Optoelectronic Devices & Communication Networks

Montreal

Toronto

Montreal

Toronto

Ottawa

Add/Drop WDM

Amplifier

Switch

WDM

λ₁

λ₂

λ₃

λₙ

λ₁

λ₂

λ₃

λₙ
Optoelectronic Devices for Communication Networks

» Optical Sources
» LED
» Laser
» Optical Diodes
» WDM
» Fiber Optics
» Optical Amplifiers
» Optical Attenuators
» Optical Isolators
» Optical Switches
» Add/Drop Devices
Optoelectronic Devices for Communication Networks

• Requirements to understand the concepts of Optoelectronic Devices:

1. We need to study concepts of light properties
2. Some concepts of solid state materials in particular semiconductors.
3. Light + Solid State Materials
Light Properties

• Wave/Particle Duality Nature of Light

• Reflection
  • Snell’s Law and Total Internal Reflection (TIR)
  • Reflection & Transmission Coefficients
  • Fresnel’s Equations
  • Intensity, Reflectance and Transmittance

• Refraction
  • Refractive Index

• Interference
  • Multiple Interference and Optical Resonators

• Diffraction
  • Fraunhofer Diffraction
  • Diffraction Grating
Light Properties

- Dispersion

- **Polarization of Light**
  - Elliptical and Circular Polarization
  - Birefringent Optical Devices
  - Electro-Optic Effects
  - Magneto-Optic Effects
Some Concepts of Solid State Materials

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Light

• The nature of light

Wave/Particle Duality Nature of Light

\[ E = h \nu = \frac{hc}{\lambda} \]
\[ E = mc^2 \]
\[ p = \frac{h}{\lambda} \]

--Particle nature of light (photon) is used to explain the concepts of solid state optical sources (LASER, LED), optical detectors, amplifiers,…

--The wave nature of light is used to explain refraction, diffraction, polarization,… used to explain the concepts of light transmission in fiber optics, WDM, add/drop/ modulators,…
The wave nature of Light

Electromagnetic Spectrum

Three basic bands: infrared (wavelengths above 0.7\(\mu\)m), visible (wavelengths between 0.4-0.7\(\mu\)m), and ultraviolet light (wavelengths below 0.4\(\mu\)m).

\[ E = h\nu = hc / \lambda; \quad c = \nu \lambda. \]

An emitted light from a semiconductor optical device has a wavelength proportional to the semiconductor band-gap.

Longer wavelengths for communication systems; \( E_g \approx 1\mu\)m. (lower Fiber loss).

Shorter wavelengths for printers, image processing,\ldots \( E_g > 1\mu\)m.
> Optical Spectrum

Optical communications

1.7 μm  0.8 μm

Visible

Red
~0.7 μm

Violet
~0.4 μm

Cosmic rays

Gamma rays

Ultraviolet

X-rays

Far Infrared

Millimetre wave

VHF

UHF

Microwave

Short wave

Standard Broadcast

Long wave

Frequency (Hz)

10^2  10^4  10^6  10^8  10^10  10^12  10^14  10^16  10^18  10^20  10^22

3000 Km  30 Km  300 m  3 m  3 cm  0.3 mm  3 μm  30 nm  0.3 nm  3 pm  0.3 pm

Wavelength
The wave nature of Light

- Polarization
- Reflection
- Refraction
- Diffraction
- Interference

To explain these concepts light can be treated as rays (geometrical optics) or as an electromagnetic wave (wave optics, studies related to Maxwell Equations).

An electromagnetic wave consist of two fields:
- Electric Field
- Magnetic Field
An electromagnetic wave is a travelling wave which has time varying electric and magnetic fields which are perpendicular to each other and the direction of propagation, \( z \).

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An electromagnetic wave consist of two components; electrical field and magnetic field components.

\( k \) is the wave vector, and its magnitude is \( \frac{2\pi}{\lambda} \)

Light can treated as an EM wave, \( E_x \) and \( B_y \) are propagating through space in such a way that they are always perpendicular to each other and to the direction of propagation \( z \).
The wave nature of Light

We can treat light as an EM wave with time varying electric and magnetic fields. $E_x$ and $B_y$ which are propagating through space in such a way that they are always perpendicular to each other and to the direction of propagation $z$. Traveling wave (sinusoidal):

$$E(r,t) = E_0 \cos(\omega t - k.r + \phi_0)$$
A plane EM wave travelling along \( z \), has the same \( E_x \) (or \( B_y \)) at any point in a given \( xy \) plane. All electric field vectors in a given \( xy \) plane are therefore in phase. The \( xy \) planes are of infinite extent in the \( x \) and \( y \) directions.

\[ E_x = E_0 \sin(\omega t - kz) \]

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The wave nature of Light

\[ E(r,t) = E_0 \cos(\omega t - kr + \phi_0) \]

\[ E_x(z,t) = \text{Re}[E_0 e^{j\phi_0} e^{j(\omega t-kz)}] \]

\[ \phi = \omega t - kz + \phi_0 = \text{constant} \]

During a time interval \( \Delta t \), a constant phase moves a distance \( \Delta z \),

Phase velocity: \[ V = \frac{\Delta z}{\Delta t} = \frac{dz}{dt} = \frac{\omega}{k} = v\lambda \]

\[ \omega = 2\pi v \]

Phase difference: \[ \Delta \phi = k\Delta z = \frac{2\pi \Delta z}{\lambda} \]
Optical Divergence

Electric field component of EM wave:

\[ E(r, t) = E_0 \cos(\omega t - kr + \phi_0) \]

\[ E = \frac{A}{r} \cos(\omega t - kr) \]

These are the solutions of Maxwell’s equation

\[ \nabla^2 E = \varepsilon\mu \frac{\partial^2 E}{\partial^2 t} \]

Maxwell’s Wave Equations

Examples of possible EM waves

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Gaussian Beams

(a) Wavefronts of a Gaussian light beam. (b) Light intensity across beam cross section. (c) Light irradiance (intensity) vs. radial distance \( r \) from beam axis \( (z) \).

\[ W_0 \text{ is called waist radius and } 2W_0 \text{ is called spot size} \]

\[ 2\theta = \frac{4\lambda}{\pi(2W_0)} \]

Is called beam divergence

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Refractive Index

If the light is traveling in dielectric medium, assuming nonmagnetic and isotropic we can use Maxwell’s equations to solve for electric field propagation, however we need to define a new phase velocity.

\[ V = \frac{1}{\sqrt{\varepsilon \mu_0}} \]  \hspace{1cm} \text{Phase velocity} \hspace{1cm} \varepsilon = \varepsilon_r \varepsilon_0

\[ V = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c \]  \hspace{1cm} \text{Speed of light}

\[ n = \frac{c}{V} = \sqrt{\varepsilon_r} \]  \hspace{1cm} \text{n and } \varepsilon_r \text{ are both depend on} \hspace{1cm} \text{The frequency of light (EM wave)}

\[ k(\text{medium}) = nk \]  \hspace{1cm} \text{What is } \varepsilon_r \text{ ??}

\[ \lambda(\text{medium}) = \frac{\lambda}{n} \]

Isotropic and anisotropic materials?; Optically isotropic/anisotropic?
Two slightly different wavelength waves travelling in the same direction result in a wave packet that has an amplitude variation which travels at the group velocity.

\[ \frac{dz}{dt} = \frac{\delta \omega}{\delta k} \quad \text{or} \quad V_g = \frac{d\omega}{dk} \quad \text{group velocity} \]
In vacuum group velocity is the same as phase velocity.

Refractive index $n$ and the group index $N_g$ of pure SiO$_2$ (silica) glass as a function of wavelength.

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$$\omega = \nu k = \left[ \frac{c}{n(\lambda)} \right] \left[ \frac{2\pi}{\lambda} \right]$$

$$\nu_g (medium) = \frac{d\omega}{dk} = \frac{c}{n-\lambda} \frac{dn}{d\lambda} = \frac{c}{N_g}$$

What is dispersion?; dispersive medium?
A plane EM wave travelling along $\mathbf{k}$ crosses an area $A$ at right angles to the direction of propagation. In time $\Delta t$, the energy in the cylindrical volume $A\nu\Delta t$ (shown dashed) flows through $A$.

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In EM wave a magnetic field is always accompanying electric field, Faraday’s Law. In an isotropic dielectric medium $E_x = \nu B_y = c/n (B_y)$, where $\nu$ is the phase velocity and $n$ is index of refraction of the medium.
The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. We use the coordinates $r, \phi, z$ to represent any point in the fiber. Cladding is normally much thicker than shown.

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Two arbitrary waves 1 and 2 that are initially in phase must remain in phase after reflections. Otherwise the two will interfere destructively and cancel each other.

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Interference of waves such as 1 and 2 leads to a standing wave pattern along the direction which propagates along $z$. 

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The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest $\theta$. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide.

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\[
E_1(y, z, t) = 2E_0 \cos \left( k_m y + \frac{1}{2} \phi_m \right) \cos \left( \omega t - \beta_m z + k_m y + \frac{1}{2} \phi_m \right)
\]

\[
E_1(y, z, t) = 2E_m(y) \cos(\omega t - \beta_m z)
\]
The electric field patterns of the first three modes \((m = 0, 1, 2)\) traveling wave along the guide. Notice different extents of field penetration into the cladding.

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Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

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Snell’s Law and Total Internal Reflection (TIR)

\[
\frac{\sin \theta_i}{V_i} = \frac{n_2}{n_1}
\]

\[
\frac{\sin \theta_t}{V_t} = \frac{n_1}{n_2}
\]

\[
V_i = V_r, \text{ therefore } \theta_i = \theta_r
\]

When \( \theta_t \) reaches 90 degree, \( \theta_i = \theta_c \) called critical angle

\[
\sin \theta_c = \frac{n_2}{n_1}, \quad \text{We have total internal reflection (TIR)}
\]
Light wave travelling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to $\theta_c$, which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected. (a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$ and total internal reflection (TIR).

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$$v = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}} \quad n = \frac{c}{v} = \sqrt{\varepsilon_r}$$

$$E_{t,\perp}(x, y, z) = e^{-\alpha_2 y} E_{i0} \exp j(\omega t - k_{iz} z)$$

$$\alpha_2 = \frac{2\pi n_2}{\lambda} \left[ \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_i - 1 \right]^{1/2}$$

$\alpha_2$ is the attenuation coefficient and $1/\alpha_2$ is called penetration depth
(a) A linearly polarized wave has its electric field oscillations defined along a line perpendicular to the direction of propagation $z$. The field vector $E$ and $z$ define a plane of polarization. (b) The $E$-field oscillations are contained in the plane of polarization. (c) A linearly polarized light at any instant can be represented by the superposition of two fields $E_x$ and $E_y$ with the right magnitude and phase.

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Transverse electric field (TE)

Transverse magnetic Field (TM)

\[ E_i = E_{i0} e^{j(\omega t - k_i \cdot r)} \]

\[ E_r = E_{r0} e^{j(\omega t - k_r \cdot r)} \]

\[ E_t = E_{t0} e^{j(\omega t - k_t \cdot r)} \]

\[ E_{t,\perp}(x, y, z) = e^{-\alpha_2 y} E_{i0} \exp j(\omega t - k_{iz} z) \]

Light wave travelling in a more dense medium strikes a less dense medium. The plane of incidence is the plane of the paper and is perpendicular to the flat interface between the two media. The electric field is normal to the direction of propagation. It can be resolved into perpendicular (\(\perp\)) and parallel (\(\parallel\)) components.
Fresnel’s Equations:

using Snell’s law, and applying boundary conditions:

\[ r_\perp = \frac{E_{r0,\perp}}{E_{i0,\perp}} = \frac{\cos \theta_i - \left[ n^2 - \sin^2 \theta_i \right]^{\frac{1}{2}}}{\cos \theta_i + \left[ n^2 - \sin^2 \theta_i \right]^{\frac{1}{2}}} \]

\[ t_\perp = \frac{E_{t0,\perp}}{E_{i0,\perp}} = \frac{2 \cos \theta_i}{\cos \theta_i + \left[ n^2 - \sin^2 \theta_i \right]^{\frac{1}{2}}} \]

\[ r_\parallel = \frac{E_{r0,\parallel}}{E_{i0,\parallel}} = \frac{\left[ n^2 - \sin^2 \theta_i \right]^{\frac{1}{2}} - n^2 \cos \theta_i}{\left[ n^2 - \sin^2 \theta_i \right]^{\frac{1}{2}} + n^2 \cos \theta_i} \]

\[ t_\parallel = \frac{E_{t0,\parallel}}{E_{i0,\parallel}} = \frac{2 n \cos \theta_i}{n^2 \cos \theta_i + \left[ n^2 - \sin^2 \theta_i \right]^{\frac{1}{2}}} \]

\[ n = \frac{n_2}{n_1} \]
Polarization angle

For incident angle close to zero:

\[ r_\parallel = r_\perp = (n_1 - n_2)/(n_1 + n_2) \]

Internal reflection: (a) Magnitude of the reflection coefficients \( r_\parallel \) and \( r_\perp \) vs. angle of incidence \( \theta_i \) for \( n_1 = 1.44 \) and \( n_2 = 1.00 \). The critical angle is \( 44^\circ \). (b) The corresponding phase changes \( \phi_\parallel \) and \( \phi_\perp \) vs. incidence angle.

\[ \tan\left( \frac{1}{2} \theta_\perp \right) = \frac{\left[ \sin^2 \theta_i - n^2 \right]^{1/2}}{\cos \theta_i} \]

and

\[ \tan\left( \frac{1}{2} \theta_\parallel + \frac{1}{2} \pi \right) = \frac{\left[ \sin^2 \theta_i - n^2 \right]^{1/2}}{n^2 \cos \theta_i} \]

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The reflection coefficients $r_{//}$ and $r_{\perp}$ vs. angle of incidence $\theta_i$ for $n_1 = 1.00$ and $n_2 = 1.44$. 

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Light intensity

\[ I = \frac{1}{2} V \varepsilon_0 \varepsilon_r E_o^2 \]

Reflectance

\[ R_\parallel = \frac{|E_{r0,\parallel}|^2}{|E_{i0,\parallel}|^2} = |r_\parallel|^2 \]
\[ R_\perp = \frac{|E_{r0,\perp}|^2}{|E_{i0,\perp}|^2} = |r_\perp|^2 \]

Transmittance

\[ T_\parallel = \frac{n_2 |E_{i0,\parallel}|^2}{|E_{i0,\parallel}|^2} = \left(\frac{n_2}{n_1}\right) |t_\parallel|^2 \]
\[ T_\perp = \frac{n_2 |E_{i0,\perp}|^2}{|E_{i0,\perp}|^2} = \left(\frac{n_2}{n_1}\right) |t_\perp|^2 \]

for normal incident

\[ R = R_\parallel = R_\perp = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \]
\[ T = T_\parallel = T_\perp = \frac{4n_1 n_2}{(n_1 + n_2)^2} \]
Illustration of how an antireflection coating reduces the reflected light intensity

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Schematic illustration of the principle of the dielectric mirror with many low and high refractive index layers and its reflectance.

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A planar dielectric waveguide has a central rectangular region of higher refractive index \( n_1 \) than the surrounding region which has a refractive index \( n_2 \). It is assumed that the waveguide is infinitely wide and the central region is of thickness \( 2a \). It is illuminated at one end by a monochromatic light source.

\[ n_1 > n_2 \]

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A light ray travelling in the guide must interfere constructively with itself to propagate successfully. Otherwise destructive interference will destroy the wave.

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\[ \Delta \Phi (AC) = k_1(AB + BC) - 2\Phi = m(2\pi), \quad k_1 = k n_1 = \frac{2\pi n_1}{\lambda} \]

\[ m=0, 1, 2, \ldots \]

\[ \left[ \frac{2\pi n_1 (2a)}{\lambda} \right] \cos \theta_m - \phi_m = m\pi \]

Waveguide condition

\[ \beta_m = k_1 \sin \theta_m = \left( \frac{2\pi n_1}{\lambda} \right) \sin \theta_m \]

\[ k_m = k_1 \cos \theta_m = \left( \frac{2\pi n_1}{\lambda} \right) \cos \theta_m \]
The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest $\theta$. It is often referred to as the glazing incidence ray. It has the highest phase velocity along the guide.

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\[
E_1(y, z, t) = 2E_0 \cos \left( k_m y + \frac{1}{2} \phi_m \right) \cos \left( \omega t - \beta_m z + k_m y + \frac{1}{2} \phi_m \right)
\]

\[
E_1(y, z, t) = 2E_m(y) \cos \left( \omega t - \beta_m z \right)
\]
The electric field patterns of the first three modes \((m = 0, 1, 2)\) traveling wave along the guide. Notice different extents of field penetration into the cladding.

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Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

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\( E_y (m) \) is the field distribution along \( y \) axis and constitute a mode of propagation.

\( m \) is called mode number. Defines the number of modes traveling along the waveguide. For every value of \( m \) we have an angle \( \theta m \) satisfying the waveguide condition provided to satisfy the TIR as well. Considering these condition one can show that the number of modes should satisfy:

\[
m = \leq \frac{(2V - \Phi)}{\pi}
\]

\( V \) is called V-number

\[
V = \frac{2\pi a \lambda}{\lambda} \left( n_1^2 - n_2^2 \right)^{\frac{1}{2}}
\]

For \( V \leq \pi/2, m= 0 \), it is the lowest mode of propagation referred to single mode waveguides.

The cut-off wavelength (frequency) is a free space wavelength for \( \nu = \pi/2 \).
Possible modes can be classified in terms of (a) transelectric field (TE) and (b) transmagnetic field (TM). Plane of incidence is the paper.

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The electric field of TE$_0$ mode extends more into the cladding as the wavelength increases. As more of the field is carried by the cladding, the group velocity increases.

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The reflected light beam in total internal reflection appears to have been laterally shifted by an amount $\Delta z$ at the interface.
When medium B is thin (thickness $d$ is small), the field penetrates to the BC interface and gives rise to an attenuated wave in medium C. The effect is the tunnelling of the incident beam in A through B to C.

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(a) A light incident at the long face of a glass prism suffers TIR; the prism deflects the light.
(b) Two prisms separated by a thin low refractive index film forming a beam-splitter cube. The incident beam is split into two beams by FTIR.

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The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. We use the coordinates \( r, \phi, z \) to represent any point in the fiber. Cladding is normally much thicker than shown.

For the step index optical fiber \( \Delta = (n_1 - n_2)/n_1 \) is called \textit{normalized index difference}.
LPs (linearly polarized waves) propagating along the fiber have either TE or TM type represented by the propagation of an electric field distribution $E_{lm}(r, \Phi)$ along $z$.

$$E_{LP} = E_{lm}(r, \Phi) \exp j(\omega t - \beta_{lm} z)$$

$E_{LP}$ is the field of the LP mode and $\beta_{lm}$ is its propagation constant along $z$. 

The electric field distribution of the fundamental mode in the transverse plane to the fiber axis $z$. The light intensity is greatest at the center of the fiber. Intensity patterns in $LP_{01}$, $LP_{11}$ and $LP_{21}$ modes.
V-number

\[ V = \frac{2\pi a}{\lambda} \left( n_1^2 - n_2^2 \right)^{\frac{1}{2}} = \frac{2\pi a}{\lambda} \left( 2n_1 n\Delta \right)^{\frac{1}{2}} \]

\[ V_{\text{cut-off}} = \frac{2\pi a}{\lambda_c} \left( n_1^2 - n_2^2 \right)^{\frac{1}{2}} = 2.405 \]

\[ \Delta = \frac{(n_1 - n_2)}{n_1} \approx \frac{(n_1^2 - n_2^2)}{2n_1^2} \]

Normalized index difference

For weakly guided fiber \( \Delta = 0.01, 0.005, .. \)

For \( V = 2.405 \), the fiber is called single mode (only the fundamental mode propagate along the fiber). For \( V > 2.405 \) the number of mode increases according to approximately

\[ M = \frac{V^2}{2} \]

Most SM fibers designed with \( 1.5 < V < 2.4 \)
\[ b = \frac{\left(\frac{\beta}{k}\right)^2 - n_2^2}{n_1^2 - n_2^2} \]

\( b \) changes between 0 and 1

\( kn_2 < \beta < kn_1 \) Propagation condition

Normalized propagation constant \( b \) vs. \( V \)-number for a step index fiber for various LP modes.

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Numerical Aperture---Maximum Acceptance Angle

\[
\sin \alpha_{\text{max}} = \frac{\left(n_1^2 - n_2^2\right)^{1/2}}{n_0}
\]

\[
NA = \left(n_1^2 - n_2^2\right)^{1/2}
\]

\[
\sin \alpha_{\text{max}} = \frac{NA}{n_0}
\]

\[
V = \frac{2\pi a}{\lambda} NA
\]

Example values for \(n_1=1.48, n_2=1.47\); very close numbers

Typical values of NA = 0.07..., 0.25
Optical waveguides display 3 types of dispersion:

These are the main sources of dispersion in the fibers.

- Material dispersion, different wavelength of light travel at different velocities within a given medium.
  - Due to the variation of $n_1$ of the core wrt wavelength of the light.

- Waveguide dispersion, $\beta$ depends on the wavelength, so even within a single mode different wavelengths will propagate at slightly different speeds.
  - Due to the variation of group velocity wrt V-number.

\[
\frac{\Delta \tau}{L} = |D| \Delta \lambda
\]

All excitation sources are inherently non-monochromatic and emit within a spectrum, $^2 \lambda$, of wavelengths. Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of $n_1$. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

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• Modal dispersion, in waveguides with more than one propagating mode. Modes travel with different group velocities.

Due to the number of modes traveling along the fiber with different group velocity and different path.

Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at different group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

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\[
\frac{\Delta \tau}{L} = |D_m| \Delta \lambda
\]

\[
D_m \approx -\frac{\lambda}{c} \left(\frac{d^2 n}{d\lambda^2}\right)
\]

Material Dispersion Coefficient

\[
\frac{\Delta \tau}{L} = |D_\omega| \Delta \lambda
\]

Waveguide Dispersion Coefficient

\[
D_\omega \approx \frac{1.984 N g^2}{(2\pi a)^2 2cn_2^2}
\]

\[
\frac{\Delta \tau}{L} = |D_p| \Delta \lambda \quad D_p \text{ is called profile dispersion; group velocity depends on } \Delta
\]

\[
\frac{\Delta \tau}{L} = |D_m + D_\omega + D_p| \Delta \lambda
\]

Material dispersion coefficient \((D_m)\) for the core material (taken as \(\text{SiO}_2\)), waveguide dispersion coefficient \((D_w)\) \((a = 4.2 \, \mu m)\) and the total or chromatic dispersion coefficient \(D_{ch} = D_m + D_w\) as a function of free space wavelength, \(\lambda\).

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Suppose that the core refractive index has different values along two orthogonal directions corresponding to electric field oscillation direction (polarizations). We can take $x$ and $y$ axes along these directions. An input light will travel along the fiber with $E_x$ and $E_y$ polarizations having different group velocities and hence arrive at the output at different times.

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Material and waveguide dispersion coefficients in an optical fiber with a core SiO$_2$-13.5\%GeO$_2$ for $a = 2.5$ to 4 \( \mu \text{m} \).

\[
\begin{align*}
D_m &= 10 \text{ ps/km.nm} \\
D_w &= -6 \text{ ps/km.nm}
\end{align*}
\]

E.g.

For $\lambda = 1.5$, and $2a = 8 \mu \text{m}$, $D_m = 10 \text{ ps/km.nm}$ and $D_w = -6 \text{ ps/km.nm}$.
Dispersion flattened fiber example. The material dispersion coefficient ($D_m$) for the core material and waveguide dispersion coefficient ($D_w$) for the doubly clad fiber result in a flattened small chromatic dispersion between $\lambda_1$ and $\lambda_2$.

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An optical fiber link for transmitting digital information and the effect of dispersion in the fiber on the output pulses.

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(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.

(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.

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Graded index (GRIN) rod lenses of different pitches. (a) Point $O$ is on the rod face center and the lens focuses the rays onto $O'$ on to the center of the opposite face. (b) The rays from $O$ on the rod face center are collimated out. (c) $O$ is slightly away from the rod face and the rays are collimated out.

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Sources of Loss and Attenuation in Fibers

Absorption depends on materials, amount of materials, wavelength, and the impurities in the substances.

It is cumulative and depends on the amount of materials, e.g. length of the fiber optics.

\[(1 - \alpha)^d\]

\(\alpha\) is the absorption per unit length and \(d\) is the distance that light travels

Lattice absorption through a crystal. The field in the wave oscillates the ions which consequently generate "mechanical" waves in the crystal; energy is thereby transferred from the wave to lattice vibrations.

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Rayleigh scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.

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Sharp bends change the local waveguide geometry that can lead to waves escaping. The zigzagging ray suddenly finds itself with an incidence angle $\theta'$ that gives rise to either a transmitted wave, or to a greater cladding penetration; the field reaches the outside medium and some light energy is lost.
Attenuation in Optical Fiber

Illustration of a typical attenuation vs. wavelength characteristics of a silica based optical fiber. There are two communications channels at 1310 nm and 1550 nm.

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\[
\alpha_{dB} = \frac{1}{L} 10 \log\left( \frac{P_{in}}{P_{out}} \right)
\]

\[
P_{out} = P_{in} \left(10^{-\alpha L/10}\right)
\]

G. Keiser (Ref. 1)

\[
\alpha_{dB} = 4.34\alpha
\]
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Solid State Optoelectronic Devices

Optical Sources; Laser, LED

Switches

Photodiodes

Photodetectors

Solar Cells

Schematic illustration of the structure of a double heterojunction stripe contact laser diode

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Type of Semiconductors

• Simple Semiconductors
• Compound Semiconductors
  • Direct Band gap Semiconductors
  • Indirect Band gap Semiconductors
### Some Properties of Some Important Semiconductors

<table>
<thead>
<tr>
<th>Compound</th>
<th>$E_g$ Gap(eV)</th>
<th>Transition $\lambda$(nm)</th>
<th>Bandgap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>5.4</td>
<td>230</td>
<td>indirect</td>
</tr>
<tr>
<td>ZnS</td>
<td>3.75</td>
<td>331</td>
<td>direct</td>
</tr>
<tr>
<td>ZnO</td>
<td>3.3</td>
<td>376</td>
<td>indirect</td>
</tr>
<tr>
<td>TiO$_2$</td>
<td>3</td>
<td>413</td>
<td>indirect</td>
</tr>
<tr>
<td>CdS</td>
<td>2.5</td>
<td>496</td>
<td>direct</td>
</tr>
<tr>
<td>CdSe</td>
<td>1.8</td>
<td>689</td>
<td>direct</td>
</tr>
<tr>
<td>CdTe</td>
<td>1.55</td>
<td>800</td>
<td>direct</td>
</tr>
<tr>
<td>GaAs</td>
<td>1.5</td>
<td>827</td>
<td>direct</td>
</tr>
<tr>
<td>InP</td>
<td>1.4</td>
<td>886</td>
<td>direct</td>
</tr>
<tr>
<td>Si</td>
<td>1.2</td>
<td>1033</td>
<td>indirect</td>
</tr>
<tr>
<td>AgCl</td>
<td>0.32</td>
<td>3875</td>
<td>indirect</td>
</tr>
<tr>
<td>PbS</td>
<td>0.3</td>
<td>4133</td>
<td>direct</td>
</tr>
<tr>
<td>AgI</td>
<td>0.28</td>
<td>4429</td>
<td>direct</td>
</tr>
<tr>
<td>PbTe</td>
<td>0.25</td>
<td>4960</td>
<td>indirect</td>
</tr>
</tbody>
</table>
Two Interpenetrating Face-Centered Cubic Lattices
Diamond or Zinc Blend Structure

- {100} Plane
- {110} Plane
- {111} Plane

Common Planes

\( a \) – Lattice Constant

For Silicon

\( a = 5.34 \text{ Å} \)
Energy Band Structure of Semiconductors

Energy band structures of GaAs.

Energy band structures of Si.
Two-dimensional representation of the breaking of a covalent bond.

Corresponding line representation of the energy band and the generation of a negative and positive charge with the breaking of a covalent bond.
Concept of positive charges in solids (holes)

Visualization of the movement of a hole in a semiconductor.
Allowed energy bands showing:
(d) an empty band
(e) a completely full band
(f) the bandgap energy between the two allowed bands
Doped Semiconductors

N-Type

$E_c$, $E_d$, $E_i$, $E_v$, $E_g$

Conduction electron

$+4$ Si, $+4$ Si, $+4$ Si, $+5$ As, $+4$ Si, $+4$ Si, $+4$ Si, $+4$ Si, $+4$ Si

Electrons in conduction band
Doped Semiconductors

P-type

Valence hole
Energy-band diagrams of a compensated semiconductor showing ionized and un-ionized donors and acceptors.

Energy-band diagram showing the redistribution of electrons when donors are added.

\[ N_d^+ = (N_d - n_d) \]
\[ N_a^- = (N_a - P_a) \]
\[ p_0 = \frac{n_i^d}{n_0} \]
The Semiconductors in Equilibrium

- The thermal equilibrium concentration of carriers is independent of time.
- The random generation-recombination of electrons-holes occur continuously due to the thermal excitation.
- In direct band-to-band generation-recombination, the electrons and holes are created-annihilated in pairs:
  - $G_{n0} = G_{p0}$, $R_{n0} = R_{p0}$
- The carriers concentrations are independent of time therefore:
  - $G_{n0} = G_{p0} = R_{n0} = R_{p0}$
Nonequilibrium Conditions in Semiconductors

• When current exist in a semiconductor device, the semiconductor is operating under nonequilibrium conditions. In these conditions *excess electrons in conduction band and excess holes in the valance band exist*, due to the external excitation (thermal, electrical, optical...) in addition to thermal equilibrium concentrations.

\[ n(t) = n_0 + \delta n(t), \quad p(t) = p_0 + \delta p(t) \]

• The behavior of the excess carriers in semiconductors (diffusion, drift, recombination, ...) which is the fundamental to the operation of semiconductors (electronic, optoelectronic, ..) is described by the ambipolar transport or Continuity equations.
Continuity Equations

\[
D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \left( E \frac{\partial p}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_p} = \frac{\partial p}{\partial t}
\]

\[
D_n \frac{\partial^2 n}{\partial x^2} + \mu_n \left( E \frac{\partial n}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_n} = \frac{\partial n}{\partial t}
\]
Nonequilibrium Conditions in Semiconductors

**Generation-Recombination Rates**

- The recombination rate is proportional to electron and hole concentrations.

\[
\frac{dn(t)}{dt} = \alpha[n_i^2 - n(t)p(t)] = \frac{d(n_0 + \delta n(t))}{dt} = \frac{d\delta n(t)}{dt}
\]

\[
n(t) = n_0 + \delta n(t) \quad \quad \quad \quad \quad \quad p(t) = p_0 + \delta p(t)
\]

\(\alpha n_i\) is the thermal equilibrium generation rate.
**Generation-Recombination Rates**

- Electron and holes are created and recombined in pairs, therefore,
- \( \delta n(t) = \delta p(t) \) and \( n_0 \) and \( p_0 \) are independent of time.

\[
\frac{d(\delta n(t))}{dt} = \alpha [n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t))] = -\alpha \delta n(t)[(n_0 + p_0) + \delta n(t)]
\]

Considering a p-type material under **low-injection** condition,

\[
\frac{d(\delta n(t))}{dt} = -\alpha p_0 \delta n(t) \quad \delta n(t) = \delta n(0)e^{-\alpha p_0 t} = \delta n(0)e^{-\frac{t}{\tau_{n0}}}
\]

\( \tau_{n0} = (\alpha p_0)^{-1} \) is the minority carrier electrons lifetime, *constant for low-injections*. 
Generation-Recombination Rates

• The recombination rate (a positive quantity) of excess minority carriers (electrons-holes) for p-type materials is:

\[ R_n^\prime = R_p^\prime = \frac{\delta n(t)}{\tau_{n0}} \]

Similarly, the recombination rate of excess minority carriers for n-type material is:

\[ R_n^\prime = R_p^\prime = \frac{\delta p(t)}{\tau_{p0}} \]

• where \( \tau_{po} \) is the minority carrier holes lifetime.
Generation-Recombination Rates

SO,

\[ \tau_{no} = (\alpha \delta p)^{-1} \text{ and } \tau_{po} = (\alpha \delta n)^{-1} \]

[For high injections which is in the case of LASER and LED operations, \( \delta n >> n_0 \) and \( \delta p >> p_0 \) ]

• During recombination process if photons are emitted (usually in direct bandgap semiconductors), the process is called \textit{radiative} (important for the operation of optical devices), otherwise is called \textit{nonradiative} recombination (takes place via surface or bulk defects and traps).
Generation-Recombination Rates

- In any carrier-decay process the total lifetime $\tau$ can be expressed as

$$\tau = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

where $\tau_r$ and $\tau_{nr}$ are the radiative and nonradiative lifetimes respectively.

The total recombination rate is given by

$$R_{total} = R_r + R_{nr} = R_{sp}$$

Where $R_r$ and $R_{nr}$ are radiative and nonradiative recombination rates per unit volume respectively and $R_{sp}$ is called the spontaneous recombination rate.
The Fermi energy of:
(a) material A in thermal equilibrium;
(b) material B in thermal equilibrium;
(c) materials A and B in contact at thermal equilibrium.
The entire semiconductor is a single-crystal material:

-- p region doped with acceptor impurity atoms

-- n region doped with donor atoms

-- the n and p region are separated by the *metallurgical junction*. 
- the potential barrier:
  - keeps the large concentration of electrons from flowing from the n region into the p region;
  - keeps the large concentration of holes from flowing from the p region into the n region;

=> The potential barrier maintains thermal equilibrium.
- the potential of the n region is positive with respect to the p region => the Fermi energy in the n region – lower than the Fermi energy in the p region;

- the total potential barrier – larger than in the zero-bias case;

- still essentially no charge flow and hence essentially no current;
- A positive voltage is applied to the p region with respect to the n region;

- The Fermi energy level – lower in the p region than in the n region;

- The total potential barrier – reduced => the electric field in the depletion region – reduced;

⇒ diffusion of holes from the p region across the depletion region into the n region;

⇒ diffusion of electrons from the n region across the depletion region into the p region;

- Diffusion of carriers => diffusion currents;
- in thermal equilibrium:
  - the n region contains many more electrons in the conduction band than the p region;
  - the built-in potential barrier prevents the large density of electrons from flowing into the p region;
  - the built-in potential barrier maintains equilibrium between the carrier distribution on either side of the junction;

\[ V_{bi} = V_t \ln \left( \frac{N_a N_d}{n_i^2} \right) \]

\[ n_{p0} = n_{n0} \exp \left( \frac{-qV_{bi}}{kT} \right) \]
- the electric field $E_{\text{app}}$ induced by $V_a$ – in opposite direction to the electric field in depletion region for the thermal equilibrium;

- the net electric field in the depletion region is reduced below the equilibrium value;

- majority carrier electrons from the n side -> injected across the depletion region into the p region;

- majority carrier holes from the p region -> injected across the depletion region into the n region;

- $V_a$ applied => injection of carriers across the depletion regions-> a current is created in the pn junction;
\[ W = x_p + x_n = \left\{ \frac{2\varepsilon_s V_{bi}}{e} \left( \frac{N_a + N_d}{N_a N_d} \right) \right\}^{1/2} \]

For zero bias

\[ W = x_n + x_p = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[ \frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} \]

For reverse biased

\[ E_{\text{max}} = -\left\{ \frac{2e(V_{bi} + V_R)}{\varepsilon_s} \left[ \frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2} = -\frac{2(V_{bi} + V_R)}{W} \]

For reverse biased

\[ V_R \gg V_{bi} \]

\[ E_{\text{max}} \approx -\left\{ \frac{2eV_R}{\varepsilon_s} \left[ \frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2} \]

\[ E_{\text{max}} = -\frac{2V_{bi}}{W} \]

For zero bias
When there is no voltage applied across the pn junction ⇔ the junction is in thermal equilibrium => the Fermi energy level – constant throughout the entire system.

\[ V_{bi} = |\Phi_{Fn}| + |\Phi_{Fp}| \]

\[ V_{bi} = |\Phi_p| + |\Phi_n| = (E_{Fi} - E_{Fp})/e + (E_{Fn} - E_{Fi})/e = \frac{kT}{e} \ln \left( \frac{N_a}{n_i} \right) + \frac{kT}{e} \ln \left( \frac{N_d}{n_i} \right) \]

\[ V_{bi} = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = \frac{kT}{e} \ln \left( \frac{p_p}{p_n} \right) = \frac{kT}{e} \left( \frac{n_n}{n_p} \right) \]
Poisson's Equation

\[
\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_s} = -\frac{dE(x)}{dx}
\]

\[
E(x) = \frac{1}{\varepsilon_s} \int_{-x_p}^{x} \rho(x) dx
\]

For Si, \( \varepsilon_s = \varepsilon_0 \cdot \varepsilon_r = (11.7)(8.85\times10^{-14}) F/cm \)

\[
E = -\frac{eN_a}{\varepsilon_s} (x + x_p), \quad -x_p \leq x \leq 0
\]

\[
E = -\frac{eN_d}{\varepsilon_s} (x_n - x), \quad 0 \leq x \leq x_n
\]

Charge neutrality:

\[
N_a x_p = N_d x_n
\]

The peak electric field is at \( x = 0 \)

\[
E_{\text{max}} = -\frac{eN_d x_n}{\varepsilon_s} = -\frac{eN_a x_p}{\varepsilon_s}
\]
Ideal I-V characteristic of a pn junction diode.

\[ J = J_s \left[ \exp \left( \frac{qV_a}{kT} \right) - 1 \right] \]  
- ideal-diode equation;

\[ J_s = \left[ \frac{qD_p p_n 0}{L_p} + \frac{qD_n n_p 0}{L_n} \right] \]
The bipolar transistor:

- tree separately doped regions
- two pn junctions;

The width of the base region – small compared to the minority carrier diffusion length;

The emitter – largest doping concentration;

The collector – smallest doping concentration;
- the bipolar semiconductor – not a symmetrical device;

-the transistor – may contain two n regions or two p regions -> the impurity doping concentrations in the emitter and collector = different;

-- the geometry of the two regions – can be vastly different;
Electromagnetic Spectrum

- Three basic bands; infrared (wavelengths above 0.7\(\mu\)m), visible (wavelengths between 0.4-0.7\(\mu\)m), and ultraviolet light (wavelengths below 0.4\(\mu\)m).

- \[ E = h\nu = \frac{hc}{\lambda}; \quad c = \nu \lambda \quad \lambda(\mu m) = 1.24 /E(eV) \]

- An emitted light from a semiconductor optical device has a wavelength proportional to the semiconductor band-gap.

- Longer wavelengths for communication systems; \( \text{Eg} \approx 1\mu m \). (lower Fiber loss).

- Shorter wavelengths for printers, image processing,... \( \text{Eg} > 1\mu m \).

- Semiconductor materials used to fabricate optical devices depend on the wavelengths required for the operating systems.
Photoluminescence & Electroluminescence

• The recombination of excess carries in direct bandgap semiconductors may result in the emission of photon. This property is generally referred to as luminescence.

• If the excess electrons and holes are created by photon absorption, then the photon emission from the recombination process is called photoluminescence.

• If the excess carries are generated by an electric current, then the photon emission from the recombination process is called electroluminescence.
Consider a two-level energy states of E1 and E2. Also consider that E1 is populated with N1 electron density, and E2 with N2 electron density.
Absorption

- dN₁ states are raised from E₁ to E₂ i.e dN₁ photons are absorbed. Electrons are created in conduction band and holes in valence band.
- When photons with an intensity of Iᵥ(x) are traveling through a semiconductor, going from x position to x + dx position (in 1-D system), the energy absorbed by semiconductor per unit of time is given by \( \alpha Iᵥ(x)dx \), where \( \alpha \) is the absorption coefficient; the relative number of photons absorbed per unit distance (cm⁻¹).

\[
Iᵥ(x + dx) - Iᵥ(x) = \frac{dIᵥ(x)}{dx} . dx = -\alpha Iᵥ(x)dx
\]

\[
\frac{dIᵥ(x)}{dx} = -\alpha Iᵥ(x)
\]

\[
Iᵥ(x) = Iᵥ₀ e^{-\alpha x}
\]

where \( Iᵥ(0) = Iᵥ₀ \)
Absorption

• Intensity of the photon flux decreases exponentially with distance.
• The absorption coefficient in semiconductor is strong function of photon energy and band gap energy.
• The absorption coefficient for $\hbar \nu < E_g$ is very small, so the semiconductor appears transparent to photons in this energy range.
Photoluminescence
Optical Absorption

• When semiconductors are illuminated with light, the photons may be absorbed (for $E_{ph} = h\nu \geq E_g = E_2 - E_1$) or they may propagate through the semiconductors (for $E_{ph} \leq E_g$).

• There is a finite probability that electrons in the lower level absorb energy from incoming electromagnetic field (light) with frequency of $\nu \geq (E_2 - E_1)/h$ and jump to the upper level.

$$\frac{dN_1}{dt}_{ab} = -B_{12}(\Phi \delta \nu)N_1$$

• $B_{12}$ is proportionality constant, $\delta \nu = \nu_2 - \nu_1$ and $\Phi \delta \nu = I_\nu$ is the photon density in the frequency range of $\delta \nu$. 
Photon Emission in Semiconductors

- When electrons in semiconductors fall from the conduction band to the valence band, called recombination process, release their energy in form of light (photon), and/or heat (lattice vibration, phonon).
- N1 and N2 are the concentrations of occupied states in level 1 (E1) and level 2 (E2) respectively.

Basic Transitions

**Radiative**
- Intrinsic emission
- Energetic carriers

**Nonradiative**
- Impurities and defect center involvement
- Auger process
 Photon Emission in Semiconductors

Diagram showing the transition of energy levels in semiconductors.
Photon Emission in Semiconductors

- **Spontaneous emission**

![Optical Gain Spectrum](image)

The energized electrons at upper level fall to a lower level by themselves without any disturbance.

The photon emission is not due to a single discrete energy but rather due to the recombination process over a range of energies.

- The emission depends on the bandgap and it is temperature sensitive.

- Large bandwidth.
Spontaneous Emission

- The released photons are not in time phase and they are incoherent.

\[
\left. \frac{dN_2}{dt} \right|_{sp} = -AN_2
\]

where \( A \) is the proportionality constant \((s^{-1})\)
Photon Emission in Semiconductors

- **Stimulated Emission**

![Diagram of stimulated emission](image)

Photons collides with electrons in one excited energy state drop them to a state with a lower energy.

- The process dependent upon the presence of photon with frequency of \( v = (E_2 - E_1)/h \)
- Energy releases in form of light.

- The new emitted photons are in time phase with existing photons and are radiated in a highly confined direction. Light is coherent.

$$\frac{dN_2}{dt} \bigg|_{st} = -B_{21} (\Phi \delta \nu) N_2$$

where $B_{21}$ is a proportionality constant, $\delta \nu = \nu_2 - \nu_1$ and $\Phi \delta \nu$ is photon density.
Photon Emission in Semiconductors
Einstein Relations

Consider the steady state of an isolated, nondegenerate two-level system in thermal equilibrium. The increase of population in the lower level must correspond to the population decrease in the upper level (no other levels exist in the system). Thus, the net rate of change of $N_1$ vs time is

$$\frac{dN_1}{dt} = \left. \frac{dN_1}{dt} \right|_{ab} - \left. \frac{dN_2}{dt} \right|_{st} - \left. \frac{dN_2}{dt} \right|_{sp} = -B_{12} \Phi \delta v N_1 + B_{21} \Phi \delta v N_2 + AN_2 = 0$$
(steady state condition).

$$
\Phi \delta \nu = \frac{AN_2}{B_{12} N_1 - B_{21} N_2} = \frac{A}{B_{21}} \frac{B_{12} N_1}{B_{21} N_2} - 1 = \frac{A}{B_{21}} e^{(E_2 - E_1)/kT} - 1
$$

which we used the Boltzmann distribution function.

Compare the above equation with Planck’s equation for black body radiation:

$$
\Phi (\nu) \delta \nu = \frac{8\pi \nu^2 \delta \nu}{c^3} \frac{1}{e^{h\nu/kT} - 1}
$$

shows that the two Einstein coefficients are the same.

$$
B_{12} = B_{21} = B \quad \text{and} \quad \frac{A}{B} = \frac{8\pi \nu^2 \delta \nu}{c^3} = \frac{8\pi \delta \nu}{\lambda^2 c}
$$

These relations are known as the Einstein relations.
Now:
\[ \frac{R_{st}}{R_{sp}} = \frac{B_{21} \Phi \delta \nu N}{AN_2} = \frac{c^3}{8 \pi \nu^2} \Phi \]

Which means to have stimulated emission exceeds spontaneous emission, a photon density built by *optical cavity* is necessary.

Also:
\[ \frac{R_{st}}{R_{ab}} = \frac{N_2}{N_1} \]

which means to have stimulated emission exceeds absorption, population inversion is necessary (non-thermal equilibrium system).

These are the two fundamental conditions for lasing.
Wave Attenuation

As we showed the photon flux intensity inside a medium decreases exponentially with distance.

\[ I_{\nu}(x) = I_{\nu_0} e^{-\alpha x} \]

which

\[ I = \frac{c}{n} \Phi \delta \nu \]

where \( c/n \) is the photon velocity in a medium with an index of refraction \( n \).

As a result of stimulated emission and absorption processes, the rate of photon intensity with time will be

\[ \frac{d(\Phi \delta \nu)}{dt} = B(N_2 - N_1) \Phi \delta \nu \]

Now,

\[ \frac{dI}{dx} = \frac{dI}{dt} \frac{dt}{dx} = \frac{n}{c} \frac{dI}{dt} = \frac{n}{c} \frac{d}{dt} \left( \frac{c}{n} \Phi \delta \nu \right) = \frac{n}{c} B(N_1 - N_2) I \]
Comparing with $I(x)$ we can see that

$$\alpha = B(N_1 - N_2) \frac{n}{c}$$

Under normal circumstances $N_1 > N_2$, and $\alpha$ is positive, i.e. the light is lost in the medium.

If the density of occupied state in level 2 is increased by some means such that $N_2 > N_1$, loss in the medium become positive. The photon flux increases rather than decreases. The medium becomes amplifier. The gain coefficient $g$ is

$$g = -\alpha = B(N_2 - N_1) \frac{n}{c}$$

This condition is known as population inversion.
Luminescent Efficiency

- Radiative transitions dominate the efficient luminescence materials.
- Quantum efficiency is defined as the ratio of the radiative recombination rate to the total recombination rate for all processes.

\[ \eta_q = \frac{R_r}{R_{total}} \]
Internal Quantum Efficiency

The 3 current components in a forward biased diode:

\[ J_n = \frac{qD_n n_p^0}{L_n} \left[ e^{\frac{qV}{kT}} - 1 \right] \]

\[ J_p = \frac{qD_p p_n^0}{L_p} \left[ e^{\frac{qV}{kT}} - 1 \right] \]

\[ J_R = \frac{2n_i W}{2\tau_0} \left[ e^{\frac{qV}{kT}} - 1 \right] \]

The recombination of electrons and holes within the space charge region is, in general, through traps near midgap and is a nonradiative.
In GaAs the injection efficiency can approach unity by using an $n^+p$ diode so $J_p$ is small.

Also for sufficiently forward biased diode, $J_R$ will be very small.

Injection efficiency is defined as:

$$
\gamma = \frac{J_n}{J_n + J_p + J_r}
$$

The radiative and nonradiative recombination rates are defined as:

$$
R_r = \Delta n/\tau_r \quad \text{and} \quad R_{nr} = \Delta n/\tau_{nr}
$$

where $\Delta n$ is the excess carrier concentration $\tau$ is the carrier lifetime.
The radiative efficiency is defined as:

\[
\eta = \frac{R_r}{R_r + R_{nr}} = \frac{1}{\frac{1}{\tau_r} + \frac{1}{\tau_{nr}}} = \frac{\tau}{\tau_r}
\]

where \( R_{nr} \sim N_t \), the concentration of nonradiative sites within the forbidden gap. As \( N_t \) increases, the \( \eta \) decreases.

In GaAs, the electroluminescence originates primarily on the p-side of the junction because the efficiency for electron injection is higher than that for hole injection.

\( R_r \sim p \) type material doping and as \( N_a \) increases, \( R_r \) increases.
Internal Quantum Efficiency

\[ \eta_i = \gamma \eta \]

External Quantum Efficiency

The fraction of generated photons that are actually emitted from a semiconductor light source is called external quantum efficiency.

Normally \( \eta_{ex} << \eta_i \)

Loss mechanisms:

- Photon absorption within the semiconductor: photons can be emitted in different directions and since \( h \nu \geq E_g \), they can be reabsorbed within the semiconductor material.
- **Fresnel loss**: light emitted from semiconductor into air.

\[ \frac{n_2 - n_1}{n_2 + n_1} \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2 \]

\[ \Gamma = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2 \]

e.g. \( n_2 = 3.66 \) for GaAs and \( n_1 = 1 \) for air, therefore \( \Gamma = 0.33 \)
- Critical Angle Loss (total reflection);

If the incident angle is larger than \( \theta_c \) (critical angle), there will be total reflection.

\[
\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right)
\]

\( \theta_c \) for GaAs is 15.9°.
Materials

- Almost all optoelectronic light sources depend on epitaxial crystal growth techniques where a thin film (a few microns) of semiconductor alloys are grown on single-crystal substrate; the film should have roughly the same crystalline quality. It is necessary to make strain-free heterojunction with good-quality substrate. The requirement of minimizing strain effects arises from a desire to avoid interface states and to encourage long-term device reliability, and this imposes a lattice-matching condition on the materials used.
Schematic illustration of the structure of a double heterojunction stripe contact laser diode

Optical Sources; Laser, LED

Switches

Photodiodes

Photodetectors

Solar Cells

Schematic illustration of the structure of a double heterojunction stripe contact laser diode

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The constraints of bandgap and lattice match force that more complex compound must be chosen. These compounds include ternary (compounds that containing three elements) and quaternary (consisting of four elements) semiconductors of the form $A_xB_{1-x}C_yD_{1-y}$; variation of $x$ and $y$ are required by the need to adjust the band-gap energy (or desired wavelength) and for better lattice matching. Quaternary crystals have more flexibility in that the band gap can be widely varied while simultaneously keeping the lattice completely matched to a binary crystal substrate. The important substrates that are available for the laser diode technology are GaAs, InP and GaP. A few semiconductors and their alloys can match with these substrates. GaAs was the first material to emit laser radiation, and its related to III-V compound alloys, are the most extensively studied developed.
Materials

III-V semiconductors

- Ternary Semiconductors; Mixture of binary-binary semiconductors; $A_xB_{1-x}C$; mole fraction, $x$, changes from 0 to 1 ($x$ will be adjusted for specific required wavelength).

  $\text{Ga}_x\text{Al}_{1-x}\text{As}$; $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$; $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$

- Vegard’s Law: The lattice constant of $A_xB_{1-x}C$ varies linearly from the lattice constant of the semiconductor AC to that of the semiconductor BC.

- The bandgap energy changes as a quadratic function of $x$.

- The index of refraction changes as $x$ changes.

- The above parameters cannot vary independently

- Quaternary Semiconductors; $A_xB_{1-x}C_yD_{1-y}$

  ($x$ and $y$ will be adjusted for specific wavelength and matching lattices).

  $\text{Ga}_x\text{In}_{1-x}\text{P}_y\text{As}_{1-y}$; $(\text{Al}_x\text{Ga}_{1-x})_y\text{In}_{1-y}\text{P}$; $\text{Al}_x\text{Ga}_{1-x}\text{As}_y\text{Sb}_{1-y}$

\[
E_g = a + bx + cx^2
\]
• **II-VI Semiconductors**

• **CdZnSe/ZnSe; visible blue lasers.**

Hard to dope $p$-type impurities at concentration larger than $2 \times 10^{18} \text{cm}^{-3}$ (due to self-compensation effect). Densities on this order are required for laser operation.
Materials

- IV-VI semiconductors
- PbSe; PbS; PbTe
- By changing the proportion of Pb atoms in these materials, semiconductor changes from n- to p-type.
- Operate around 50 K°
- PbTe/Pb$_{1-x}$Eu$_x$Se$_y$Te$_{1-y}$ operates at 174 K°
Materials
• In the **near infrared** region, the most important and certainly the most extensively characterized semiconductors are GaAs, AlAs and their ternary derivatives Al$_x$Ga$_{1-x}$As.

• At **longer wavelengths**, the materials of importance are InP and ternary and quaternary semiconductors lattice matched to InP. The smaller band-gap materials are useful for application in the long wavelength range.
Energy Band Structure of Semiconductors

**Gallium Arsenide**
Conduction Band:
Lowest Valley: Γ-valley
Upper Valley: L-valley

**Indium Arsenide**

**Silicon**
Γ point: \( k = (0,0,0) \)
X point: \( k = \frac{2\pi}{a} (1,0,0) \)
L point: \( k = \frac{2\pi}{a} (1,1,1) \)

**Germanium**

**Aluminum Arsenide**
Bandgap energy $E_g$ and lattice constant $a$ for various III-V alloys of GaP, GaAs, InP and InAs. A line represents a ternary alloy formed with compounds from the end points of the line. Solid lines are for direct bandgap alloys whereas dashed lines for indirect bandgap alloys. Regions between lines represent quaternary alloys. The line from $X$ to InP represents quaternary alloys $\text{In}_{1-x}\text{Ga}_x\text{As}_{1-y}\text{P}_y$ made from $\text{In}_{0.535}\text{Ga}_{0.465}\text{As}$ and InP which are lattice matched to InP.

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3.6 III-V compound semiconductors in optoelectronics  Figure 3Q6 represents the bandgap $E_g$ and the lattice parameter $a$ in the quarternary III-V alloy system. A line joining two points represents the changes in $E_g$ and $a$ with composition in a ternary alloy composed of the compounds at the ends of that line. For example, starting at GaAs point, $E_g = 1.42$ eV and $a = 0.565$ nm, and $E_g$ decreases and $a$ increases as GaAs is alloyed with InAs and we move along the line joining GaAs to InAs. Eventually at InAs, $E_g = 0.35$ eV and $a = 0.606$ nm. Point $X$ in Figure 3Q6 is composed of InAs and GaAs and it is the ternary alloy $\text{In}_x\text{Ga}_{1-x}\text{As}$. It has $E_g = 0.7$ eV and $a = 0.587$ nm which is the same $a$ as that for InP. $\text{In}_x\text{Ga}_{1-x}\text{As}$ at $X$ is therefore lattice matched to InP and can hence be grown on an InP substrate without creating defects at the interface.
Further, \( \text{In}_x\text{Ga}_{1-x}\text{As} \) at \( X \) can be alloyed with InP to obtain a quarternary alloy \( \text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y} \) whose properties lie on the line joining \( X \) and InP and therefore all have the same lattice parameter as InP but different bandgap. Layers of \( \text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y} \) with composition between \( X \) and InP can be grown epitaxially on an InP substrate by various techniques such as liquid phase epitaxy (LPE) or molecular beam expitaxy (MBE) .

The shaded area between the solid lines represents the possible values of \( E_g \) and \( a \) for the quarternary III-V alloy system in which the bandgap is direct and hence suitable for direct recombination.

The compositions of the quarternary alloy lattice matched to InP follow the line from \( X \) to InP.

a Given that the \( \text{In}_x\text{Ga}_{1-x}\text{As} \) at \( X \) is \( \text{In}_{0.535}\text{Ga}_{0.465}\text{As} \) show that quarternary alloys \( \text{In}_{1-x}\text{Ga}_x\text{As}_y\text{P}_{1-y} \) are lattice matched to InP when \( y = 2.15x \).

b The bandgap energy \( E_g \), in eV for \( \text{In}_x\text{Ga}_{1-x}\text{As}_y\text{P}_{1-y} \) lattice matched to InP is given by the empirical relation,

\[
E_g \, (\text{eV}) = 1.35 - 0.72y + 0.12y^2
\]

Find the composition of the quarternary alloy suitable for an emitter operating at 1.55 mm.
Materials
Materials

\[ E_g = 1.35 - 0.72y + 0.12y^2 \]

\[ y = 2.2x \]
Basic Semiconductor Luminescent Diode Structures

LEDs (Light Emitting Diode)

• Under forward biased when excess minority carriers diffuse into the neutral semiconductor regions where they recombine with majority carriers. If this recombination process is direct band-to-band process, photons are emitted. The output photon intensity will be proportional to the ideal diode diffusion current.

• In GaAs, electroluminescence originated primarily on the p-side of the junction because the efficiency for electron injection is higher than that for hole injection.

• The recombination is spontaneous and the spectral outputs have a relatively wide wavelength bandwidth of between 30 – 40 nm.

• \[ \lambda = \frac{hc}{E_g} = \frac{1.24}{E_g} \]
Photon Emission in Semiconductors
A schematic illustration of typical planar surface emitting LED devices. (a) $p$-layer grown epitaxially on an $n^+$ substrate. (b) First $n^+$ is epitaxially grown and then $p$ region is formed by dopant diffusion into the epitaxial layer.

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pn Homojunctions (made of degenerately doped semiconductors)
Basic Semiconductor Luminescent Diode Structures

pn Heterojunctions
(a) A double heterostructure diode has two junctions which are between two different bandgap semiconductors (GaAs and AlGaAs).

(b) A simplified energy band diagram with exaggerated features. $E_F$ must be uniform.

(c) Forward biased simplified energy band diagram.

(d) Forward biased LED. Schematic illustration of photons escaping reabsorption in the AlGaAs layer and being emitted from the device.
In single quantum well (SQW) lasers electrons are injected by the forward current into the thin GaAs layer which serves as the active layer. Population inversion between $E_1$ and $E'_1$ is reached even with a small forward current which results in stimulated emissions.

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Free space wavelength coverage by different LED materials from the visible spectrum to the infrared including wavelengths used in optical communications. Hatched region and dashed lines are indirect $E_g$ materials.

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(a) Photon emission in a direct bandgap semiconductor. (b) GaP is an indirect bandgap semiconductor. When doped with nitrogen there is an electron trap at $E_N$. Direct recombination between a trapped electron at $E_N$ and a hole emits a photon. (c) In Al doped SiC, EHP recombination is through an acceptor level like $E_a$.

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LED Characteristics

(a) Energy band diagram with possible recombination paths. (b) Energy distribution of electrons in the CB and holes in the VB. The highest electron concentration is $(1/2) k_B T$ above $E_c$. (c) The relative light intensity as a function of photon energy based on (b). (d) Relative intensity as a function of wavelength in the output spectrum based on (b) and (c).

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(a) A typical output spectrum (relative intensity vs wavelength) from a red GaAsP LED. (b) Typical output light power vs. forward current. (c) Typical I-V characteristics of a red LED. The turn-on voltage is around 1.5V.

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The output spectrum from AlGaAs LED. Values normalized to peak emission at 25°C.
(a) Some light suffers total internal reflection and cannot escape. (b) Internal reflections can be reduced and hence more light can be collected by shaping the semiconductor into a dome so that the angles of incidence at the semiconductor-air surface are smaller than the critical angle. (b) An economic method of allowing more light to escape from the LED is to encapsulate it in a transparent plastic dome.

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Light is coupled from a surface emitting LED into a multimode fiber using an index matching epoxy. The fiber is bonded to the LED structure.

A microlens focuses diverging light from a surface emitting LED into a multimode optical fiber.

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Schematic illustration of the structure of a double heterojunction stripe contact edge emitting LED

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A multiple quantum well (MQW) structure. Electrons are injected by the forward current into active layers which are quantum wells.

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Light from an edge emitting LED is coupled into a fiber typically by using a lens or a GRIN rod lens.

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LASERS

• The sensitivity of most photosensitive material is greatly increased at wavelength < 0.7 μm; thus, a laser with a short wavelength is desired for such applications as printers and image processing. The sensitivity of the human eye range between the wavelengths of 0.4 and 0.8μm and the highest sensitivity occur at 0.555μm or green so it is important to develop laser in this spectral regime for visual applications.

• Lasers with wavelength between 0.8 – 1.6 μm are used in optical communication systems.
The semiconductor laser diode is a forward bias $p$-$n$ junction. The structure appears to be similar to the LED as far as the electron and holes are concerned, but it is quite different from the point of view of the photons. Electrons and holes are injected into an active region by forward biasing the laser diode. At low injection, these electrons and holes recombine (radiative) via the spontaneous process to emit photons. However, the laser structure is so designed that at higher injections the emission process occurs by stimulated emission. As we will discuss, the stimulated emission process provides spectral purity to the photon output, provides coherent photons, and offers high-speed performance.

The exact output spectrum from the laser diode depends both on the nature of the optical cavity and the optical gain versus wavelength characteristics.

Lasing radiation is only obtained when optical gain in the medium can overcome the photon loss from the cavity, which requires the diode current $I$ to exceed a threshold value $I_{th}$ and $g_{op}>g_{th}$.

Laser-quality crystals are obtained only with lattice mismatches $<0.01\%$ relative to the substrate.
Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, modes, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. \( R \) is mirror reflectance and lower \( R \) means higher loss from the cavity.

\[ m \left( \frac{\lambda}{2} \right) = L \quad m=1,2,3,\ldots \]

\[ I_{cavity} = \frac{I_0}{(1-R)^2 + 4R \sin^2(KL)} \]

\[ \nu_m = m\left( \frac{c}{2L} \right) = m \nu_f \]

\[ \nu_f = \frac{c}{2L} \quad \text{Lowest frequency; } m=1 \]
Transmitted light through a Fabry-Perot optical cavity.

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\[ I_{\text{max}} = \frac{I_0}{(1 - R)^2} \]

\[ F = \frac{\pi R^{1/2}}{1 - R} \]

Finesse measures the loss in the cavity, \( F \) increases as loss decreases.

\[ F = \frac{\Delta \nu_f}{\delta \nu_m} \]

= ratio of the mode separation to spectral width

\[ I_{\text{transmitted}} = I_{\text{incident}} \frac{(1 - R)^2}{(1 - R)^2 + 4R \sin^2(kL)} \]

It is maximum when \( kL = m\pi \)
LASER Diode Modes of Threshold Conditions

Lasing Conditions:

\[
\begin{align*}
I(z) = I(0)e^{\left[\Gamma g(hv) - \bar{\alpha}(hv)\right]z}
\end{align*}
\]

- Population Inversion
- Fabry-Perot cavity
- Gain (of one or several modes) > optical loss

In one round trip i.e. \( z = 2L \) gain should be > loss for lasing;
During this round trip only \( R_1 \) & \( R_2 \) fractions of optical radiation are reflected from the two laser ends 1 & 2.

\[
R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2
\]

\( I \) – optical field intensity
\( g \) – gain coefficient in F.P. cavity
\( \bar{\alpha} \) - effective absorption coefficient
\( \Gamma \) – optical field confinement factor. (the fraction of optical power in the active layer)
LASER Diode Modes of Threshold Conditions

\[ E(z) = E(o)e^{-j\beta z} \]

From the laser conditions: \( I(2L) = I(o) \), \( e^{-j2\beta L} = 1 \) \((2\beta L = 2\pi m)\)

\[ I(2L) = I(0)R_1R_2e^{2L[\Gamma g - \bar{\alpha}]} = I(0) \]

\[ e^{2L[\Gamma g - \bar{\alpha}]} = \frac{1}{R_1R_2} \]

\[ 2L(\Gamma g - \bar{\alpha}) = \ln \frac{1}{R_1R_2} \]

Lasing threshold is the point at which the optical gain is equal to the total loss \( \alpha_t \)

Thus the gain \( \Gamma g_{th} = \alpha_t = \bar{\alpha} + \alpha_{end} \)

\[ g \geq g_{th} = \beta J_{th} \]

\( M \) is constant and depends on the specific device construction.
**Laser Diode Rate Equations**

The relationship between optical output and the diode drive current:

Rate Equations govern the interaction of photons and electrons in the active region.

**Variation of photon concentration:**

\[
\frac{d\phi}{dt} = Bn\phi + R_{sp} - \frac{\phi}{\tau_{ph}} = \text{stimulated emission} + \text{spontaneous emission} - \text{photon loss}
\]

\[
\frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau_{sp}} - Bn\phi = \text{injection} - \text{spontaneous recombination} - \text{stimulated emission}
\]

(shows variation of electron concentration \(n\)).

- \(d\) - is the depth of carrier-confinement region
- \(B\) - is a coefficient (Einstein’s) describing the strength of the optical absorption and emission interactions;
- \(R_{sp}\) - is rate of spontaneous emission into the lasing mode;
- \(\tau_{ph}\) – is the photon lifetime;
- \(\tau_{sp}\) – is the spontaneous recombination lifetime;
- \(J\) – is the injection current density;
Solving the above Equations for a steady-state condition yields an expression for the output power.

\[
\text{Steady-state} \Rightarrow \frac{d\Phi}{dt} = 0 \quad \text{and} \quad \frac{dn}{dt} = 0
\]

n must exceed a threshold value \( n_{th} \) in order for \( \Phi \) to increase. In other words \( J \) needs to exceed \( J_{th} \) in steady-state condition, when the number of photons \( \Phi = 0 \).

\[
\frac{J_{th}}{qd} = \frac{n_{th}}{\tau_{sp}} \quad \text{No stimulating emission}
\]

This expression defines the current required to sustain an excess electron density in the laser when spontaneous emission is the only decay mechanism.
Laser Diode Rate Equations

Now, under steady-state condition at the lasing threshold:

\[
\begin{align*}
Bn_{th}\phi_s + R_{sp} - \frac{\phi_s}{\tau_{ph}} &= 0 \\
-Bn_{th}\phi_s - \frac{n_{th}}{\tau_{sp}} + \frac{J}{qd} &= 0
\end{align*}
\]

\(\phi_s\) is the steady-state photon density.

Adding these two equations:

\[
R_{sp} - \frac{\phi_s}{\tau_{ph}} - \frac{n_{th}}{\tau_{sp}} + \frac{J}{qd} = 0
\]

\[\phi_s = R_{sp}\tau_{ph} - \frac{n_{th}}{\tau_{sp}}\tau_{ph} + \frac{J}{qd}\tau_{ph}\]

but \[\frac{n_{th}}{\tau_{sp}} = \frac{J_{th}}{qd}\]

\[\phi_s = \frac{\tau_{ph}}{qd}\left( J - J_{th} \right) + R_{sp}\tau_{ph}\]

\# of photon resulting from stimulated emission

The power from the first term is generally concentrated in one or few modes;
The second term generates many modes, in order of 100 modes.
**Laser Diode Rate Equations**

To find the optical power $P_0$:

$$P_0 = \frac{\left(\frac{1}{2} \phi_s\right) \text{(volume)} (hv) (1 - R)}{\Delta t}$$

$$\Delta t = \frac{nL}{c} \quad \text{time for photons to cross cavity length } L.$$  

$$\frac{1}{2} \phi_s \quad - \text{is the part travels to right or left (toward output face)}$$

$$P_0 = \left[ \frac{hc^2 \tau_{ph} W (1 - R)}{2qn\lambda} \right] [J - J_{th}]$$  

$R$ is part of the photons reflected and $1-R$ part will escape the facet

$W$ is the width of active layer
Laser Characteristics

\[ P_0 \] lasing output power \( \equiv \Phi_s \)

Threshold population inversion

\[ J_{th} \]
Typical output optical power vs. diode current ($I$) characteristics and the corresponding output spectrum of a laser diode.

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Laser Characteristics
Resonant Frequency

So: \[ 2\beta L = 2\pi m \quad \beta = \frac{2\pi n}{\lambda} \]

\[ 2\left(\frac{2\pi n}{\lambda}\right)L = 2\pi m \quad ; \quad m = \frac{L}{\frac{\lambda}{2n}} \quad ; \quad m = \frac{2nL}{\lambda} = \frac{2nLv}{c} \]

Remember that inside the optical cavity for zero phase difference:

\[ e^{-j2\beta L} = 1 \]

This states that the cavity resonates (i.e. a standing wave pattern exists within it) when an integer number \( m \) of \( \lambda/2 \) spans the region between the mirrors. Depending on the laser structures, any number of freq. can satisfy

\[ I(2L) = I(0) \quad \& \quad e^{-j2\beta L} = 1 \]

Thus some lasers are single - & some are multi-modes. The relationship between gain & freq. can be assumed to have Gaussian form:

\[ g(\lambda) = g(0)e^{-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}} \]

where \( \lambda_0 \) is the wavelength at the center of spectrum; \( \sigma \) is the spectrum width of gain & maximum \( g(0) \) is proportional to the population inversion.
Laser Characteristics
Spacing between the modes:

\[ m = \frac{2Ln}{c} v_m \quad \frac{m\lambda}{2n} = L \]

\[ m - 1 = \frac{2Ln}{c} v_{m-1} \]

\[ \frac{2Ln}{c} (v_m - v_{m-1}) = \frac{2Ln}{c} \Delta v = 1 \]

\[ \Delta v = \frac{c}{2Ln} \]

\[ c = v\lambda \quad v = \frac{c}{\lambda} \Rightarrow \Delta v = -\frac{c\Delta\lambda}{\lambda^2} \]

\[ \frac{c\Delta\lambda}{\lambda^2} = \frac{c}{2Ln} \Rightarrow \Delta\lambda = \frac{\lambda^2}{2Ln} \]

or

\[ \lambda = \frac{2Ln}{m} \]

\[ \Delta\lambda_m = \frac{2Ln}{m} - \frac{2Ln}{m+1} = \frac{2Ln}{m^2} = \frac{\lambda^2}{2Ln} \]
Laser Characteristics
Internal & External Quantum Efficiency

Quantum Efficiency (QE) = # of photons generated for each EHP injected into the semiconductor junction \( \equiv \text{a measure of the efficiency of the electron-to-photon conversion process.} \)

If photons are counted at the junction region, QE is called internal QE (\( \eta_{\text{int}} \)), which depends on the materials of the active junction and the neighboring regions. For GaAs \( \eta_{\text{int}} = 65\% \) to 100%.

If photons are counted outside the semiconductor diode QE is external QE (\( \eta_{\text{ext}} \)).

Consider an optical cavity of length L, thickness W and width S. Defining a threshold gain \( g_{\text{th}} \) as the optical gain needed to balance the total power loss, due to various losses in the cavity, and the power transmission through the mirrors.
The optical intensity due to the gain is equal to:
\[ I = I_0 \exp(2Lg_{th}), \]
There will be lost due to the absorption and the reflections on both ends by

\[ R_1R_2 \exp(-2L\alpha) \]

So: \[ I = I_0 \exp(2Lg_{th}) \{ R_1R_2 \exp(-2L\alpha) \} = I_0 \] (at threshold). Therefore:

\[ R_1R_2 e^{2L(g_{th} - \alpha)} = 1 \]

where \( R_1 \) and \( R_2 \) are power reflection coefficients of the mirrors, \( \alpha \) is attenuation constant.
Laser Characteristics
Internal & External Quantum Efficiency

\[ g_{th} = \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \]

\( g \) is gain constant of the active region and is roughly proportional to current density \( (g = \beta J) \). \( \beta \) is a constant.

\[ J_{th} = \left( \frac{1}{\beta} \right) \left( \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) \]

By measuring \( J_{th}, \alpha, L, R_1 \) and \( R_2 \) one can calculate \( \beta \) (dependent upon the materials and the junction structure).

\[ \frac{1}{2L} \ln \frac{1}{R_1 R_2} \]
\[ \alpha + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \equiv \frac{P_{ra}}{P_{total}} \]

The ratio of the power radiated through mirrors to the total power generated by the semiconductor junction is
Laser Characteristics
Internal & External Quantum Efficiency

Therefore $\eta_{\text{ext}} = \eta_{\text{int}} (P_{\text{ra}} / P_{\text{total}})$ which can be determined experimentally from PI characteristic.

For a given $I_a$ (in PI curve current at point a) the number of electrons injected into the active area/sec = $I_a / q$ and the number of photons emitted /second = $P_a / h\nu$

\[
\eta_{\text{ext}} = \frac{P_a}{I_a / q} \quad \eta_{\text{ext}} = \frac{P_b}{I_b / q} \quad \eta_{\text{ext}} = \frac{q(P_a - P_b)}{(I_a - I_b)h\nu} = \frac{q \Delta P}{h\nu \Delta I}
\]

i.e. $\eta_{\text{ext}}$ is proportional to slope of PI curve in the region of $I > I_{\text{th}}$.

If we choose $I_b = I_{\text{th}}$, $P_b \approx 0$ and

$h\nu \approx E_g$ and $h\nu / q = E_g / q$ gives voltage across the junction in volts.
At dc or low frequency the equivalent circuit to a LASER diode may be viewed as an ideal diode in series with $r_s$.

Therefore the power efficiency is given by

$$\eta_p = \frac{\text{optical power output}}{\text{dc electrical power input}}$$

$$\eta_p = \frac{P}{I(E_g / q) + I^2 r_s}$$

$$P = \frac{h \nu}{q} (I - I_{th}) \eta_{ext}$$

$$\eta_p = \frac{\eta_{ext} (I - I_{th}) E_g / q}{I(E_g / q) + I^2 r_s}$$
A schematic illustration of a GaAs homojunction laser diode. The cleaved surfaces act as reflecting mirrors.

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Typical output optical power vs. diode current ($I$) characteristics and the corresponding output spectrum of a laser diode.

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(a) A double heterostructure diode has two junctions which are between two different bandgap semiconductors (GaAs and AlGaAs).

(b) Simplified energy band diagram under a large forward bias. Lasing recombination takes place in the $p$-GaAs layer, the active layer.

(c) Higher bandgap materials have a lower refractive index.

(d) AlGaAs layers provide lateral optical confinement.
Schematic illustration of the the structure of a double heterojunction stripe contact laser diode

Oxide insulator
Stripe electrode
Substrate
Current paths
Elliptical laser beam
Active region where $J > J_{th}$
(Emission region)

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Oxide insulation

$p^+\text{-AlGaAs}$ (Contacting layer)

$p\text{-AlGaAs}$ (Confining layer)

$n\text{-AlGaAs}$

$p\text{-GaAs}$ (Active layer)

$n\text{-AlGaAs}$ (Confining layer)

$n\text{-GaAs}$ (Substrate)

Schematic illustration of the cross sectional structure of a buried heterostructure laser diode.

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The laser cavity definitions and the output laser beam characteristics.

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Output spectra of lasing emission from an index guided LD. At sufficiently high diode currents corresponding to high optical power, the operation becomes single mode. (Note: Relative power scale applies to each spectrum individually and not between spectra)

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Typical optical power output vs. forward current for a LED and a laser diode.

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(a) Distributed Bragg reflection (DBR) laser principle. (b) Partially reflected waves at the corrugations can only constitute a reflected wave when the wavelength satisfies the Bragg condition. Reflected waves $A$ and $B$ interfere constructive when $q(\lambda_B/2n) = \Lambda$.

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(a) Distributed feedback (DFB) laser structure. (b) Ideal lasing emission output. (c) Typical output spectrum from a DFB laser.

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Fiber Bragg grating

\[ \lambda_B = 2n_{\text{eff}} \Lambda \]
Fiber Bragg grating fabrication
Phase Mask: Direct Imprinting

$\Lambda_{PM}$

$248 \text{ nm Laser}$

Ge doped Fiber

Diffraction
$m = -1$

(Suppressed)

$0^{th}$ order

Diffraction
$m = +1$
Cleaved-coupled-cavity (C³) laser

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A quantum well (QW) device. (a) Schematic illustration of a quantum well (QW) structure in which a thin layer of GaAs is sandwiched between two wider bandgap semiconductors (AlGaAs). (b) The conduction electrons in the GaAs layer are confined (by $^{2}E_{c}$) in the x-direction to a small length $d$ so that their energy is quantized. (c) The density of states of a two-dimensional QW. The density of states is constant at each quantized energy level.

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In single quantum well (SQW) lasers electrons are injected by the forward current into the thin GaAs layer which serves as the active layer. Population inversion between $E_1$ and $E'_1$ is reached even with small forward current which results in stimulated emissions.

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A multiple quantum well (MQW) structure. Electrons are injected by the forward current into active layers which are quantum wells.

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