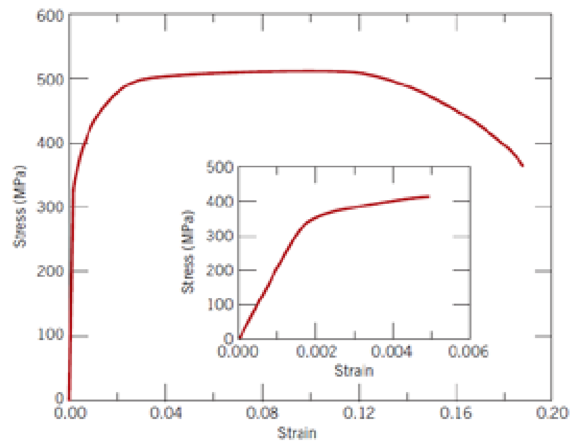
Question 1:

A steel alloy specimen having a rectangular cross section of dimensions 10 mm × 5 mm has the stress–strain behavior shown on the right. If this specimen is subjected to a tensile force of 20,000 N then:

(a) Determine the elastic and plastic strain values.

(b) If its original length is 350 mm, what will be its final length after the load in part (a) is applied and then released?



Solution:

(a) Determining the applied stress:

$$\sigma = \frac{F}{A_0} = \frac{F}{b_0 d_0} = \frac{20000N}{(10 \times 10^{-3}) \times (5 \times 10^{-3})} = 400 \times 10^6 \; N/m^2 = 400MPa$$

From the Figure, this point is in the plastic region so the specimen will be both elastic and plastic strains. The total strain at this point, Ɛt, is about 0.01. We are able to estimate the amount of permanent strain recovery Ɛe from Hooke's law, as

$$e_e = \frac{s}{E}$$

And, since E = 207 GPa for steel (Table 6.1)

$$\varepsilon_e = \frac{400 \; MPa}{207 \times 10^3 \; MPa} \approx 0.002$$

Plastic strain is just the difference between the total and elastic strains; that is

$\varepsilon_p = \varepsilon_t - \varepsilon_e = 0.01 - 0.002 = 0.008$

(b) If the initial length is 350 mm then the final specimen length l_i may be determined from a rearranged form of Equation 6.2 using the plastic strain value as:

$l_i = l_0(1 + \varepsilon_p) = (350 \text{ mm})(1 + 0.008) = 352.8$ mm

Question 2:

A cylindrical rod 120 mm long and having a diameter of 15.0 mm is to be deformed using a tensile load of 35,000 N. It must not experience either plastic deformation or a diameter reduction of more than $1.2\text{x}10^{-2}$ mm. Of the materials listed below, which are possible candidates? Justify your choice(s).

Material	Modulus of Elasticity (GPa)	Yield Strength (MPa)	Poisson's Ratio
Aluminum Alloy	70	250	0.33
Titanium Alloy	105	850	0.36
Steel Alloy	205	550	0.27
Magnesium Alloy	45	170	0.35

Solution:

This problem asks that we assess the four alloys relative to the two criteria presented. The first criterion is that the material not experience plastic deformation when the tensile load of 35,000 N is applied; this means that the stress corresponding to this load not exceed the yield strength of the material. Upon computing the stress

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi(\frac{d}{s})^2} = \frac{35000\ N}{\pi \times \frac{15 \times 10^{-3} m}{2}} = 200 \times 10^6 \frac{N}{m^2} = 200\ Mpa$$

Of the alloys listed in the table, the Al, Ti and steel alloys have yield strengths greater than 200 MPa. Relative to the second criterion, it is necessary to calculate the change in diameter Δd for these two alloys. From Equation 6.8

$$v = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_0}{\sigma / E}$$

Now, solving for Δd from this expression,

$$\Delta d = -\frac{v \sigma d_0}{E}$$

For the aluminum alloy

$$\Delta d = -\frac{(0.33)(200\ MPa)(15\ mm)}{70 \times 10^3\ MPa} = -1.41 \times 10^{-2}\ mm$$

Therefore, the Al alloy is not a candidate. For the steel alloy:

$$\Delta d = -\frac{(0.27)(200\ MPa)(15\ mm)}{250 \times 10^3\ MPa} \approx -0.40 \times 10^{-2}\ mm$$

Therefore, the steel is a candidate. For the Ti alloy:

$$\Delta d = -\frac{(0.36)(200\ MPa)(15\ mm)}{105 \times 10^3\ MPa} \approx -1.0 \times 10^{-2}\ mm$$

Hence, the titanium alloy is also a candidate.

Question 3:

For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking:

Engineering Stress (MPa)	Engineering Strain
235	0.194
250	0.296

On the basis of this information, compute the *engineering* stress necessary to produce an *engineering* strain of 0.25.

Solution:

For this problem we first need to convert engineering stresses and strains to true stresses and strains so that the constants **K** and **n** in Equation (6.19) may be determined.

Since $\sigma_T = \sigma(1 + \varepsilon)$ then

$$\sigma_{T1} = (235 \text{ MPa})(1 + 0.194) = 280 \text{ MPa}$$

$$\sigma_{T2} = (250 \text{ MPa})(1 + 0.296) = 324 \text{ MPa}$$

Similarly for strains, since $\varepsilon_T = \ln(1 + \varepsilon)$ then

$$\varepsilon_{T_1} = \ln(1 + 0.194) = 0.177$$

$$\varepsilon_{T_2} = \ln(1 + 0.296) = 0.259$$

Taking the logarithm of Equation (6.19), we get: $\log \sigma_T = \log K + n \log \varepsilon_T$

which allows us to set up two simultaneous equations for the above pairs of true stresses and true strains, with K and n as unknowns. Thus

$$\log(280) = \log K + n \log(0.177)$$

$$\log(324) = \log K + n \log(0.259)$$

Solving for these two expressions yields K = 543 MPa and n = 0.383.

Now, converting $\varepsilon = 0.25$ to true strain, $\varepsilon_T = \ln(1 + 0.25) = 0.223$

The corresponding σ_T to give this value of ε_T is just

$$\sigma_T = K\,\varepsilon_T^{\,n} = (543 \text{ MPa})(0.223)^{0.383} = 306 \text{ MPa}$$

Now converting this σ_T to an engineering stress: $\sigma = \sigma_T/1+\varepsilon = \dfrac{306 \text{ MPa}}{1 + 0.25} = 245 \text{ MPa}$

Question 4:

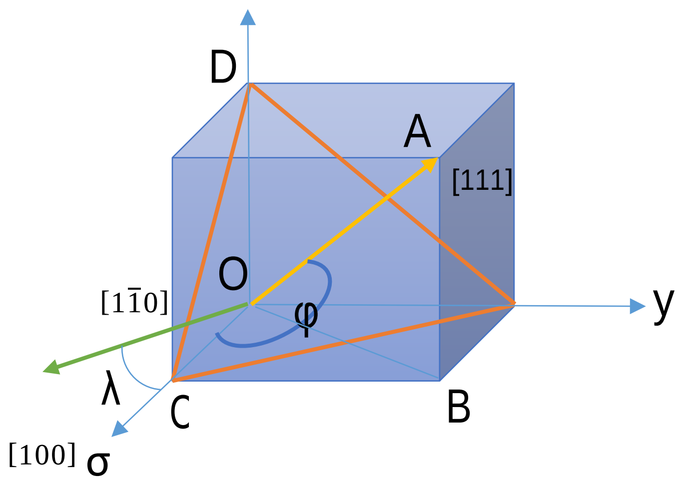
A single crystal of a metal that has the FCC crystal structure is oriented such that a tensile stress is applied parallel to the [100] direction. If the critical resolved shear stress for this material is 0.5 MPa, calculate the magnitude of applied stress necessary to cause slip to occur on the (111) plane in the $[1\bar{1}0]$ direction.

Solution:

The critical shear stress is:

$$\tau_{cr} = \sigma \, \mathrm{Cos}\phi \, \mathrm{Cos}\, \lambda$$

.



According to the above image, the angle λ is the angle between the tensile axis i.e., along the [100] direction and the slip direction which is $[1\bar{1}0]$. Since there is FCC unit cell, each face can be assumed as a square. So it will be 45°.
Furthermore, φ is the angle between the tensile axis the [100] direction and the normal to the slip plane i.e., the (111) plane; for this case this normal is along a [111] direction. To find φ, first we can find the length of CA in OAC triangle and then go for φ. To find the length of CA, we know that, CA is diagonal of one face in FCC unit cell and its length is 4R. Also OC is a side of FCC unit cell and according to the Pythagoras theory it is equal to 2√2 R.
$\tan^{-1}\varphi$ = CA/OC= 4R/(2√2 R) = 1.41 So φ = 54.7°
And λ = 45°
And, finally, the an applied force to cause slip occur on the plan (111) is equal to

$\mathrm{Cos}45 \times \mathrm{Cos}\ 54.7 = 0.7 \times 0.57 = 0.4$

Hence $\sigma = \frac{\tau_{cr}}{\mathrm{Cos}\theta\ \mathrm{Cos}\,\lambda} = \frac{0.5\ Mpa}{0.4} = 1.25 Mpa$

Question 5:

(a) A 10-mm-diameter Brinell hardness indenter produced an indentation 1.62 mm in diameter in a steel alloy when a load of 500 kg was used. Compute the HB of this material.

(b) What will be the diameter of an indentation to yield a hardness of 450 HB when a 500 kg load is used?

Solution:

(a) Using the equation in Table 6.5 for HB, where P = 500 kg, d = 1.62 mm, and D = 10 mm:

$$HB = \left[\frac{2P}{\pi D\left(D - \sqrt{D^2 - d^2}\right)}\right] = \frac{2 \times 500}{\pi \times 10\left(10 - \sqrt{10^2 - 1.62^2}\right)} = 241$$

(b) Solving for d from the equation in Table 6.5:

$$d = \sqrt{D^2 - \left[D - \frac{2P}{(HB)\pi D}\right]^2} = \sqrt{(10)^2 - \left[10 - \frac{2 \times 500}{\pi \times 10 \times 450}\right]^2} = 1.19mm$$