Question 1

Derive the force equations for F, N, F_s, and F_n as functions of cutting and thrust forces using Merchant's circle.

Solution: Eq. (21.9): In Figure 23.11, construct a line starting at the intersection of $\mathbf{F_t}$ and $\mathbf{F_c}$ that is perpendicular to the friction force $\mathbf{F}$. The constructed line is at an angle α with $\mathbf{F_c}$. The vector $\mathbf{F}$ is divided into two line segments, one of which = $\mathbf{F_c} \sin \alpha$ and the other = $\mathbf{F_t} \cos \alpha$.
Thus, $\mathbf{F} = \mathbf{F_c} \sin \alpha + \mathbf{F_t} \cos \alpha$. Q.E.D.

Eq. (21.10): In Figure 23.11, translate vector $\mathbf{N}$ vertically upward until it coincides with the previously constructed line, whose length = $\mathbf{F_c} \cos \alpha$. Next, translate vector $\mathbf{F_t}$ to the right and downward at an angle α until its base is at the arrowhead of $\mathbf{F}$. $\mathbf{F_t}$ now makes an angle α with $\mathbf{F}$. The arrowhead of $\mathbf{F_t}$ will now be at the base of the translated base of $\mathbf{N}$. The distance along the previously constructed line between the $\mathbf{F_t}$ arrowhead (base of translated $\mathbf{N}$ vector) and $\mathbf{F}$ is $\mathbf{F_t}$ $\sin\alpha$.
Hence, $\mathbf{N} = \mathbf{F_c} \cos \alpha - \mathbf{F_t} \sin \alpha$ Q.E.D.

Eq. (21.11): In Figure 23.11, extend vector $\mathbf{F_s}$ in the opposite direction of its arrowhead, and from the intersection of $\mathbf{F_t}$ and $\mathbf{F_c}$ construct a line that is perpendicular to vector $\mathbf{F_s}$. A right triangle now exists in which $\mathbf{F_c}$ is the hypotenuse and the two sides are (1) the extended $\mathbf{F_s}$ vector and (2) the constructed line that runs between $\mathbf{F_s}$ and the intersection of $\mathbf{F_c}$ and $\mathbf{F_t}$. The extended $\mathbf{F_s}$ vector is related to $\mathbf{F_c}$ as $\mathbf{F_c} \cos \phi$. The length difference between the extended $\mathbf{F_s}$ vector and the original $\mathbf{F_s}$ vector is $\mathbf{F_t} \sin \phi$.
Thus $\mathbf{F_s}$ (original) = $\mathbf{F_c} \cos \phi - \mathbf{F_t} \sin \phi$ Q.E.D.

Eq. (21.12): In Figure 23.11, construct a line from the intersection of $\mathbf{F_t}$ and $\mathbf{F_c}$ that is perpendicular to and intersects with vector $\mathbf{F_n}$. Vector $\mathbf{F_n}$ is now divided into two line segments, one of which = $\mathbf{F_t} \cos \phi$ and the other = $\mathbf{F_c} \sin \phi$.
Hence, $\mathbf{F_n} = \mathbf{F_c} \sin \phi + \mathbf{F_t} \cos \phi$ Q.E.D.

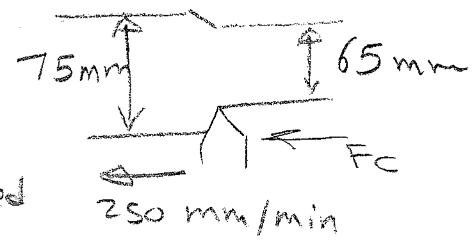
Question 2

A 200 mm long, 75 mm diameter titanium alloy rod is being reduced in diameter to 6.5 mm by turning on a lathe. The spindle rotates at 400 rpm, and the tool is traveling at an axial velocity of 250 mm/min. Calculate the cutting speed, material removal rate, time of cut, power required and cutting force. (hint: the specific energy of titanium ranges from 3.0 to 4.1 W.s/mm^3)

Solution:

$N = 400 \text{ rpm}$

$d = \dfrac{75-65}{2} = 5 \text{ mm}$ (depth of cut)



$V_c = N \pi D_o \Rightarrow$ maximum cutting speed

$= 400 * 3.14 * 75 = \underline{94200} \text{ mm/min}$

$MRR = \pi D_{avg} * d * f * N, \quad D_{avg} = 70 \text{ mm}$

f = how many mm/rev the cutter makes

$\dfrac{mm}{rev} = \dfrac{mm}{min} * \dfrac{min}{rev} = 250 * \dfrac{1}{400} = 0.625 \text{ mm/rev}$

$\Rightarrow MRR = \pi * 70 * 5 * 0.625 * 400 = 2.75 * 10^5 \text{ mm}^3/\text{min}$

$= \underline{4580} \text{ mm}^3/\text{sec}$

$time = l/speed = \dfrac{200}{250} = 0.8 \text{ min} = \underline{48 \text{ sec}}$

$U = \dfrac{3. + 4.1}{2} = 3.5 \dfrac{W.s}{mm^3}$

$P = MRR * U = 4580 \dfrac{mm^3}{sec} * 3.5 \dfrac{W.Sec}{mm^3} = 16030 W$

$= 16 KW$

$P = F_c * V_c = T * \omega \quad \omega = \underset{N}{400} * \dfrac{2\pi}{60} = 41.88 \text{ rad/sec}$

$\Rightarrow T = \underline{383} \text{ N.m}$

$F_c = \dfrac{P}{V_c \Rightarrow average} = \dfrac{16030 W}{87.92 m/min} * 60 \dfrac{sec}{min} = \underline{10940 N}$

Question 3

In a surface grinding operation performed on hardened plain carbon steel, the grinding wheel has a diameter = 200 mm and width = 25 mm. The wheel rotates at 2400 rev/min, with a depth of cut (in-feed) = 0.05 mm/pass and a cross-feed = 3.50 mm. The reciprocating speed of the work is 6 m/min, and the operation is performed dry. Determine: (a) the length of contact between the wheel and the work, (b) the volume rate of metal removed. (c) If C = 0.64 active grits/mm^2, estimate the number of chips formed per unit time. (d) What is the average volume per chip? (e) If the tangential cutting force on the work = 30 N, compute the specific energy in this operation?

Solution:

Solution: (a) $l_c = (200 \times 0.05)^{0.5}$ = **3.16 mm**

(b) MRR = $v_w wd$ = (6 m/min)(10^3 mm/m)(3.5 mm)(0.05 mm) = **1050 mm^3/min**

(c) $n_c = vwC$

$v = N\pi D$ = (2400 rev/min)(200π mm/rev) = 1,507,964 mm/min

n_c = (1,507,964 mm/min)(3.5 mm)(0.64 grits/mm^2) = **3,377,840 grits/min (= chips/min).**

(d) 3,377,840 grits/min. = 3,377,840 chips/min.

Average volume per chip = (1050 mm^3/min)/(3,377,840 chips/min) = **0.00031 mm^3/chip**

(e) $U = F_c v$/MRR

v = 1,507,964 mm/min = 1,508 m/min

U = 30(1508)/1050 = **43.1 N-m/mm^3**

Question 4

A slab-milling operation is being carried out on a 30 in. long, 6 in. wide high strength steel block at a feed of 0.01 in./tooth and a depth of cut of 0.15 in. The cutter has a diameter of 3 in. has eight straight cutting teeth, and rotates at 150 rpm. Calculate the material removal rate and the cutting time, and estimate the power required.

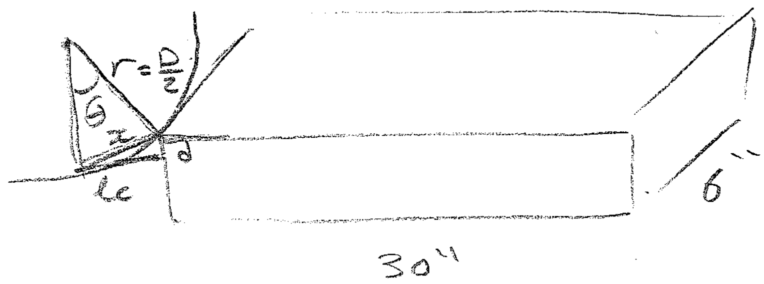
Solution:

$f = 0.01''/\text{tooth}$

$d = 0.15''$

$D_{cutter} = 3''$

$n = 8$ teeth $\quad N = 150\,rpm$



$MRR = w * d * v_c$?

$v_c = f * N * n = 0.01 * 150 * 8 = 12\ in/min$

$MRR = 6 * 0.15 * 12 = 10.8\ in^3/min$

$P = 1.1\ hp\text{-}\cancel{min}/\cancel{in^3} * 10.8 \frac{\cancel{in^3}}{\cancel{min}} = 11.9\ hp$

$t = \frac{l}{v_c} = \frac{30''}{12''}\ min = \underline{2.56\ min}$

Question 5:

An orthogonal cutting operation is being carried out under the following conditions: depth of cut = 0.15 mm, width of cut = 5 mm, chip thickness = 0.2 mm, cutting speed = 2 m/s, rake angle = 15°, cutting force = 500 N, and thrust force = 200 N. Calculate the percentage of the total energy that is dissipated in the shear plane during cutting.

Solution:

$d = 0.15\ mm$

$w = 5\ mm$

$t_c = 0.2\ mm$

$V_c = 2\ m/s$

$\alpha = 15°$

$F_c = 500N$

$F_t = 200\ N$

$P_{total} = F_c * V_c = 500N * 2\ \frac{m}{s} = 1000\ N.m/s$

$P_{shear} = F_s * V_s$

$$\boxed{\begin{aligned} F_s &= R\cos(\phi + \beta - \alpha) \\ R &= \sqrt{F_c^2 + F_t^2} = 538\ N \end{aligned}}$$

$r = \frac{t_o}{t_c} = \frac{0.15}{0.2} = 0.75$

$\phi = \tan^{-1} \frac{r\cos\alpha}{1 - r\sin\alpha} = 42°$

$F_c = R\cos(\beta - \alpha) \Rightarrow \beta = \cos^{-1}\left(\frac{F_c}{R}\right) + \alpha = 36.7°$

$F_s = R\cos(\phi + \beta - \alpha) = 238\ N$

$V_s = \frac{V\cos\alpha}{\cos(\phi - \alpha)} = \frac{2\cos 15°}{\cos(42 - 15)} = 2.17\ m/s$

$\Rightarrow P_{shear} = F_s * V_s = 238 * 2.17 = 516\ \frac{N.m}{s}$

$\frac{516}{1000} = 0.516$ or 51.6%

Question 6:

i) A series of turning tests are performed to determine the parameters *n*, *m*, and *K* in the expanded version of the Taylor's equation. The following data were obtained during the tests: (1) $v = 2.0$ m/s, $f = 0.20$ mm/rev, $T = 12$ min; (2) $v = 1.5$ m/s, $f = 0.20$ mm/rev, $T = 40$ min; and (3) $v = 2.0$ m/s, $f = 0.3$ mm/rev, $T = 10$ min. (a) Determine *n*, *m*, and *K*. (b) Using your equation, compute the tool life when $v = 1.5$ m/s and $f = 0.3$ mm/rev.

ii) Using the Taylor equation for tool wear and letting $n = 0.4$, calculate the percentage increase in tool life if the cutting speed is reduced by (a) 20% and (b) 50%.

i) **Solution**: Three equations to be solved simultaneously:

(1) $(2 \times 60)(12)^n(0.2)^m = K$

(2) $(1.5 \times 60)(40)^n(0.2)^m = K$

(3) $(2 \times 60)(10)^n(0.3)^m = K$

(1) and (2): $\ln 120 + n \ln 12 + m \ln 0.2 = \ln 90 + n \ln 40 + m \ln 0.2$

$\ln 120 + n \ln 12 = \ln 90 + n \ln 40$

$4.7875 + 2.4849\, n = 4.4998 + 3.6889\, n$

$0.2877 = 1.204\, n$ **$n = 0.239$**

(1) and (3): $\ln 120 + 0.239 \ln 12 + m \ln 0.2 = \ln 120 + 0.239 \ln 10 + m \ln 0.3$

$0.5939 + m(-1.6094) = 0.5503 + m(-1.2040)$

$-0.4054\, m = -0.0436$ **$m = 0.108$**

(1) $K = 120(12)^{0.239}(0.2)^{0.108} = 120(1.811)(0.8404) = $ **$K = 182.65$**

(b) $v = 1.5$ m/s, $f = 0.3$ mm/rev

$(1.5 \times 60)(T)^{0.239}(0.3)^{0.108} = 182.65$

$90(T)^{0.239}(0.8781) = 182.65$

$(T)^{0.239} = 2.311$

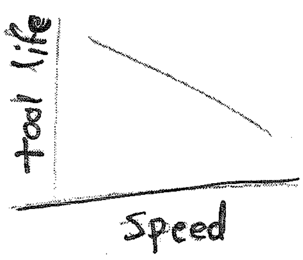
$T = 2.311^{1/.239} = $ **33.3 min.**

ii)

$T^n V = C$

$T_1^n V_1 = T_2^n V_2$

$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{1/n} = \left(\frac{V_2}{V_1}\right)^{2.5}$



(a) $V_2 = 0.8 V_1 \Rightarrow \frac{T_1}{T_2} = (0.8)^{2.5} = 0.57$

$T_2 = \frac{T_1}{0.57} = 1.75 T_1$

75% increase in life

(b) $V_2 = 0.5 V_1 \Rightarrow \frac{T_1}{T_2} = (0.5)^{2.5} = 0.1768$

$T_2 = 5.657 T_1$

466% increase in tool life

Question 7:

A gun-drilling operation is used to drill a 7/16-in diameter hole to a certain depth. It takes 4.5 minutes to perform the drilling operation using high pressure fluid delivery of coolant to the drill point. The cutting conditions are: N = 3000 rev/min at a feed = 0.002 in/rev. In order to improve the surface finish in the hole, it has been decided to increase the speed by 20% and decrease the feed by 25%. How long will it take to perform the operation at the new cutting conditions?

Solution:

Solution: f_r = 3000 rev/min(0.001 in/rev) = 3.0 in/min.
Hole depth d = 4.5 min(3.0 in/min.) = 13.5 in.
New speed v = 3000(1 + 0.20) = 3600 rev/min.
New feed f = 0.001(1- 0.25) = 0.00075 in/min.
New feed rate f_r = 3600(0.00075) = 2.7 in/min.
New drilling time T_m = 13.5 in/2.7 in/min = **5.0 min.**