## Question 1:

The density of polystyrene is $1.05 \mathrm{~g} / \mathrm{cm}^{3}$. Calculate the volume of air space in the walls of an expanded polystyrene foam cup of $50 \mathrm{~kg} / \mathrm{m}^{3}$ density.

## Solution:

$1 \mathrm{~g} / \mathrm{cm}^{3}=1000 \mathrm{~kg} / \mathrm{m}^{3}$
$\rho=$ mass $/$ volume $\rightarrow$ dense polystyrene $1050 \mathrm{~kg} / \mathrm{m}^{3}$
The foam under question has $50 \mathrm{~kg} / \mathrm{m}^{3} \boldsymbol{\rightarrow} 50 / 1050=0.0476$
or the relative density is $4.75 \% \rightarrow$ the volume of air $=100-4.76=95.24 \%$

## Question 2:

(i) The green compact shown in the accompanied figure (part a) has a hub of 0.960 in height and a flange of 0.300 in thickness. Calculate the position of the lower punches for filling the die cavity (figure b) if the density of loose powder is $38 \%$ and the density of the green compact is $78 \%$ of theoretical density.
(ii) The green compact shown in the accompanied figure (part a) is sintered to $96 \%$ of the theoretical density. The composition is $\mathrm{Fe}-8 \mathrm{Cu}-2 \mathrm{C}$. Calculate (1) the theoretical density and (2) shrinkage of the 0.960 -in dimension. (Hint: the densities of the constituent elements are Fe: 7.87; Cu: 8.96; C (graphite): $2.25 \mathrm{~g} / \mathrm{cm}^{3}$.

## Solution:

(i) The mass of loose powder (after leveling, Fig. c) must be equal to the mass of green compact. Powder cannot be counted upon to flow from hub to flange or vice versa; therefore, the same mass of powder remains in the flange (and in the hub) before and after pressing. Since mass is (volume) x (relative density), we may write $V_{l}($ rel. dens. loose $)=V_{g}($ rel. dens. green $)$
Since no powder flows from flange to hub and vice versa, we may write $V_{l}=A h_{l}$ and $V_{g}=A h_{g}$
where $h_{l}$ is the height of loose powder column and $h_{g}$ is the thickness of the green compact.
Consequently,
$h_{l}=h_{g}$ (rel. dens. green)/(rel. dens. loose)
For the hub, $h_{l}=0.960(0.78 / 0.38)=1.97 \mathrm{in}$; for the flange, $h_{l}=0.300(0.78 / 0.38)=$ 0.645 in.
(ii) -1) The volume of $100-\mathrm{g}$ ponder is
$(90 / 7.87)+(8 / 8.96)+(2 / 2.25)=13.21 \mathrm{~cm}^{3}$
Theoretical density is $100 / 13.21=7.566 \mathrm{~g} / \mathrm{cm}^{3}$
sintered density is 0.96 theoretical density. A cube of 1 -em side length and 0.78 density contains material of $0.78 \mathrm{~cm}^{3}$ volume. Since the mass is unchanged during sintering, it will now occupy a volume of $0.78 / 0.96-0.8125 \mathrm{~cm}^{3}$. The side of this shrunken cube is $(0.8125)^{1 / 3}=0.933 \mathrm{~cm}$, and linear shrinkage is $1-0.933=0.067$ or $6.7 \%$. Thus, the 0.960 -in dimension shrinks to $0.960(0.933)$ - 0.896 in .

(d) Powder forced into upper punch cavity


(e) Compact pressed

## Question 3:

The time dependence of crystallization can be described by Avrami equation:

$$
1-y=\exp \left(-k t^{n}\right)
$$

Where, y is the fraction crystallized, k and n are time-independent constants whose values depend on the crystallizing system. Using the accompanied figure, determine $k$ and $n$.

Plot of normalized fraction crystallized versus the logarithm of time for polypropylene at constant temperatures of $140^{\circ} \mathrm{C}, 150^{\circ} \mathrm{C}$, and $160^{\circ} \mathrm{C}$.


## Solution:

In this problem we are asked to determine the values of the constants $n$ and $k$ for the crystallization of polypropylene at $150^{\circ} \mathrm{C}$. One way to solve this problem is to take two values of percent recrystallization [which is just 100 y in the equation] and their corresponding time values, then set up two simultaneous equations, from which n and k may be determined. In order to expedite this process, we will rearrange and do some algebraic manipulation of the equation. First of all, we rearrange as follows:

$$
1-y=\exp \left(-k t^{n}\right)
$$

Now taking natural logarithms

$$
\ln (1-y)=-k t^{n}
$$

Or

$$
-\ln (1-y)=k t^{n}
$$

which may also be expressed as

$$
\ln \left(\frac{1}{1-y}\right)=k t^{n}
$$

Now taking natural logarithms again, leads to

$$
\ln \left[\ln \left(\frac{1}{1-y}\right)\right]=\ln k+n \ln t
$$

which is the form of the equation that we will now use. From the $150^{\circ} \mathrm{C}$ curve, let us arbitrarily choose two percent crystallized values of $20 \%$ and $80 \%$ (i.e., $\mathrm{y}_{1}=0.20$ and $\mathrm{y}_{2}$ $=0.80$ ). Their corresponding time values are $\mathrm{t}_{1}=220 \mathrm{~min}$ and $\mathrm{t}_{2}=460 \mathrm{~min}$ (realizing that the time axis is scaled logarithmically). Thus, our two simultaneous equations become

$$
\begin{aligned}
& \ln \left[\ln \left(\frac{1}{1-0.2}\right)\right]=\ln k+n \ln (220) \\
& \ln \left[\ln \left(\frac{1}{1-0.8}\right)\right]=\ln k+n \ln (460)
\end{aligned}
$$

from which we obtain the values $\mathrm{n}=2.68$ and $\mathrm{k}=1.2 \times 10^{-7}$.
Haider: Students are encouraged to find n and k for other temperatures and to compare the results. Also, it is recommended to compare the experimental (given) curves with the calculated ones.

## Question 4:

An extruder has a barrel diameter $=5.0$ inch and length $=12 \mathrm{ft}$. The extruder screw rotates at $50 \mathrm{rev} / \mathrm{min}$; it has channel depth $=0.30$ in and flight angle $=17.7^{\circ}$. The plastic melt has a shear viscosity $=100 \times 10^{-4} \mathrm{lb}-\mathrm{sec} / \mathrm{in}^{2}$. Find: (a) the extruder characteristic, (b) the values of Q and p at the operating point, given that the die characteristic is $\mathrm{Q}_{\mathrm{x}}=$ 0.00150 p .

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Solution: (a) \(\mathrm{Q}_{\max }=0.5 \pi^{2}(5)^{2}(50 / 60)(.3) \sin 17.7 \cos 17.7=30.84(0.3040)(0.9527)=8.93 \mathrm{in}^{3} / \mathrm{sec}\)
\(\mathrm{p}_{\max }=6 \pi(5)(50 / 60)(144)(.01)(\cot 17.7) /(0.3)^{2}=3937.6 \mathrm{lb} / \mathrm{in}^{2}\)
\(\mathrm{Q}_{\mathrm{x}}=\mathrm{Q}_{\max }-\left(\mathrm{Q}_{\max } / \mathrm{p}_{\max }\right) \mathrm{p}=8.93-0.002268 \mathrm{p}\)
(b) Given: die characteristic \(\mathrm{Q}_{\mathrm{x}}=\mathbf{0 . 0 0 1 5} \mathrm{p}\)
\(\mathrm{Q}_{\mathrm{x}}=8.93-0.002268 \mathrm{p}=0.0015 \mathrm{p}\)
\(0.00377 \mathrm{p}=8.93 \quad \mathbf{p}=\mathbf{2 3 7 0} \mathbf{~ l b} / \mathbf{i n}^{2}\)
\(\mathrm{Q}_{\mathrm{x}}=8.93-0.002268(2370)=\mathbf{3 . 5 5} \mathbf{~ i n}^{3} / \mathrm{sec}\)
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## Question 5:

(i) A rotational molding operation is to be used to mold a hollow playing ball out of polyethylene. The ball will be 1.5 ft in diameter and its wall thickness should be $1 / 16 \mathrm{in}$. What weight of PE powder should be loaded into the mold in order to meet these specifications? The specific gravity of the PE grade is 0.95 .
(ii) The extrusion die for a polyethylene parison used in blow molding has a mean diameter $=$ 16.0 mm . The size of the ring opening in the die $=1.5 \mathrm{~mm}$. The mean diameter of the parison is observed to swell to a size of 20.5 mm after exiting the die orifice. If the diameter of the blow molded container is to be 100 mm , determine (a) the corresponding wall thickness of the container and (b) the wall thickness of the parison.

Solution: Density $\rho=0.95(62.4 / 1728)=0.0343 \mathrm{lb} / \mathrm{in}^{3}$
Volume $=\pi\left(\mathrm{D}_{0}{ }^{3}-\mathrm{D}_{\mathrm{i}}{ }^{3}\right) / 6=0.16667 \pi\left[(1.5 \times 12)^{3}-(1.5 \times 12-2 / 16)^{3}\right]=63.18 \mathrm{in}^{3}$
Weight $=(63.18)(0.0343)=\mathbf{2 . 1 7} \mathbf{l b}$.
(ii)

Solution: (a) $\mathrm{r}_{\text {sd }}=20.5 / 160=1.281$
$\mathrm{t}_{\mathrm{m}}=(1.281)^{2}(1.5)(16.0) / 100.0=\mathbf{0 . 5 0 5} \mathrm{mm}$
(b) $t_{p}=(1.281)^{2}(1.5)=\mathbf{2 . 4 6 2 ~ m m}$

## Question 6:

An extruder barrel has a diameter and length of 100 mm and 2.8 m , respectively. The screw rotational speed $=50 \mathrm{rev} / \mathrm{min}$, channel depth $=7.5 \mathrm{~mm}$, and flight angle $=17^{\circ}$. The plastic melt has a shear viscosity = 175 Pas. Determine: (a) the extruder characteristic, (b) the shape factor $K_{s}$ for a circular die opening with diameter $=3.0 \mathrm{~mm}$ and length $=12.0 \mathrm{~mm}$, and (c) the operating point ( $Q$ and $p$ ).

## Solution:

$$
\begin{aligned}
& \mathrm{Q}_{\max }=\mathrm{Q}_{\mathrm{d}}=0.5 \pi^{2}(.1)^{2}(50 / 60)\left(7.5 \times 10^{-3}\right) \sin 17 \cos 17=308.4 \times 10^{-6}(0.2924)(0.9563) \\
& \quad=86.2 \times 10^{-6} \mathrm{~m}^{3} / \mathrm{s} \\
& \mathrm{P}_{\max }=6 \pi(.1)(50 / 60)(2.8)(175)(\cot 17) /\left(7.5 \times 10^{-3}\right)^{2}=44.75 \times 10^{6} \mathrm{~Pa}=44.75 \mathrm{MPa} \\
& \mathrm{Q}_{\mathbf{x}}=86.2 \times 10^{-6}-\mathbf{1 . 9 2 6} \times 10^{-12} \mathbf{p}, \text { where } \mathrm{p} \text { has units of } \mathrm{Pa} \\
& \text { (b) Given: } \mathrm{D}_{\mathrm{d}}=3 \mathrm{~mm}, \mathrm{~L}_{\mathrm{d}}=12 \mathrm{~mm} . \\
& \mathrm{K}_{\mathbf{s}}=\pi\left(3 \times 10^{-3}\right)^{4} / 128(175)\left(12 \times 10^{-3}\right)=\mathbf{0 . 9 4 6 7 \times 1 0 ^ { - 1 2 }} \\
& \left(\text { (c) } 0.9467 \times 10^{-12} \mathrm{p}=86.2 \times 10^{-6}-1.926 \times 10^{-12} \mathrm{p}\right. \\
& 2.8727 \times 10^{-12} \mathrm{p}=86.2 \times 10^{-6} \\
& \mathrm{p}=\mathbf{3 0 . 0} \times 10^{6} \mathbf{~ P a}=\mathbf{3 0} \mathbf{~ M P a} \\
& \mathrm{Q}_{\mathrm{x}}=0.9467 \times 10^{-12}\left(30 \times 10^{6}\right)=\mathbf{2 8 . 4} \times \mathbf{1 0}^{-6} \mathbf{m}^{\mathbf{3}} / \mathbf{s}
\end{aligned}
$$

Check with extruder characteristic: $\mathrm{Q}_{\mathrm{x}}=86.2 \times 10^{-6}-1.926 \times 10^{-12}\left(30 \times 10^{6}\right)=28.4 \times 10^{-6} \mathrm{~m}^{3} / \mathrm{s}$.

## Question 7:

The foreman in the injection molding department says that a polyethylene part produced in one of the operations has greater shrinkage than the calculations indicate it should have. The important dimension of the part is specified as $112.5 \pm 0.25 \mathrm{~mm}$. However, the actual molded part measures 112.02 mm . (a) As a first step, the corresponding mold cavity dimension should be checked. Compute the correct value of the mold dimension, given that the shrinkage value for polyethylene is 0.025 (from Table 13.1 of Groover's text). (b) What adjustments in process parameters could be made to reduce the amount of shrinkage.

Solution: (a) Given: $\mathrm{S}=0.025, \mathrm{D}_{\mathrm{c}}=112.5+112.5(.025)+112.5(.025)^{2}=\mathbf{1 1 5 . 3 8 3} \mathbf{~ m m}$ (b) Adjustments to reduce shrinkage include: (1) increase injection pressure, (2) increase compaction time, and (3) increase molding temperatures.

