Outline

- Stress Analysis of Cracks
- Fracture Toughness
- Effect of Geometry on Fracture Toughness
- Plane Strain Fracture Toughness
- Design Using Fracture Toughness
- Examples
  - *Pressure Vessels*
  - *Hydraulic Actuator*

Modes of Crack Displacement

- Three modes that a load can act on a crack:
  - ***mode I*** is tensile (most common)
  - *mode II and III* are sliding or tearing actions

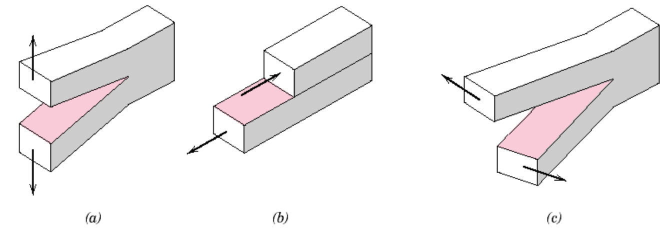
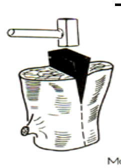
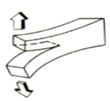


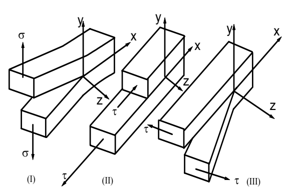
FIGURE 8.9 The three modes of crack surface displacement. (*a*) Mode I, opening or tensile mode; (*b*) mode II, sliding mode; and (*c*) mode III, tearing mode.

Stress distribution at crack tips depends on how crack is being extended *(Mode of cracking)*.

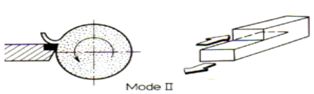
Modes of Crack Displacement

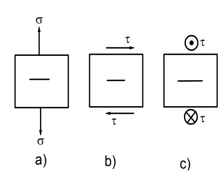




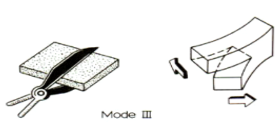


Mode I is opening or tensile mode where the crack surfaces move directly apart.





Mode II is sliding or in-plane shear mode where the crack surfaces slide over one another in a direction perpendicular to the leading edge of the crack.



Mode III is tearing and antiplane shear mode where the crack surfaces move relative to one another and parallel to the leading edge of the crack.

Modes of Crack Displacement

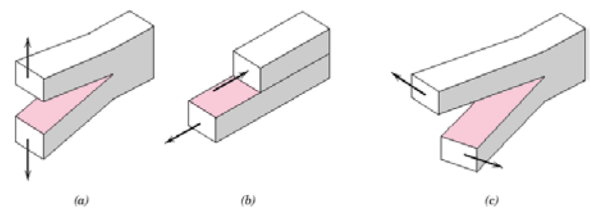


FIGURE 8.9 The three modes of crack surface displacement. (*a*) Mode I, opening or tensile mode; (*b*) mode II, sliding mode; and (*c*) mode III, tearing mode.

- In practice, crack propagation is not limited to the three basic modes and cracks often propagate under so called mixed modes, which are the …………… of the above mentioned modes, such as I-II, I-III, II-III and so on.
- In practice, however, crack propagation under ……….. is the most dangerous. Under Mode I, it is easier for crack propagation to trigger a brittle fracture, so it has been studied extensively.

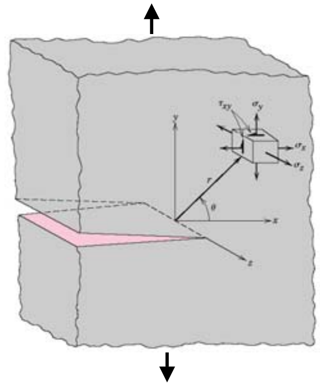
Stress Analysis of Cracks

- in mode I, the stresses acting on the crack as a function of radial distance r and angle θ are:

$$\sigma_x = \frac{K}{\sqrt{2\pi r}} f_x(\theta)$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} f_y(\theta)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} f_{xy}(\theta)$$



- K is the …………………. *factor* and defines the stress around a crack or flaw (*similar to but NOT the same as the stress concentration factor* K_t)

$$f_x(\theta) = \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$f_y(\theta) = \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$f_{xy}(\theta) = \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

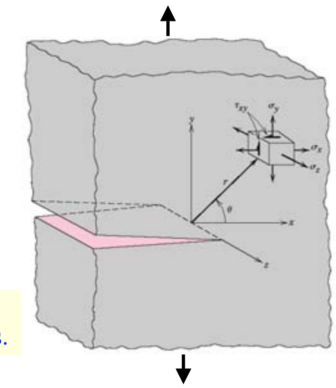
Fracture Toughness

- when the plate is thin, *plane stress* exists, i.e. $\sigma_z=0$
- when the plate is thick, $\sigma_z=\nu(\sigma_x+\sigma_y)$ and a state of *plane strain exists,* $\varepsilon_z= 0$
- **stress intensity factor** is related to the applied stress and crack length by:

$$K = Y\sigma\sqrt{\pi a}$$

Where,
Y is a function of the crack and specimen size and geometry (*dimensionless parameter*)

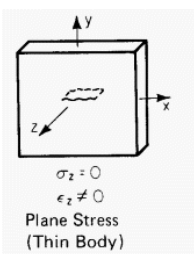
units of K are MPa m$^{1/2}$

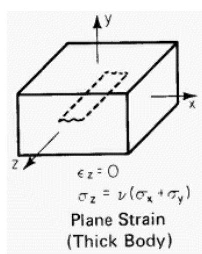


Thicker, more rigid pieces of a give material have a **………** fracture toughness than thin ones.

What is the difference between plane stress or plane strain conditions?

Plane Stress and Plane Strain Conditions





Plane stress condition:

- In a thin body, the stress through the thickness (σ_z) cannot vary appreciably due to the thin section.
- Because there can be no stresses normal to a free surface, $\sigma_z = 0$ throughout the section and a biaxial state of stress results.

Plane strain condition:

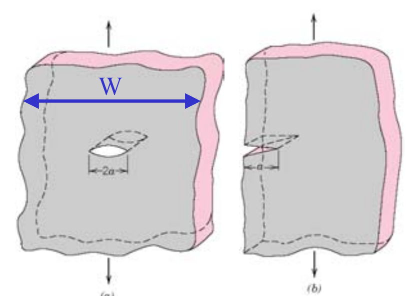
- In a thick body, the material is constrained in the z direction due to the thickness of the cross section and $\varepsilon_z = 0$.
- Due to Poisson`s effect, a stress, σ_z, is developed in the z direction.

Fracture Toughness

***Fracture Toughness,* K$_c$** is a critical value of stress intensity factor, K, at which **brittle fracture** will occur.

- When stress level reaches some critical level, σ_c, have crack propagation and fracture
- critical stress intensity factor, K_c, at the crack tip exists:

$$K_c = Y(a/W)\sigma_c\sqrt{\pi a}$$



- Y is a function of a and W (component width)
- for wide plates and short cracks a/W → 0

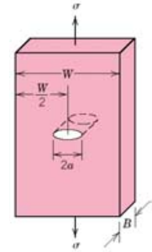
a) Y(a/W) = 1 (infinite plate)

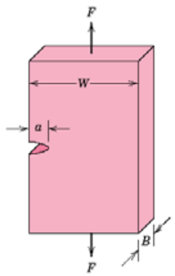
b) Y(a/W) ≈ 1.1 (semi-infinite plate)

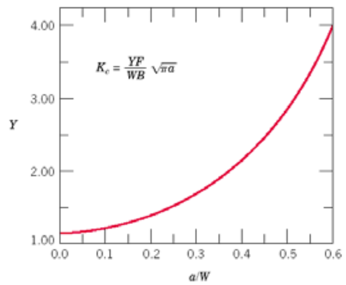
Effect of Specimen Geometry

- For a central, through thickness crack:

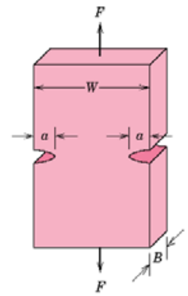
$$Y(a/W) = \left(\frac{W}{\pi a}\right)\tan\frac{\pi a}{W}$$

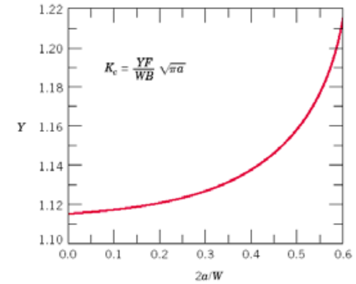


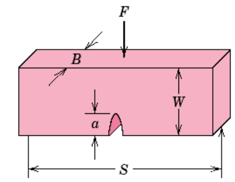


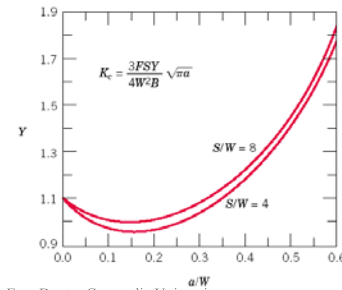


Effect of Specimen Geometry







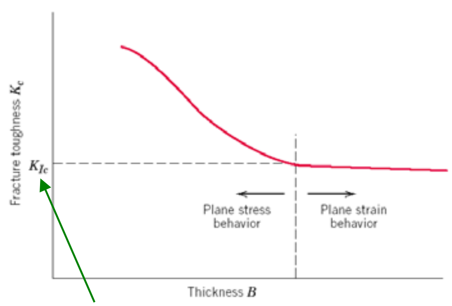


Plane Strain Fracture Toughness, K_{Ic}

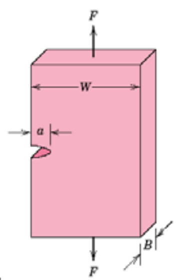
- K_c is a function of specimen thickness, B
- when K_c in *mode I* becomes independent of B, then have *plane strain fracture toughness, K_{Ic}*:

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$

pronounced as "kay-.......-see"



minimum value → safer for design



✓ plane strain condition exists when:

$$B \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

Design Using Fracture Mechanics

- Important parameters are: K_c or K_{Ic}, applied stress σ and crack size a
- If K_{Ic} and a are fixed then the critical stress is:

$$\sigma_c \leq \frac{K_{Ic}}{Y\sqrt{\pi a}}$$

- If design involves a maximum allowable crack size then:

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma Y}\right)^2$$

Table 8.1 **Room-Temperature Yield Strength and Plane Strain Fracture Toughness Data for Selected Engineering Materials**

	Yield Strength		K_{Ic}	
Material	*MPa*	*ksi*	*MPa√m*	*ksi√in.*
Metals				
Aluminum Alloy[a] (7075-T651)	495	72	24	22
Aluminum Alloy[a] (2024-T3)	345	50	44	40
Titanium Alloy[a] (Ti-6Al-4V)	830	120	55	50
Alloy Steel[a] (4340 tempered @ 260°C)	1640	238	50.0	45.8
Alloy Steel[a] (4340 tempered @ 425°C)	1420	206	87.4	80.0
Ceramics				
Concrete	—	—	0.2–1.4	0.18–1.27
Soda-Lime Glass	—	—	0.7–0.8	0.64–0.73
Aluminum Oxide	—	—	2.7–4.2	2.5–3.8
Polymers				
Polystyrene (PS)	—	—	0.7–1.1	0.64–1.0
Polymethyl Methacrylate (PMMA)	53.8–73.1	7.8–10.6	0.7–1.6	0.64–1.5
Polycarbonate (PC)	62.1	9.0	2.2	2.0

Summary

- Stress intensity factor, K, describes the stress intensity felt by that material under a particular loading condition, so if load varies, or specimen shape varies etc., then K will vary
  - *in a similar way that the stress on a component can vary*
- When the stress intensity in a brittle material reaches a particular value, K_{Ic} then something happens - i.e. ……… *fracture occurs*
  - *in a similar way to yield strength being the stress when a material starts plastically deforming*
- Tests are used to measure K_{Ic} using Mode I crack opening and calculating the Y scale parameter.
- K_{Ic} is a fundamental material property that is affected by:
  - temperature (K_{Ic} …. as T↑)
  - strain rate (K_{Ic} ….. as SR↑)
  - strengthening (usually K_{Ic} …… as σ_y↑)
  - microstructure (K_{Ic} ….. as grain size ↓)

Design using Fracture Mechanics

$$K_{Ic} = Y\sigma\sqrt{\pi a}$$

Material Property: so can select material with appropriate value of K_{Ic}

This could be design stress (including safety factor) or applied stress

This is the allowable flaw size or smallest flaw that can be detected

✓ During design, we have to decide which parameters are constrained by the application and which can be controlled by **design**.

✓ For example, K_{Ic} may be fixed because of the need for certain material; density, corrosion resistance etc

✓ Or flaw size may be limited by detection equipment available.

BUT ONCE TWO PARAMETERS ARE FIXED SO IS THE THIRD!

Design using Fracture Mechanics

- Example: If K_{Ic} and "*a*" are fixed because a particular material is required, then the design (or *critical*) stress is limited as:

$$\sigma_c \leq \frac{K_{Ic}}{Y\sqrt{\pi a}}$$

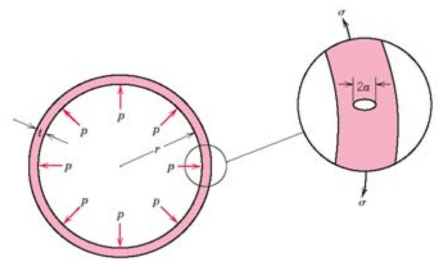
- Or if the stress level is fixed and material has been chosen (K_{Ic} is fixed) then the maximum allowable flaw size in the material is given by:

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma Y}\right)^2$$

So manufacturing techniques must be good enough to produce flaws less than this size and NDT (non-destructive testing) techniques must be **good enough** to measure flaws this size.

Example: Pressure Vessels

Use of fracture mechanics to design pressure vessels.



Schematic diagram showing the cross section of a ***spherical*** *tank that is subjected to an internal pressure* p*, and that has a radial crack of length* 2a *in its wall.*

- one method is for wall to ……. before failure so plastic deformation can be observed before formation of crack of critical size and fast fracture occurs. *Requires observation/inspection to notice yielding etc.*
- second method is design for ………-*before-burst*. Ensure that critical crack size for fast fracture is greater than wall thickness. (i.e. a_c = t, allowing for safety). So crack can grow through thickness of wall without fast fracture (bursting) and *vessel will leak allowing detection of leak by pressure drop and presence of fluid*.

Example: Pressure Vessels

Method 1: require circumferential wall stress less than yield strength.

Assume wall stress given by : $\sigma = \frac{pr}{2t}$

and using a safety factor, N, then :

$$K_{Ic} = Y\left(\frac{\sigma_y}{N}\right)\sqrt{\pi a_c}$$

and then solving for a_c :

$$a_c = \frac{N^2}{Y^2\pi}\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

So look for materials with best ratio of $(K_{Ic}/\sigma_y)^2$

Table 8.2 Ranking of Several Metal Alloys Relative to Critical Crack Length (Yielding Criterion) for a Thin-Walled Spherical Pressure Vessel

Material	$\left(\frac{K_{Ic}}{\sigma_y}\right)^2$ (mm)
Medium carbon (1040) steel	43.1
AZ31B magnesium	19.6
2024 aluminum (T3)	16.3
Ti-5Al-2.5Sn titanium	6.6
4140 steel (tempered @ 482°C)	5.3
4340 steel (tempered @ 425°C)	3.8
Ti-6Al-4V titanium	3.7
17-7PH steel	3.4
7075 aluminum (T651)	2.4
4140 steel (tempered @ 370°C)	1.6
4340 Steel (tempered @ 260°C)	0.93

Example: Pressure Vessels

$$K_{Ic} = Y\sigma\sqrt{\pi a} \qquad \sigma = \frac{pr}{2\,t}$$

Method 2: Using a = t, then and substituting for t from above:

$$K_{Ic} = Y\sigma\sqrt{\pi t}$$

$$p = \frac{2}{Y^2\pi r}\left(\frac{K_{Ic}^2}{\sigma_y}\right)$$

∴ different parameters for optimization.

Table 8.3 Ranking of Several Metal Alloys Relative to Maximum Allowable Pressure (Leak-Before-Break Criterion) for a Thin-Walled Spherical Pressure Vessel

Material	$\frac{K_{Ic}^2}{\sigma_y}$ (MPa-m)
Medium carbon (1040) steel	11.2
4140 steel (tempered @ 482°C)	6.1
Ti-5Al-2.5Sn titanium	5.8
2024 aluminum (T3)	5.6
4340 steel (tempered @ 425°C)	5.4
17-7PH steel	4.4
AZ31B magnesium	3.9
Ti-6Al-4V titanium	3.3
4140 steel (tempered @ 370°C)	2.4
4340 steel (tempered @ 260°C)	1.5
7075 aluminum (T651)	1.2

Example:

A 7049- T73 Al forging is the material of choice for an 8 cm-internal diameter, hydraulic actuator cylindrical housing that has a wall thickness of 1 cm. After manufacture, each cylinder is subjected to a safety check, involving a single fluid over-pressurization that generates a hoop stress no higher than 50% σ_{ys}. The component design calls for an operating internal fluid pressure, corresponding to a hoop stress no higher than 25% σ_{ys}. Prior to over-pressurization, a 2mm-deep semicircular surface flaw that was oriented normal to the hoop stress direction was discovered in one cylinder. Given that σ_{ys} = 460 MPa and K_{Ic} = 23 MPa.m$^{1/2}$,

a) Would the cylinder have survived the over-pressurization test?

b) Would the cylinder experience a leak-before-break condition?

c) Also, what were the fluid pressure levels associated with the overpressurization cycle and design stress?

Example:

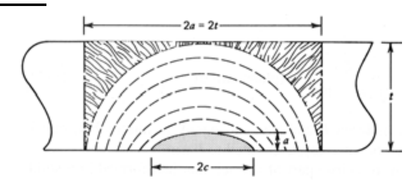
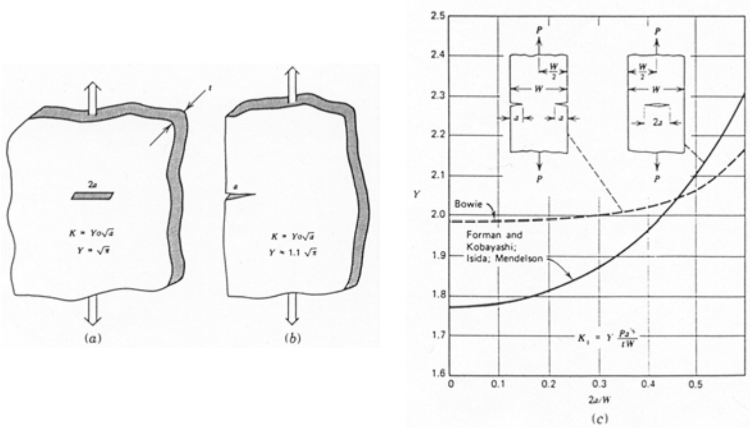
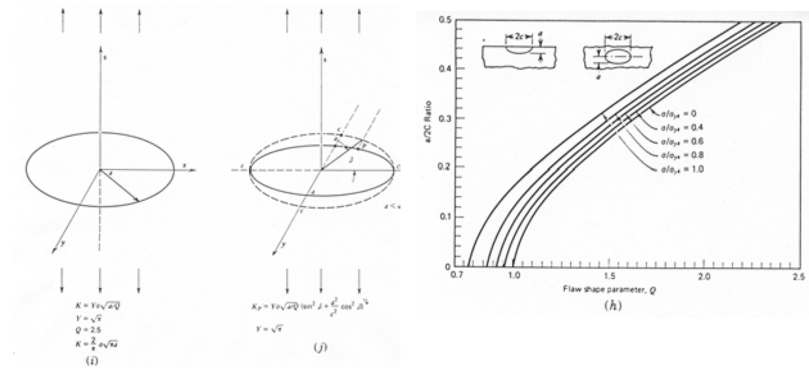


Diagram showing growth of semi elliptical surface flaw to semicircular configuration. At leak condition *(a = t),* unbroken ligaments (shaded areas) break open to form through-thickness crack.

Example:



Example:



Next Time

Impact Test and Fatigue