created when the sapphire is doped with chromium oxide.
   - c) Sapphire has a very small electron energy band gap (< 1.8 eV) which is significantly increased to >3.1 eV when it is turned into ruby.
   - [✓ d)] The band gap of Sapphire is >3.1 eV but when chromium oxide is added the band gap is altered to absorb orange, green and yellow light.
   - e) Sapphire has an electron energy band gap > 3.1 eV but when chromium oxide is added the band gap is increased which absorbs red light making the ruby turn red.

20. Fibre Optic cables can carry huge amounts of data (one cable can carry 24,000 simultaneous phone calls). The data can be carried by light waves because:
   - a) The data is split into different colours (red, orange, yellow, etc) which allows many more signals.
   - b) One fibre optic cable is made from 12,000 individual very thin (12 x 10^{-6} m) glass fibres bundled into one cable and each fibre can carry two signals as binary (one-zero).
   - c) The electron energy bang gap of the glass fibre is eliminated by doping with Cu, Fe and V which allows vast numbers of adjacent electron energy levels to transmit data as free electrons.
   - d) Light rays in the fibre optic travels faster than free electrons in traditional copper cable thus more signals per minute can be carried.
   - [✓ e)] The refractive index of the fibre optic is designed so that light is contained by total internal reflection allowing the light signals to be contained when the cable is curved.

21. The linear coefficient of thermal expansion of a material:
   - a) is exponentially related to the temperature, T (in Kelvin) through an Arrhenius relationship similar to that for the diffusion coefficient.
   - [✓ b)] is governed by the asymmetric curvature of the potential energy trough (dip in the curve of potential energy vs interatomic distance), a more asymmetric curve giving a higher thermal expansion coefficient.

Question continues on next page

c) is governed by the asymmetric curvature of the potential energy trough (dip in the curve of potential energy vs interatomic distance), a more symmetric curve giving a higher thermal expansion coefficient.
d) can be calculated by calculating the cube root of the volume coefficient of thermal expansion.
e) indicates the amount of extension (strain) that one mole of material goes through per joule of heating.

22. Materials such as metals and ceramics are held together by atomic bonds between atoms. The equilibrium separation distance between adjacent atoms is the distance where:
a) the net force between the atoms is highest
b) the net force between the atoms is lowest
c) the potential energy between the atoms is zero
[✓ d)] the potential energy between the atoms is lowest
e) the potential energy between the atoms is highest

23. The electrical conductivity of a piece of metal generally decreases with:
a) increasing deformation
b) increasing impurity content
c) increasing temperature
d) a and b
[✓ e)] all of the above.

24. The metal rhodium has an FCC crystal structure. If the angle of diffraction (*2θ*) for the (311) set of planes occurs at 36.12° (first order reflection) when monochromatic x-ray having a wavelength of 0.0711 nm is used, compute the atomic radius for a rhodium atom. $n\lambda = 2d\sin\theta$
a) 0.1147 nm
[✓ b)] 0.1345 nm
c) 0.3300 nm
d) 0.3804 nm
e) The whole X-ray spectrum is needed to solve this question

$d_{hkl} = \frac{a}{\sqrt{h^2+k^2+l^2}}$

$a = \frac{\lambda\sqrt{h^2+k^2+L^2}}{2\sin\theta} = 3.8 \times 10^{-10}$

$r^2 = a^2/8 \qquad r = \sqrt{a^2/8}$

$r = 0.1345 \times 10^{-9}\,m$

25. For a p-type semiconductor,
a) The Fermi level is located in the middle of the band gap
[✓ b)] Electron concentration < Hole concentration
c) Electron concentration = Hole concentration
d) Electron concentration > Hole concentration
e) This occurs only in ionic materials with high concentration of cations

26. The property that enables a material to absorb elastic strain energy is:
a) strength b) toughness c) elasticity [✓ d)] resilience e) hardness

27. Which of the following will increase the thermal shock resistance of a ceramic material?
a) decreasing the modulus of elasticity
b) decreasing the material's thermal expansion coefficient
c) increasing the material's ability to reduce internal temperature gradients (i.e. material's thermal conductivity)
d) increasing the material's fracture strength
[✓ e)] all of the above

28. Copper has an atomic radius of 0.128 nm, an FCC crystal structure and an atomic weight of 63.5 g/mol. The theoretical density of copper is thus:
    - [✓ a) 8.89 g/cm^3]
    - b) 4.445 g/cm^3
    - c) 8.89 kg/cm^3
    - d) 17.78 Mg/m^3
    - e) 17.78 g/cm^3

$$\rho_c = \frac{nA}{V_c N_c} = \frac{4 \times 63.5}{6.023 \times 10^{23} \cdot (a^3)}$$

$$a = \sqrt{8}\, r$$

29. Devices such as transistors are made by doping semiconductors with different dopants to generate regions that have p- or n-type semiconductivity. The diffusion coefficient of phosphorus (P) in Si is 6.5×10^{-13} cm^2/s at 1100°C. Assume that: the P source provides a surface concentration of 10^{20} atoms/cm^3, the diffusion time is one hour and that the silicon wafer contains no P to begin with. Calculate the depth at which the concentration of P will be 10^{18} atoms/cm^3.
    - [✓ a) 1.76 µm]
    - b) 2.06×10^2 µm
    - c) 1.53 mm
    - d) 3.8 µm
    - e) 0.01 cm

$$\frac{C_x - C_o}{C_s - C_o} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$C_s = 10^{20}$

$C_x = 10^{18}$

$C_o = 0$

$t = 60$

~~4~~ $D = 6.5 \times 10^{-13}$ cm^2/s.

30. In question 29, if the temperature is increased to 1500°C, what would be the time required to complete the same amount of diffusion given that the diffusion coefficient at this temperature is 1.3×10^{-12} cm^2/s?

a) 1 hour b) ¾ hour [✓ c) ½ hour] d) ¼ hour e) 5 minutes

z	erf(z)	z	erf(z)	z	erf(z)
0	0	0.55	0.5633	1.3	0.9340
0.025	0.0282	0.60	0.6039	1.4	0.9523
0.05	0.0564	0.65	0.6420	1.5	0.9661
0.10	0.1125	0.70	0.6778	1.6	0.9763
0.15	0.1680	0.75	0.7112	1.7	0.9838
0.20	0.2227	0.80	0.7421	1.8	0.9891
0.25	0.2763	0.85	0.7707	1.9	0.9928
0.30	0.3286	0.90	0.7970	2.0	0.9953
0.35	0.3794	0.95	0.8209	2.2	0.9981
0.40	0.4284	1.00	0.8427	2.4	0.9993
0.45	0.4755	1.1	0.8802	2.6	0.9998
0.50	0.5205	1.2	0.9103	2.8	0.9999

Questions continue on next page

For **Questions 31 – 50,** decide whether the following statements are **True** or **False** and shade in the appropriate box on the **Answer sheet. (Each correct answer is worth 1 mark).**

T 31. Time affects the mechanical behavior of polymers significantly. For example, at higher strain rate, thermoplastics become stronger and less ductile.

F 32. Engineering stress is defined as the instantaneous load divided by the instantaneous cross-sectional area, whereas true stress is defined as the instantaneous load divided by the original cross-sectional area.

T 33. Thermal conductivities are higher for crystalline than for non-crystalline ceramics because, for non-crystalline materials, phonon scattering, and thus the resistance to heat transport, is much more effective due to their highly disordered and irregular atomic structure.

T 34. Isotactic polymers, in which side groups are regularly arranged on the same side are more likely to produce polymers with higher crystallinity than atactic polymers in which the side groups are arranged randomly.

T 35. In ceramics, for a specific coordination number there is a minimum cation-anion radius ratio for which the contact between the cations and anions can be maintained.

F 36. Rubbers have relatively rigid chains; this is why they could be stretched to 900% or more without breaking.

F 37. Ceramics often fracture above their theoretical strength because of their strong ionic bonding.

T 38. In metals, tensile strength (TS) is always higher than or equal to yield strength (YS) but in polymers, TS can be greater or smaller than YS.

T 39. The tensile strength of polymers can increase significantly by increasing the molecular chain entanglement.

F 40. For noncrystalline ceramics, plastic deformation occurs by the motion of dislocations.

T 41. During a tensile test on a semicrystalline polymer, once a neck forms in the gauge section of the specimen, continued specimen elongation proceeds by neck propagation along the specimen gauge length.

F 42. Because carbon atoms diffuse in FCC-Fe by the substitutional mechanism, the diffusion coefficient of carbon in FCC-Fe at 500°C should be larger than the diffusion coefficient of carbon in FCC-Fe at 1000°C.

F 43. Boron, having 3 valence electrons creates an *n-type extrinsic semiconductor* when added to a silicon (4 valence electrons) substrate.

T 44. Diamond and fullerene are two different polymorphs of carbon.

T 45. Plastic deformation of metals by dislocation slip occurs preferentially on the most densely packed planes in the most densely packed directions.

F 46. Elastic deformation results from the breaking and reforming of atomic bonds.

F 47. If elements Si and N have electronegativities of 1.8 and 3.0 respectively, the material Si_3N_4 is approximately 70% ionic in character.

F 48. Covalent bonding is commonly found in ceramics but not in polymers or metals.

T 49. The interstitial diffusion coefficient for a small atom diffusing in a metal is usually larger than the diffusion coefficient for an atom that diffuses in that same metal via vacancy diffusion.

T 50. The temperature required to carburize a specimen of γ-iron for 2 hours to produce the same diffusion result as a carburization at 900°C for 15 hours, will be higher.

Attempt all questions: **Where appropriate put your final answer in the answer box provided**

Question 51: *{10 marks}*

A research lab prepares a ceramic material with very precise and sophisticated techniques. The material produced is fully dense and has a Modulus of Elasticity of 337GPa. The strength coefficients, σ_0 and n, have values of 238 MPa and 3.75 respectively. A company wants to use this material and sets up a production line for pressing and sintering the material. The resulting industrial material is 94% dense, the remainder being porosity. Two types of bar are made from this material: one with a square cross-section (5 mm x 5 mm) and one with a round cross-section (diameter = 7.5 mm). One can assume that the fracture strength of the ceramic is independent of the shape of the sample. The samples are tested in three-point bending with an outer span of 50 mm.

i. What is the actual Modulus of Elasticity of this porous industrial material? $P = 0.06$ ①

$$E = E_0(1 - 1.9P + 0.9P^2)$$

$$= 337(1 - (1.9 \times 0.06) + (0.9 \times 0.06^2)) \quad \boxed{300 \text{ GPa}}$$

①

$$= 299.67 \text{ GPa}$$

ii. What is the fracture strength of the porous industrial ceramic material?

$$\sigma = \sigma_0 e^{-nP}$$

$$= 238 e^{-(0.06 \times 3.75)} \quad \boxed{190 \text{ MPa}}$$

①

$$= 190 \text{ MPa}$$

iii. **What is the force required to fracture the square bars?**

2

$$\sigma_{sq} = \frac{3\,FL}{2\,bd^2} = \frac{3FL}{2b^3}$$

$$\boxed{316.7\ N}$$

$$F_{sq} = \frac{2\sigma}{3L} \cdot b^3 = \frac{2(190\times10^6)(5\times10^{-3})^3}{3\times(50\times10^{-3})} = 316.7\ N$$

iv. **What is the force required to fracture the round bars?**

$\phi = 7.5\times10^{-3}\ m$

$r = 3.75\times10^{-3}\ m$

$$\sigma_{RD} = \frac{FL}{\pi R^3}, \quad F_{RD} = \frac{\sigma}{L}\pi R^3$$

$$\boxed{629.5\ N}$$

2

$$F_{RD} = \frac{(190\times10^6)\pi(3.75\times10^{-3})^3}{50\times10^{-3}}$$

$$F_{RD} = 629.5\ N$$

v. **Why is there usually some porosity present in most sintered ceramics?**

Sintered ceramics are made from powders which are assembled together in the "green" form, usually by pressing. This means there are may "gaps" between particles. These gaps close up during sintering (diffusion) as grains join together and gaps get smaller and smaller as density increases but there is usually some residual porosity as the time required to completely remove all porosity would be very large

3

Question 52: *{25 marks}*

The Phase diagram below shows the system between Barium (Ba) and a soon-to-be-discovered element from the ice-plant Hoth called Rafinium (Rf). As shown, there are two terminal solid solutions (α, β) and an intermetallic compound with a eutectic reaction between each of the solid solutions and the intermetallic compound. Using this phase diagram answer the following questions putting your final answer in the box provided.

a) What are the melting points of Barium, Rafinium and the intermetallic compound (IM)? 3

$T_{m\,(Ba)}$ = 720 °C

$T_{m\,(Rf)}$ = 490 °C (490 - 500 °C)

$T_{m\,(IM)}$ = 595° C

b) What is the stoichiometric formula for the Intermetallic compound between Ba and Rf? $\boxed{BaRf}$ 1

50 at% Ba

50 at% Rf ⇒ BaRf

c) Knowing this, label the two-phase regions of the phase diagram (on the diagram). 2

(α + L) (L + BaRf) (L + BaRf) (L + β)

d) Give the composition of the alloy which has 100% liquid phase at the lowest temperature. (wt %) $\boxed{90\%\,Rf\ 10\%\,Ba}$ 1

e) What is the maximum amount of Rafinium that can be dissolved into the α solid solution? (wt %) $\boxed{15\%\,Rf}$ 1

f) 15 kg of alloy containing 25wt% Rf (75 wt% Ba) is heated to 560 °C and held to equilibrate. Give the following:

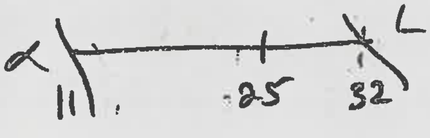
(i) the phases present and their compositions:

α : 11 wt% Rf + 89 wt% Ba

L : 32 wt% Rf + 68 wt% Ba

2

α + L



(ii) the amounts (in kg) of these phases present:

5 kg α	10 kg Liquid

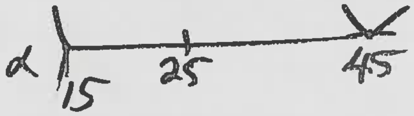
4

$$\alpha = \frac{32-25}{32-11} \times 15$$

$$= 5\ \text{kgs}$$

$$L = \frac{25-11}{32-11} \times 15 = 10\ \text{kg}$$

g) If this alloy from part (f) is cooled to just above its eutectic temperature, give the weight of solid phase and the weight of liquid phase that will be present.



Solid (kg) = 10

Liquid (kg) = 5

2

$\alpha = \frac{45-25}{45-15} \times 15 = 10$ kg

$Liq = \frac{25-15}{45-15} \times 15 = 5$ kg

h) If this alloy is then cooled to just below its eutectic temperature what amount of eutectic mixture will be present (kg)?

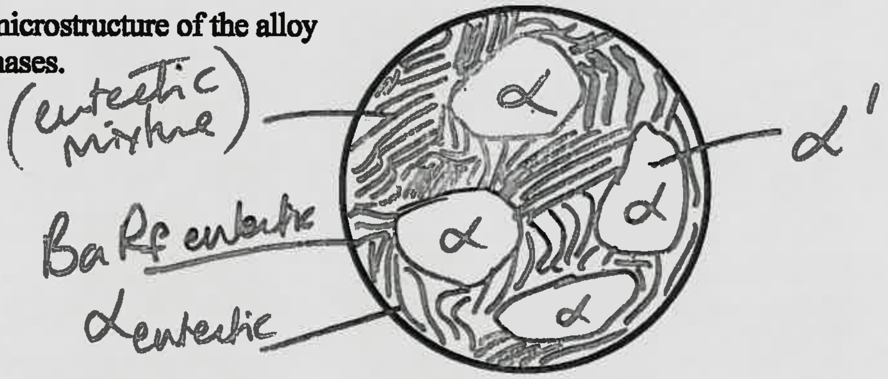
what is liquid just above becomes eutectic mix below ⇒ 5 kg.

$Liq = \frac{25-15}{45-15} \times 15 = 5$ kg.

Eutectic mixture (kg) = 5 kg

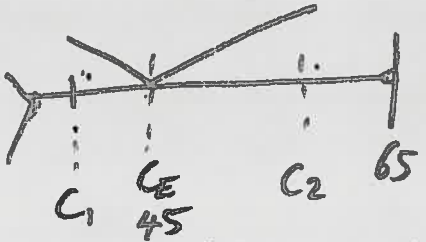
2

i) Draw a schematic of the microstructure of the alloy in part (h) and label the phases.



3

j) Is there another alloy composition that will give the same mass of this eutectic mixture as for the alloy in part (h)? If so, what is the composition?



On other side of eutectic. Will be BaRf + Liq → BaRf + eutectic mixture.

[✓ YES] / NO

Circle the correct one

1

%Liq ⇒ % Eut. Mix = $\frac{1}{3}$

$\frac{1}{3} = \frac{65-C_2}{65-45}$, $C_2 = 65 - \frac{20}{3}$

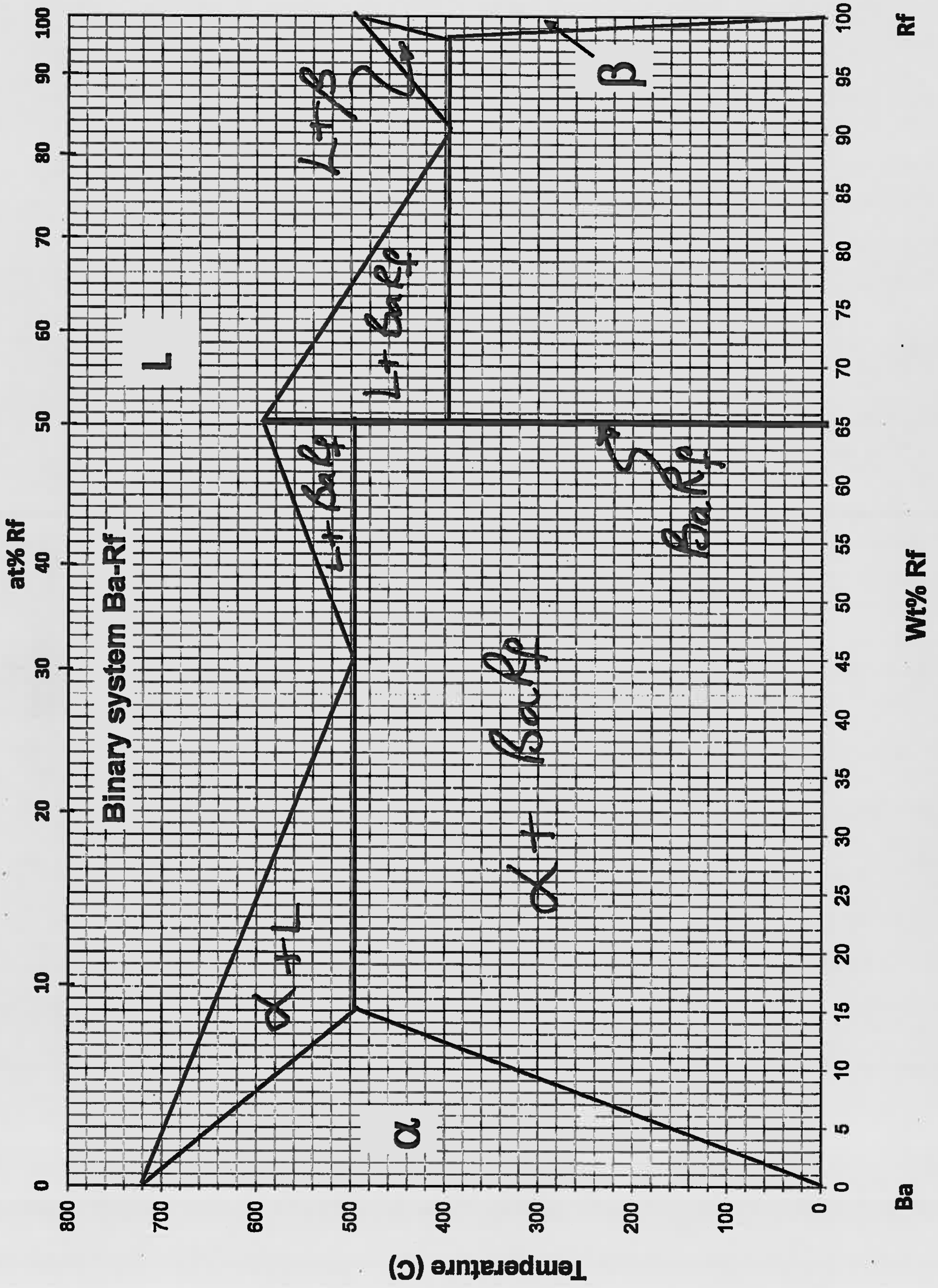
$= 58.3$ wt% Rf

If YES above then:

wt%Rf = 58.3

wt%Ba = 41.7

3


Binary system Ba-Rf
at% Rf
0
10
20
30
40
50
60
70
80
90
100
Wt% Rf
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Ba
Rf
Temperature (C)
800
700
600
500
400
300
200
100
0
L
α
β
α+L
L+BaRf
L+BaRf
α+BaRf
L+β
BaRf

Equation Sheet

$$F_{Net} = F_A + F_R \qquad E = \int F dr \qquad E_{Net} = E_A + E_R \qquad \rho = \frac{nA}{V_c N_A} \qquad n\lambda = 2d_{hkl} \sin\theta$$

$$\%ionicity = \left\{1 - \exp\left(-0.25[X_A - X_B]^2\right)\right\} x100 \qquad N_v = N \exp\left(\frac{-Q}{kT}\right) \qquad D = D_o \exp\left(\frac{-Q}{RT}\right)$$

$$\frac{C_X - C_0}{C_s - C_0} = 1 - erf\left(\frac{x}{2\sqrt{Dt}}\right) \qquad d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \qquad N = 2^{n-1} \qquad J = -\frac{M}{At}$$

$$J = -D\frac{dc}{dx} \qquad \frac{dC}{dt} = D\frac{d^2c}{dx^2} \qquad C_1 = \frac{m_1}{m_1 + m_2} x100 \qquad \tau = \frac{F}{A_0} = G\gamma$$

$$E = 2G(1+\nu) \qquad \%El = \left(\frac{l_f - l_0}{l_o}\right) x100 \qquad \nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z} \qquad \%AR = \left(\frac{A_o - A_i}{A_o}\right) x100$$

$$U_r = \frac{1}{2}\sigma_y \varepsilon_y = \frac{\sigma_y^2}{2E} \qquad \tau = G\gamma \qquad \%crystallinity = \frac{\rho_c(\rho_s - \rho_a)}{\rho_s(\rho_c - \rho_a)} \qquad TS = TS_\infty - \frac{A}{M_n}$$

$$K_{1c} = Y\sigma\sqrt{\pi a} \qquad \sigma_{fs} = \sigma_o \exp(-nP) \qquad E = E_o(1 - 1.9P + 0.9P^2)$$

$$\sigma_{fs} = \frac{3F_f L}{2bd^2} \qquad \sigma_{fs} = \frac{F_f L}{\pi R^3} \qquad E = \frac{F}{\delta}\frac{L^3}{4bd^3} \qquad E = \frac{F}{\delta}\frac{L^3}{12\pi R^4}$$

$$\overline{M}_n = \sum x_i M_i \qquad \overline{M}_w = \sum w_i M_i \qquad \Delta V = IR \qquad R = \frac{\rho L}{A} = \frac{L}{\sigma A} \qquad \rho_{total} = \rho_{thermal} + \rho_{impurity} + \rho_{def}$$

$$\rho_{impurity} = Ac_i(1 - c_i) \qquad \rho_{thermal} = \rho_o + aT \qquad \sigma = n|e|\mu_e + p|e|\mu_h \qquad C = \frac{dQ}{dT} \qquad \frac{\Delta L}{L_0} = \alpha(T_1 - T_2)$$

$$\sigma_{th} = E\alpha(T_0 - T_f) \qquad TSR \cong \frac{\sigma_f k}{E\alpha_l} \qquad q = -k\frac{dT}{dx}$$

$$c = \lambda v \qquad E = hv = \frac{hc}{\lambda} \qquad I_0 = I_T + I_A + I_R \qquad n = \frac{c}{v} \qquad E_g(\max) = \frac{hc}{\lambda(\min)} \qquad E_g(\min) = \frac{hc}{\lambda(\max)}$$

Constants

N_A = 6.023 x 10^{23} atoms/mol speed of light = 3 x 10^8 m/s
k = 8.62 x 10^{-5} eV/atom-K or 1.38 x 10^{-23} J/atoms-K
Planck's constant = 6.63 x 10^{-34} J-s = 4.13 x 10^{-15} eV-s
R = 8.314 J/(mol.K)