Photoconductors

\[ \sigma_0 = q(\mu_n n_0 + \mu_p p_0) \]

\[ \sigma = q[\mu_n (n_0 + \delta n) + \mu_p (p_0 + \delta p)] = \sigma_0 + \delta \sigma \]

\[ \delta \sigma = q(\delta p)(\mu_n + \mu_p) \]

\[ G_n = \frac{\delta n}{\tau_n} \quad \& \quad G_p = \frac{\delta p}{\tau_p} \]

\[ J = (J_0 + J_L) = (\sigma_0 + \delta \sigma)E \]

\[ \text{Gain} = \frac{\tau_p}{t_n} \left(1 + \frac{\mu_p}{\mu_n}\right) \quad t_n = \frac{\ell}{\mu_n E} \]
Photodiodes

(a) A schematic diagram of a reverse biased photodiode. (b) Net space charge across the diode in the depletion region. \( N_d \) and \( N_a \) are the donor and acceptor concentrations in the \( p \) and \( n \) sides. (c) The field in the depletion region.

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\[ v_e = \mu_e E \]
\[ v_p = \mu_p E \]
\[ t_e = \frac{L - l}{v_e} \]
\[ t_p = \frac{l}{v_p} \]

work = \int qEdx = Vi_e(t)dx

\[ E = V / L \quad \& \quad v_e = dx / dt \]

\[ i_e(t) = \frac{qV_e}{L} \quad t < t_e \quad \& \quad i_p(t) = \frac{qV_p}{L} \quad t < t_p \]

\[ Q_{collected} = \int_0^{t_e} i_e(t)dt + \int_0^{t_p} i_p(t)dt = \frac{qV_e t_e}{L} + \frac{qV_p t_p}{L} = q \]

So, the collected charge is one electron charge.
Power absorption

\[ P(X) = p_0 e^{-\alpha(\lambda)x} \]

Power generation

\[ P(X) = p_0(1 - e^{-\alpha(\lambda)x}) \]

Absorption coefficient (\( \alpha \)) vs. wavelength (\( \lambda \)) for various semiconductors
(Data selectively collected and combined from various sources.)

Figure 5.3
\[ P(X) = p_0(1 - e^{-\alpha x}) \] \hspace{1cm} \text{Power generation}

\[ P(X) = p_0e^{-\alpha x} \] \hspace{1cm} \text{Power absorption}

\[ \eta_{ext} = \frac{\# \text{of EHP generated}}{\# \text{of incident photons}} = \frac{I_p}{q \frac{P_0}{hc}} \] \hspace{1cm} \text{External Q. Efficiency}

\[ I_p = qA \int_0^w G(x)dx = qA \Phi_0 \alpha_s \int_0^w e^{-\alpha_s x} dx = qA \Phi_0 [1 - e^{-\alpha_s w}] = qA \frac{P_0(1 - R_f)}{hc \nu A} [1 - e^{-\alpha_s w}] \]

\[ G(x) = \Phi_0 \alpha_s e^{-\alpha_s x} \] \hspace{1cm} \text{EHP generation rate} \hspace{1cm} \Phi_0 = \frac{P_0(1 - R_f)}{Ah \nu} \hspace{1cm} \text{Incident photon flux/unit area}

\[ R = \frac{\text{photocurrent} \ (A)}{\text{incident optical power} \ (W)} = \frac{I_p}{P_0} = \eta_{ext} \frac{q \lambda}{hc} \] \hspace{1cm} \text{Responsivity of the photodiode}
(a) Photon absorption in a direct bandgap semiconductor. (b) Photon absorption in an indirect bandgap semiconductor (VB, valence band; CB, conduction band)

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Photodiodes

External Q. Efficiency

$$\eta_{ext} = \frac{\# \text{of EHP generated}}{\# \text{of incident photons}} = \frac{I_p}{q} \frac{P_0}{h\nu}$$

$$I_p = qA \int_0^w G(x) \, dx = qA \Phi_0 \alpha_s \int_0^w e^{-\alpha_s x} \, dx = qA \Phi_0 [1 - e^{-\alpha_s w}] = qA \frac{P_0(1 - R_f)}{h\nu A} [1 - e^{-\alpha_s w}]$$

$$G(x) = \Phi_0 \alpha_s e^{-\alpha_s x} \quad \text{EHP generation rate} \quad \Phi_0 = \frac{P_0(1 - R_f)}{Ah\nu} \quad \text{Incident photon flux/unit area}$$

$$R = \frac{\text{photocurrent (A)}}{\text{incident optical power (W)}} = \frac{I_p}{P_0} = \eta_{ext} \frac{q\lambda}{hc} \quad \text{Responsivity of the photodiode}$$

(a) A schematic diagram of a reverse biased pn junction photodiode; (b) Net space charge across the diode in the depletion region. $N_d$ and $N_a$ are the donor and acceptor concentrations in the $p$ and $n$ sides. (c) The field in the depletion region.
Responsivity ($R$) vs. wavelength ($\lambda$) for an ideal photodiode with QE = 100% ($\eta = 1$) and for a typical commercial Si photodiode.

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The schematic structure of an idealized pin photodiode (b) The net space charge density across the photodiode. (c) The built-in field across the diode. (d) The pin photodiode in photodetection is reverse biased.

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A reverse biased pin photodiode is illuminated with a short wavelength photon that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the $i$-layer and drifted across.

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Avalanche photodiode (APD)

(a) A schematic illustration of the structure of an avalanche photodiode (APD) biased for avalanche gain. (b) The net space charge density across the photodiode. (c) The field across the diode and the identification of absorption and multiplication regions.

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(a) A pictorial view of impact ionization processes releasing EHPs and the resulting avalanche multiplication. (b) Impact of an energetic conduction electron with crystal vibrations transfers the electron's kinetic energy to a valence electron and thereby excites it to the conduction band.

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(a) A Si APD structure without a guard ring. (b) A schematic illustration of the structure of a more practical Si APD

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Simplified schematic diagram of a separate absorption and multiplication (SAM) APD using a heterostructure based on InGaAs-InP. $P$ and $N$ refer to $p$ and $n$-type wider-bandgap semiconductor.

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Simplified schematic diagram of a more practical mesa-etched SAGM layered APD.

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Energy band diagram of a staircase superlattice APD (a) No bias. (b) With an applied bias.

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The principle of operation of the photodiode. SCL is the space charge layer or the depletion region. The primary photocurrent acts as a base current and gives rise to a large photocurrent in the emitter-collector circuit.
A photoconductor with ohmic contacts (contacts not limiting carrier entry) can exhibit gain. As the slow hole drifts through the photoconductors, many fast electrons enter and drift through the photoconductor because, at any instant, the photoconductor must be neutral. Electrons drift faster which means as one leaves, another must enter.

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In $pn$ junction and $pin$ devices the main source of noise is shot noise due to the dark current and photocurrent.

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The responsivity of a commercial Ge pn junction photodiode

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The responsivity of two commercial Si *pin* photodiodes

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The responsivity of an InGaAs pin photodiode

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An infinitesimally short light pulse is absorbed throughout the depletion layer and creates an EHP concentration that decays exponentially.

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Drift velocity vs. electric field for holes and electrons in Si.

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(a) A linearly polarized wave has its electric field oscillations defined along a line perpendicular to the direction of propagation, \( z \). The field vector \( \mathbf{E} \) and \( z \) define a plane of polarization. (b) The \( E \)-field oscillations are contained in the plane of polarization. (c) A linearly polarized light at any instant can be represented by the superposition of two fields \( E_x \) and \( E_y \) with the right magnitude and phase.

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A right circularly polarized light. The field vector $E$ is always at right angles to $z$, rotates clockwise around $z$ with time, and traces out a full circle over one wavelength of distance propagated.

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Examples of linearly, (a) and (b), and circularly polarized light (c) and (d): (c) is right circularly and (d) is left circularly polarized light (as seen when the wave directly approaches a viewer)

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(a) Linearly polarized light with $E_{yo} = 2E_{xo}$ and $\phi = 0$. (b) When $\phi = \pi/4$ (45°), the light is right elliptically polarized with a tilted major axis. (c) When $\phi = \pi/2$ (90°), the light is right elliptically polarized. If $E_{xo}$ and $E_{yo}$ were equal, this would be right circularly polarized light.

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Field distributions in plane E&M waves
Adding two linearly polarized waves
Elliptically polarized light
Adding linearly polarized waves
Randomly polarized light is incident on a Polarizer 1 with a transmission axis $\text{TA}_1$. Light emerging from Polarizer 1 is linearly polarized with $\mathbf{E}$ along $\text{TA}_1$, and becomes incident on Polarizer 2 (called "analyzer") with a transmission axis $\text{TA}_2$ at an angle $\theta$ to $\text{TA}_1$. A detector measures the intensity of the incident light. $\text{TA}_1$ and $\text{TA}_2$ are normal to the light direction.

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7.6 Modulation by Malus’s law  Suppose that a linearly polarized light is passed through a polarizer placed with its transmission axis at angle $\pi/4$ (45°) to the incoming optical field. Suppose now that we “rotationally modulate” the transmission axis of the polarizer by small amounts $\varphi$ about $\pi/4$ (45°). Show that the change in the transmission intensity is

$$\Delta l \propto -\varphi + \frac{2}{3} \varphi^3 - ...$$

where $\varphi$ is in radians. What is the extent of change (in degrees) in $\varphi$ so that the second term is only 1% of the first term? What is your conclusion.
A line viewed through a cubic sodium chloride (halite) crystal (optically isotropic) and a calcite crystal (optically anisotropic).

Optically isotropic materials like glasses, and cubic crystals

Optically anisotropic materials (birefringent) like noncubic crystals, calcite (CaCO$_3$, Quartz, ice, tournaline,...
Two polaroid analyzers are placed with their transmission axes, along the long edges, at right angles to each other. The ordinary ray, undeflected, goes through the left polarizer whereas the extraordinary wave, deflected, goes through the right polarizer. The two waves therefore have orthogonal polarizations.

**Biaxial** crystals like mica have 3 distinct principal indices.

**Uniaxial** crystals like calcite, and quartz have two of their indices equal to each other.

The velocity of light and state of its polarization depends on the direction of propagation of light.
Light (EM wave) entering an isotropic crystal splits into two \( \perp \) linearly polarized waves traveling with different phase velocities. In uniaxial crystals are called ordinary (o) and extraordinary (e) waves.

The o-wave behaves like an ordinary light. Has the same phase velocity in all direction and the field is perpendicular to propagation direction.

The e-wave has a phase velocity that depends on its direction of propagation and the electric field is not necessarily perpendicular to the propagation direction.

The two waves propagate with same velocity only along a special direction called the optic axis.

The o-wave is always perpendicularly polarized to the optic axis and obeys the Snell’s law.
(a) Fresnel's ellipsoid

(b) An EM wave propagating along $OP$ at an angle $\theta$ to optic axis.

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$E_o = E_{o\text{-wave}}$ and $E_e = E_{e\text{-wave}}$ (a) Wave propagation along the optic axis. (b) Wave propagation normal to optic axis

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(a) Wavevector surface cuts in the $xz$ plane for $o$- and $e$-waves.  (b) An extraordinary wave in an anisotropic crystal with a $k_e$ at an angle to the optic axis. The electric field is not normal to $k_e$. The energy flow (group velocity) is along $S_e$ which is different than $k_e$.

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An EM wave that is off the optic axis of a calcite crystal splits into two waves called ordinary and extraordinary waves. These waves have orthogonal polarizations and travel with different velocities. The $o$-wave has a polarization that is always perpendicular to the optical axis.

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(a) A birefringent crystal plate with the optic axis parallel to the plate surfaces. (b) A birefringent crystal plate with the optic axis perpendicular to the plate surfaces.

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• **7.7 Birefringence** Consider a negative uniaxial crystal such as calcite \((n_e < n_o)\) plate that has the optic axis (taken along \(z\)) parallel to the plate faces. Suppose that a linearly polarized wave is incident at normal incidence on a plate face. If the optical field is at an angle \(45^\circ\) to the optic axis, sketch the wavefronts and the rays through the calcite plate.
A retarder plate. The optic axis is parallel to the plate face. The \( o \)- and \( e \)-waves travel in the same direction but at different speeds.

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Half wavelength plate: $\phi = \hat{s}$

Optic axis

Input

$\alpha = \text{arbitrary}$

Output

$\alpha < 45^\circ$

Quarter wavelength plate: $\phi = \hat{s}/2$

$0 < \alpha < 45^\circ$

$\alpha = 45^\circ$

Input and output polarizations of light through (a) a half-wavelength plate and (b) through a quarter-wavelength plate.

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\[ \phi_1 = \frac{2\pi}{\lambda} (n_e d + n_o D) \]

\[ \phi_2 = \frac{2\pi}{\lambda} (n_o d + n_e D) \]

\[ \phi = \frac{2\pi}{\lambda} (n_e - n_o)(D - d) \]
• **7.8 Wave plates** Calculate and compare the thickness of quarter-wave plates made from calcite, quartz and LiNbO3 crystals all operating at a wavelength of $\lambda \approx 590$ nm. What is your conclusion? Assuming little relative change in the indices, what are the thicknesses at double the wavelength?
7.9 Soleil Compensator: Consider a Soleil compensator as in Figure 7Q9 that uses a quartz crystal. Given a light wave with a wavelength $\lambda \approx 600$ nm, a lower plate thickness of 5 mm, calculate the range of $d$ values in Figure 7Q9 that provide a retardation from 0 to $\pi$ (half-wavelength).
The Wollaston prism is a beam polarization splitter. $E_1$ is orthogonal to the plane of the paper and also to the optic axis of the first prism. $E_2$ is in the plane of the paper and orthogonal to $E_1$. 

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• **7.10 Quartz Wollaston prism**  Draw a quartz Wollaston prism and clearly show and identify the directions of orthogonally polarized waves traveling through the prisms. How would you test the polarization states of the emerging rays? Consider two identical Wollaston prisms, one from calcite and the other from quartz. Which will have a greater beam splitting ability? (Explain).
Optical Activity

An optically active material such as quartz rotates the plane of polarization of the incident wave: The optical field $\mathbf{E}$ rotated to $\mathbf{E}'$. If we reflect the wave back into the material, $\mathbf{E}'$ rotates back to $\mathbf{E}$.

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Vertically polarized wave at the input can be thought of as two right and left handed circularly polarized waves that are symmetrical, i.e. at any instant $\alpha = \beta$. If these travel at different velocities through a medium then at the output they are no longer symmetric with respect to $y$, $\alpha \neq \beta$, and the result is a vector $E'$ at an angle $\theta$ to $y$.

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$$\theta = \frac{\pi}{\lambda} (n_R - n_L) L$$
Pockels Effect

(a) Cross section of the optical indicatrix with no applied field, \( n_1 = n_2 = n_o \)  (b) Th applied external field modifies the optical indicatrix. In a KDP crystal, it rotates the principal axes by 45° to \( x' \) and \( y' \) and \( n_1 \) and \( n_2 \) change to \( n_1' \) and \( n_2' \). (c) Applied field along \( y \) in \( \text{LiNbO}_2 \) modifies the indicatrix and changes \( n_1 \) and \( n_2 \) change to \( n \) and \( n_2' \).

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\[
\begin{align*}
n_1' & \approx n_1 + \left( \frac{1}{2} \right) n_1^3 r_{22} E_a \\
n_2' & \approx n_2 - \left( \frac{1}{2} \right) n_2^3 r_{22} E_a
\end{align*}
\]

\[
\Delta \phi = \frac{2\pi n_1'}{\lambda} L - \frac{2\pi n_2'}{\lambda} L = \frac{2\pi}{\lambda} n_0^3 r_{22} \frac{L}{d} V
\]
Tranverse Pockels cell phase modulator. A linearly polarized input light into an electro-optic crystal emerges as a circularly polarized light. $E_a$ is the applied field parallel to $y$.

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Left: A transverse Pockels cell intensity modulator. The polarizer $P$ and analyzer $A$ have their transmission axis at right angles and $P$ polarizes at an angle 45° to $y$-axis. Right: Transmission intensity vs. applied voltage characteristics. If a quarter-wave plate ($QWP$) is inserted after $P$, the characteristic is shifted to the dashed curve.

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(a) An applied electric field, via the Kerr effect, induces birefringences in an otherwise optically isotropic material. (b) A Kerr cell phase modulator.

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Kerr Effect

\[ \Delta n = \lambda KE_a^2 \]

\[ \Delta \phi = \frac{2\pi LKV^2}{d^2} \]
Integrated tranverse Pockels cell phase modulator in which a waveguide is diffused into an electro-optic (EO) substrate. Coplanar strip electrodes apply a transverse field $E_a$ through the waveguide. The substrate is an $x$-cut LiNbO$_3$ and typically there is a thin dielectric buffer layer (e.g. ~200 nm thick SiO$_2$) between the surface electrodes and the substrate to separate the electrodes away from the waveguide.

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An integrated Mach-Zender optical intensity modulator. The input light is split into two coherent waves $A$ and $B$, which are phase shifted by the applied voltage, and then the two are combined again at the output.

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(a) Cross section of two closely spaced waveguides $A$ and $B$ (separated by $d$) embedded in a substrate. The evanescent field from $A$ extends into $B$ and vice versa. Note: $n_A$ and $n_B > n_s$ (= substrate index).

(b) Top view of the two guides $A$ and $B$ that are coupled along the $z$-direction. Light is fed into $A$ at $z = 0$, and it is gradually transferred to $B$ along $z$. At $z = L_o$, all the light been transferred to $B$. Beyond this point, light begins to be transferred back to $A$ in the same way.

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Transmission power ratio from guide A to guide B over the transmission length $L_o$ as a function of mismatch $\Delta \beta$. 

\[ \frac{P_B(L_o)}{P_A(0)} \]

$\Delta \beta \propto V$

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An integrated directional coupler. Applied field $E_a$ alters the refractive indices of the two guides and changes the strength of coupling.

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Traveling acoustic waves create a harmonic variation in the refractive index and thereby create a diffraction grating that diffracts the incident beam through an angle $2\theta$.

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Consider two coherent optical waves $A$ and $B$ being "reflected" (strictly, scattered) from two adjacent acoustic wavefronts to become $A'$ and $B'$. These reflected waves can only constitute the diffracted beam if they are in phase. The angle $\theta$ is exaggerated (typically this is a few degrees).

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Magneto-Optic Effects (Faraday Effect)

The sense of rotation of the optical field $E$ depends only on the direction of the magnetic field for a given medium (given Verdet constant). If light is reflected back into the Faraday medium, the field rotates a further $\theta$ in the same sense to come out as $E''$ with a $2\theta$ rotation with respect to $E$.

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$$\theta = \nu BL$$