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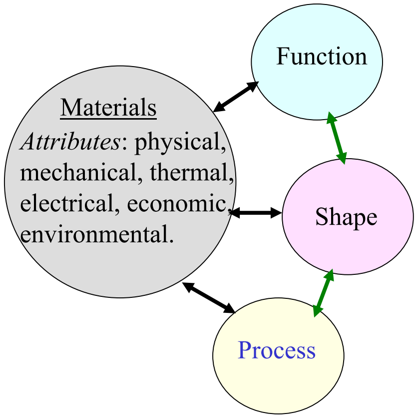
Materials Selection and Design: *Introduction*



Function

Materials

Attributes: physical, mechanical, thermal, electrical, economic, environmental.

Shape

Process

For selection, one must establish a link between materials and function, with shape and process playing also an important role (*We will focus on the function part*)

Design Requirements

Function: support a load, withstand temperature, transmit heat, etc.

What does the component do ?

Objective: make thing cheaply, light weight, increase safety, etc., or combinations of these.

What is to be maximized or minimized ?

Constraints: make thing cheaply, light weight, increase safety, etc., or combinations of these.

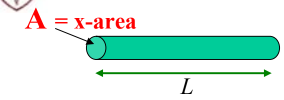
What essential conditions to be met ?

Free variables:

Which design variables are free ?

From which we obtain:
- ***Screening Criteria:*** go / no-go criteria
- ***Ranking Criteria:*** an ordering of the materials that “go”

Example 1: Strong and light Tie-Rod



- Tie-rod is common mechanical component.
- Tie-rod must carry tensile force, F, w/o failure.
- L is usually fixed by design.
- While strong, need lightweight, or low mass.

Function: Tie-rod

Objective: Minimize mass:

$M = A\,L\,\rho$ ……………………. (1)

Constraints:
- L = fixed length
- must not fail under load F (A can carry the load)

$F/A \le \sigma_f$ ……………………. (2)

Free variables:
- Material choice
- Section area A

Performance index: *Eliminate A in (1) and (2):*

fixed

$$m = \boxed{FL}\left(\frac{\rho}{\sigma_f}\right)$$

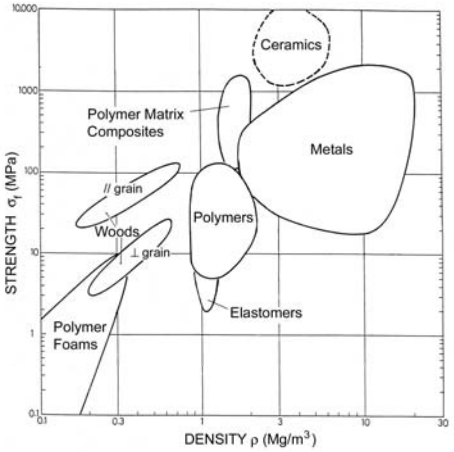
Chose materials with smallest ρ/σ_f

Example 1: Strong and light Tie-Rod

Maximize P = σ_f / ρ ***P is performance index***

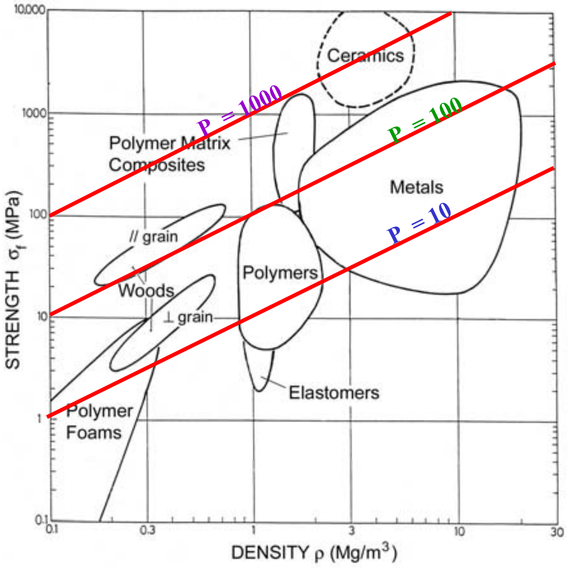
Performance Index where a higher number gives better performance

Consider $\log \sigma_f$ vs $\log \rho$



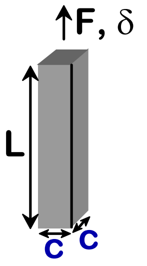
For fixed P: $\log P = \log \sigma_f - \log \rho = \text{constant} = C$

Example 1: Strong and light Tie-Rod



- For fixed P, look at
  $\log \sigma_f = \log \rho + C$
- For fixed P, you look for Lines of Slope = 1.
- Each Line of Slope = 1 have the same P values!
- But NOT the same materials properties (σ_f or ρ) e.g. some less dense (lighter).

Example 2: Stiff & Light Tension Members



- Bar must not lengthen by more than δ under force F; must have initial length L.

Stiffness relation:

$(\sigma = E\varepsilon)$ $\frac{F}{c^2} = E\frac{\delta}{L}$

Mass of bar:

$M = \rho L c^2$

- Eliminate the "free" design parameter, c:

$$M = \frac{FL^2}{\delta}\,\frac{\rho}{E}$$

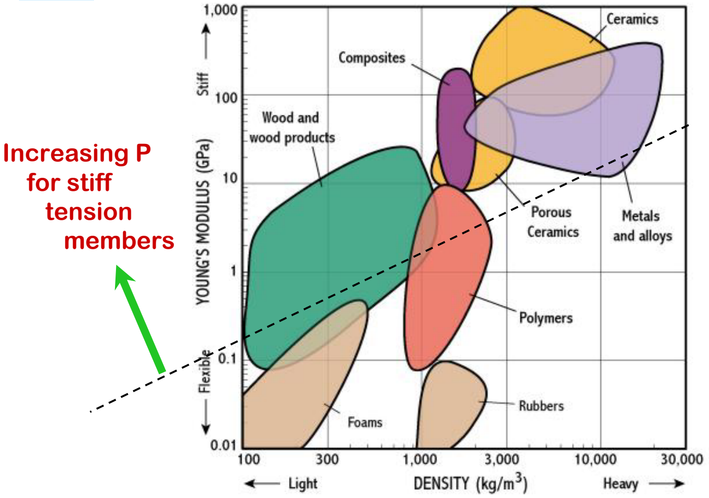
specified by application

minimize for small M

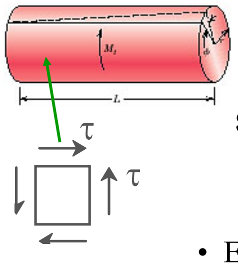
- **Maximize the Performance Index:** $P = \frac{E}{\rho}$
  (*stiff, light tension members*)

Example 2: Stiff & Light Tension Members

$P = \frac{E}{\rho}$ $\log P = \log E - \log \rho = C$



Example 3: Strong & Light Torsion Members



- Bar must carry a moment, M_t
- must have a length L.

Strength relation:

$$\frac{\tau_f}{N} = \frac{2M_t}{\pi R^3}$$

N is a safety factor

Mass of bar:

$$M = \rho \pi R^2 L$$

- Eliminate the "free" design parameter, R:

$$M = \left(2\sqrt{\pi}\, N M_t\right)^{2/3} L \frac{\rho}{\tau_f^{2/3}}$$

specified by application

minimize for small M

- **Maximize the Performance Inde** $P = \frac{\tau_f^{2/3}}{\rho}$

  (*strong, light torsion members*)

Example 3: Torsionally stressed shaft

Maximize Performance Material Index $P = \left[\frac{\tau_{ys}^{2/3}}{\rho}\right]$

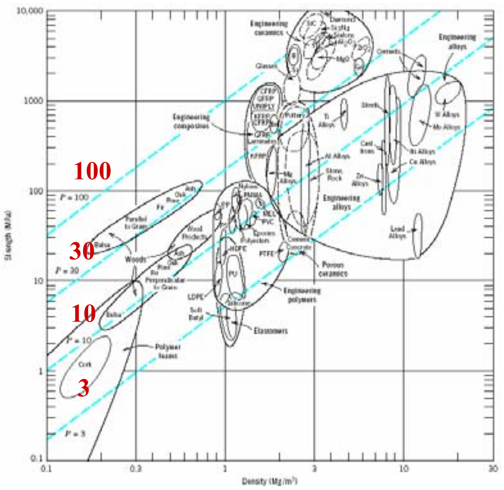
log τ = **3/2** [log ρ + log P]

Need to consider lines of **constant P** with **slope of 3/2 on log-scale**

Example 3: Torsionally stressed shaft

All parallel lines have same performance index.

However, P=30 has 1/3 the mass of P=10 (mass ∝ 1/P ∝ ρ).



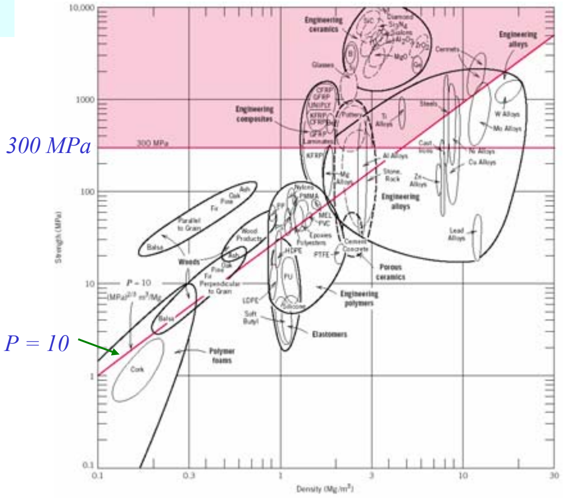
All materials that lie on these lines will perform equally for strength-per-mass basis.
However, each line has a different Materials **M index**, or overall Performance **P index**.

Example 3: Torsionally stressed shaft

Additional constraint

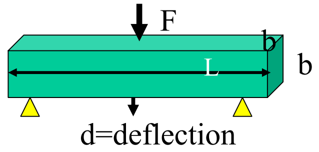
E.g. require P ≥ 10 and $\tau_f \geq 300$ MPa.

This identifies search region- shaded



Strength vs Density

Example 4: Light and Stiff Beam



Bending is common mode of loading in engineering, e.g., golf clubs, wing spars, floor joists.

Light, square beam ($A=b^2$) with length ***L***, loaded in bending must meet a constraint on its stiffness, S, so that it does not deflect more than ***d*** with load ***F***.

Stiffness Constraint $S=\frac{F}{d}\geq\frac{C_1E\mathbf{I}}{L^3}=\frac{C_1E}{L^3}\left(\frac{b^4}{12}\right)=\frac{C_1E}{L^3}\left(\frac{A^2}{12}\right)$ **Mass Constraint** $m = AL\rho.$

Eliminating Area, A: $m\geq\left(\frac{12S}{C_1L}\right)^{1/2}(L^3)\left(\frac{\rho}{E^{1/2}}\right)$ *(note L in first bracket)*

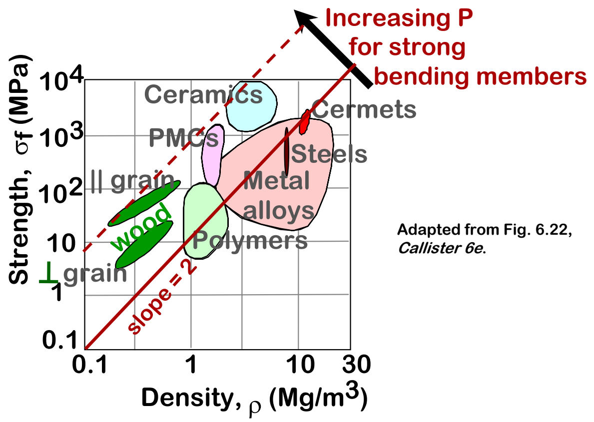
If beam remains square, <u>the Light, Stiff Beam</u> is one with largest **P =** $\left(\frac{E^{1/2}}{\rho}\right)$

If only beam height can change (not A), then $P = (E^{1/3}/\rho)$ (Car door) **$I \propto b^3w$**

If only beam width can change (not A), then $P = (E/\rho)$

Example 5: Strong & Light Bending Members

- **Maximize the Performance Index:** $P=\frac{\sigma^{1/2}}{\rho}$



Adapted from Fig. 6.22, *Callister 6e.*

Examples of Materials Indices

Function, Objective, and Constraint	Index	
Tie, minimum weight, stiffness	E/ρ	
Beam, minimum weight, stiffness	$E^{1/2}/\rho$	
Beam, minimum weight, strength	$\sigma^{2/3}/\rho$	
Beam, minimum cost, stiffness	$E^{1/2}/C_m\rho$	C_m =cost/mass
Beam, minimum cost, strength	$\sigma^{2/3}/C_m\rho$	
Column, minimum cost, buckling load	$E^{1/2}/C_m\rho$	
Spring, minimum weight for given energy storage	$\sigma_{YS}^2/E\rho$	
Thermal insulation, minimum cost, heat flux	$1/(\alpha\, C_m\rho)$	α =thermal cond
Electromagnet, maximum field, temperature rise	$\kappa\, C_p\rho$	κ =elec. cond

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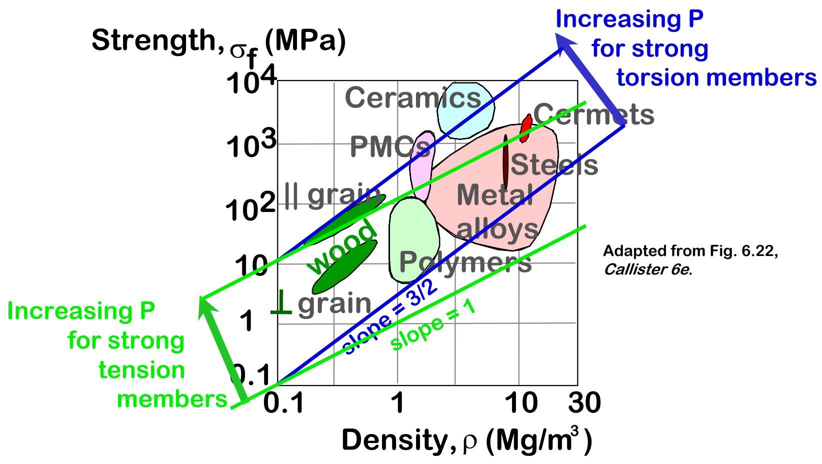
Strong & Light Tension vs. torsion Members



Adapted from Fig. 6.22, *Callister 6e.*

Next time:

Continue Materials Selection