

Lecture 12, Feb. 26, 2007

## Encoders

In the last lecture, we introduced the decoders. An encoder does the inverse function of a decoder. That is, an encoder converts a symbol (say, one of  $2^n$  possible symbols) into a bit stream of  $n$  bits.

A familiar example of an encoder is an ASCII encoder. An ASCII encoder transforms an alphanumeric symbol such as a computer keyboard's keys into a 7-bit output suitable for the computer.

When you type the letter A, for example, the output will be 1000001, and letter

W will correspond to 1010111. The symbol

> (greater than) is represented by 01111110, etc.

In general, an encoder will have  $2^n$  (or less) inputs denoted by  $D_0, D_1, \dots$  and up to  $n$  outputs.

Let's consider the example of the octal (8-input) encoder. This encoder needs to have 8 inputs  $D_0, D_1, \dots, D_7$  and three outputs  $x, y$  and  $z$ . The value of the output represents the input that is active for example if  $D_0 = 1$  and other inputs are 0. Then  $xyz = 000$ .

The truth table for the octal encoder is:

$D_0$	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	$D_7$	$x$	$y$	$z$
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

You may notice that the implementation of the encoder is quite simple. It can be implemented using OR gates:

$$z = D_1 + D_3 + D_5 + D_7$$

$$y = D_2 + D_3 + D_6 + D_7$$

$$x = D_4 + D_5 + D_6 + D_7$$

Note that all is well as long as one and only one input is selected.

As soon as two inputs become 1, the output becomes ambiguous. It is like pressing two keys on the key-board at the same time. Say, if we press 3 and 6 at the same time. Then  $D_3 = D_6 = 1$  and  $z = y = x = 1$ . But, 111 represents  $D_7$ .

So, we need to put in some logic so that when two inputs are high (equal to 1) only one of them be given priority and be accepted. Also note that when there is no input, the output is 000 which is the same as the code for  $D_0 = 1$ .

To avoid these problems, we need:

- An extra output indicating whether any of the inputs is on or not.
- A priority logic deciding to

choose one of the inputs when more than one input is high.

Example: As an example, we take a 4-input encoder. Here, instead of two outputs  $x$  and  $y$ , we have three outputs  $x$ ,  $y$  and  $V$ , where  $V=1$  signifies the presence of input.  $V=0$  means that no input is selected. We also add priority by giving priority to the input with the largest index. For example if  $D_0$  and  $D_2$  are both 1, we assume that the input is  $D_2=1$ .

The truth table for this priority encoder is:

$D_0$	$D_1$	$D_2$	$D_3$	$x$	$y$	$V$
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

Note that  $V = D_0 + D_1 + D_2 + D_3$ .

The K-maps for  $x$  and  $y$  are:

$D_2 D_1 \backslash D_2 D_3$	00	01	11	10
00	X	1	1	1
01	0	1	1	1
11	0	1	1	1
10	0	1	1	1

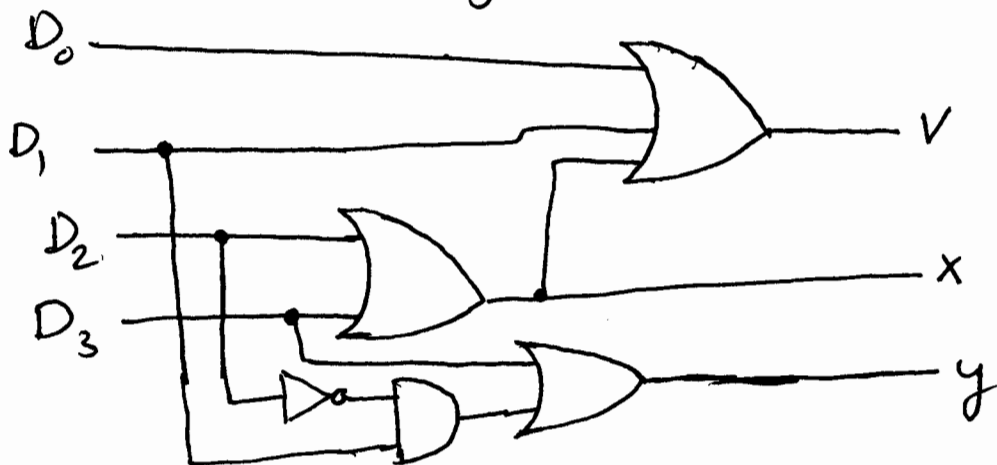
$D_2$  (points to the rightmost column)

$D_3$  (bracket under the last three columns)

$$X = D_2 + D_3$$

$D_2 D_1 \backslash D_2 D_3$	00	01	11	10
00	X	1	1	0
01	1	1	1	0
11	1	1	1	0
10	0	1	1	0

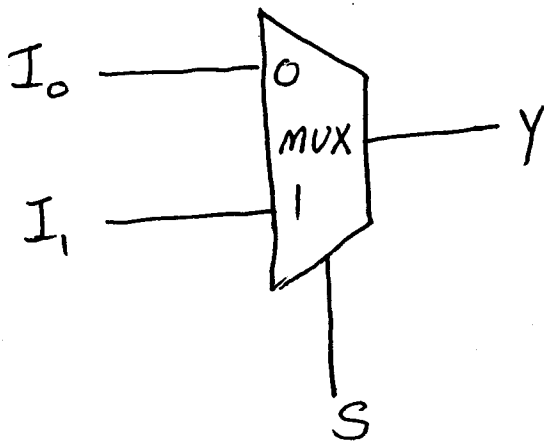
$$Y = D_3 + D_1 D_2'$$



## Multiplexers

A multiplexer has several (usually  $2^n$ ) inputs and one output. It works like many-to-one switch. If the multiplexer has  $2^n$  input, it needs  $n$  control (or select) inputs to say which input should be selected, i. e., connected to the output.

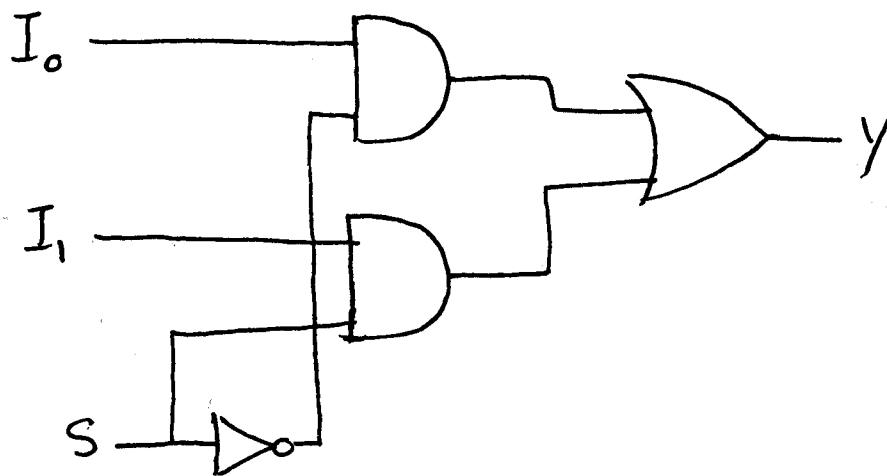
Example: A two-input multiplexer:



Here, the select bit decides which input should be connected to the output.

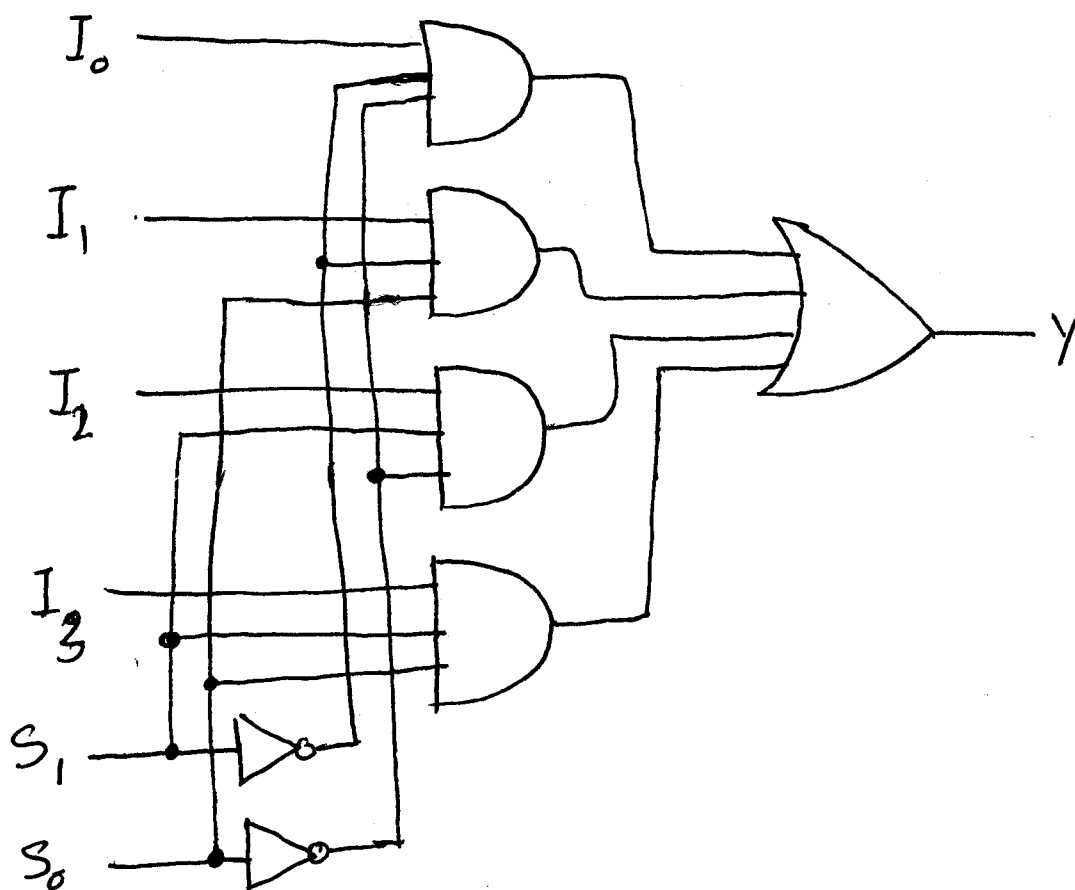
If  $S = 0$ , then  $Y = I_0$  and when  $S = 1$ , we have  $Y = I_1$ .

The 2-input multiplexer can be implemented as:



Example: A 4-to-1 multiplexer.

Here, we need  $\log_2 4 = 2$  select inputs:



## Boolean function implementation using multiplexers

A Boolean function with  $n$  variables, can be implemented using a multiplexer with  $2^{n-1}$  inputs. Such a multiplexer has  $n-1$  selection lines. The first  $n-1$  variables of the function are connected to the  $n-1$  selection inputs of the multiplexer. The remaining variable, say,  $z$ , will be used for data inputs. Depending on the function, the inputs will receive  $z, z', 1$  or  $0$ .

Example: Implement

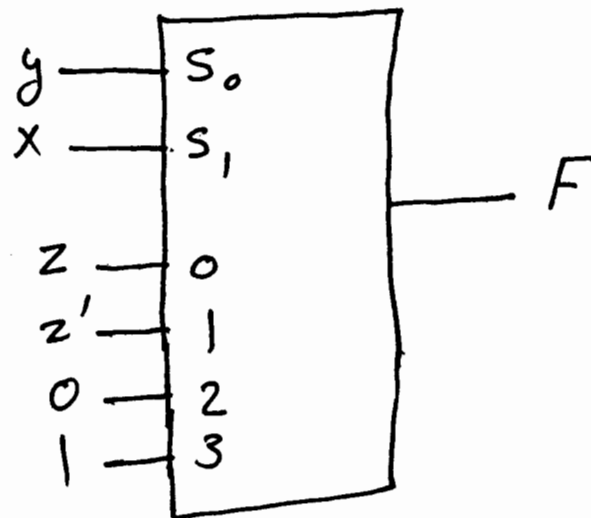
$$F(x, y, z) = \sum(1, 2, 6, 7)$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$F=z$  (for  $x=0$ )  
 $F=z'$  (for  $x=0$ )  
 $F=0$  (for  $x=1$ )  
 $F=1$  (for  $x=1$ )



We connect  $x$  and  $y$  to the 2 selection inputs of 4-to-1 MUX.



When  $xy = 00$ ,  $F = D_0$ . From the truth table, we see that for  $xy = 0$ ,  $F = z$ .

So to  $D_0$  we should connect  $z$

When  $xy = 01$ ,  $F = D_1$ , But, from the truth table  $xy = 01 \Rightarrow F = z'$ . So, connect  $z'$  to  $D_1$ .

Similarly, connect  $0$  to  $D_2$  and  $1$  to  $D_3$ .

Example: Implement

$$F(A, B, C, D) = \sum (1, 3, 4, 11, 12, 13, 14, 15)$$

using a multiplexer.

Since there are 4 variables, we need an 8-to-1 multiplexer.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

