**Question 1:**
Two previously undeformed cylindrical specimens of an alloy are to be strain hardened by reducing their cross-sectional areas (while maintaining their circular cross sections). For one specimen, the initial and deformed radii are 16 mm and 11 mm, respectively. The second specimen, with an initial radius of 12 mm, must have the same deformed hardness as the first specimen; compute the second specimen's radius after deformation.

**Solution:**
In order for these two cylindrical specimens to have the same deformed hardness, they must be deformed to the same percent cold work (Equation 7.8). For the first specimen

\[
\%CW = \frac{A_0 - A_d}{A_0} \times 100 = \frac{\pi r_0^2 - \pi r_d^2}{\pi r_0^2} \times 100
\]

\[
= \frac{\pi (16 \text{ mm})^2 - \pi (11 \text{ mm})^2}{\pi (16 \text{ mm})^2} \times 100 = 52.7 \% CW
\]

For the second specimen, the deformed radius is computed using the above equation and solving for \(r_d\) as

\[
r_d = r_0 \sqrt{1 - \frac{\%CW}{100}} =
\]

\[
= (12 \text{ mm}) \sqrt{1 - \frac{52.7\%CW}{100}} = 8.25 \text{ mm}
\]

**Question 2:**
The lower yield point for an iron that has an average grain diameter of \(5 \times 10^{-2} \text{ mm}\) is 135 MPa. At a grain diameter of \(8 \times 10^{-3} \text{ mm}\), the yield point increases to 260 MPa. At what grain diameter will the lower yield point be 205 MPa?

**Solution:**
The best way to solve this problem is to first establish two simultaneous expressions of Equation 7.7, solve for \(\sigma_0\) and \(k_f\), and finally determine the value of \(d\) when \(\sigma_y = 205 \text{ MPa}\). The data pertaining to this problem may be tabulated as follows:

<table>
<thead>
<tr>
<th>(\sigma_y) (MPa)</th>
<th>(d) (mm)</th>
<th>(d^{1/2}) (mm)^{1/2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>140 MPa</td>
<td>(6 \times 10^{-2})</td>
<td>4.08</td>
</tr>
<tr>
<td>270 MPa</td>
<td>(9 \times 10^{-3})</td>
<td>10.54</td>
</tr>
</tbody>
</table>

The two equations thus become
140 MPa = σ₀ + (4.08) kᵣ

270 MPa = σ₀ + (10.54) kᵣ

Which yield the values, σ₀ = 57.9 MPa and kᵣ = 20.12 MPa(mm)\(^{1/2}\). At a yield strength of 200 MPa

200 MPa = 57.9 MPa + [20.12 MPa (mm)\(^{1/2}\)] t\(^{-1/2}\)

or \(d\)\(^{1/2}\) = 7.06 (mm)\(^{-1/2}\), which gives \(d = 2 \times 10^{-2}\) mm.

**Question 3:**

The average grain diameter for a brass material was measured as a function of time at 650°C, which is tabulated below at two different times:

(a) What was the original grain diameter?
(b) What grain diameter would you predict after 150 min at 650°C?

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Grain Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3.9 × 10⁻²</td>
</tr>
<tr>
<td>90</td>
<td>6.6 × 10⁻²</td>
</tr>
</tbody>
</table>

**Solution:**

(a) Using the data given, we may set up two simultaneous equations with \(d₀\) and \(K\) as unknowns; thus

\[(3.9 \times 10^{-2} \text{ mm})^2 - d₀^2 = (30 \text{ min})K\]

\[(6.6 \times 10^{-2} \text{ mm})^2 - d₀^2 = (90 \text{ min})K\]

Solution of these expressions yields a value for \(d₀\), the original grain diameter, of \(d₀ = 0.01\) mm, and a value for \(K\) of \(4.73 \times 10^{-5}\) mm\(^²\)/min

(b) At 150 min, the diameter \(d\) is computed as:

\[d = \sqrt{d₀^2 + Kt}\]

\[= \sqrt{(0.01 \text{ mm})^2 + (4.73 \times 10^{-5} \text{ mm}^2/\text{min}) (150 \text{ min})} = 0.085 \text{ mm}\]
Question 4:

We have a piece of aluminum-copper alloy with the composition of Al – 4 wt% Cu. The alloy is supposed to be strengthened via both precipitation hardening and strain hardening. Use the Al-Cu phase diagram and design the fabrication procedure for the alloy.

Solution:

The specimen must be heated up to $T_0 \approx 540 \, ^\circ C$ in the $\alpha$-phase region and then quenched to room temperature. Then the sample must be cold worked to reach the desired shape. After cold working, it must be aged at $T_1 \approx 450 \, ^\circ C$ in the $\alpha$-$\theta$ region to let the $\theta$-phase precipitate in the $\alpha$-matrix. In this case the strength of the specimen has improved due to cold working and precipitation hardening procedures.

Question 5:

(a) List four major differences between deformation by twinning and deformation by slip relative to mechanism, conditions of occurrence, and final result.

(b) Briefly explain why HCP metals are typically more brittle than FCC and BCC metals.

(c) Briefly cite the differences between recovery and recrystallization processes.

(d) What is the driving force for recrystallization and grain growth?
Solution:

(a) Four major differences between deformation by twinning and deformation by slip are as follows: (1) with slip deformation there is no crystallographic reorientation, whereas with twinning there is a reorientation; (2) for slip, the atomic displacements occur in atomic spacing multiples, whereas for twinning, these displacements may be other than by atomic spacing multiples; (3) slip occurs in metals having many slip systems, whereas twinning occurs in metals having relatively few slip systems; and (4) normally slip results in relatively large deformations, whereas only small deformations result for twinning.

(b) Hexagonal close packed metals are typically more brittle than FCC and BCC metals because there are fewer slip systems in HCP.

(c) For recovery, there is some relief of internal strain energy by dislocation motion; however, there are virtually no changes in either the grain structure or mechanical characteristics. During recrystallization, on the other hand, a new set of strain-free grains forms, and the material becomes softer and more ductile.

(d) The driving force for recrystallization is the difference in internal energy between the strained and unstrained material. The driving force for grain growth is the reduction in grain boundary energy as the total grain boundary area decreases.