DIGITAL DESIGN, COEN 312	Lecturers: A. J. Al-Khalili,
Final Dec. 14, 2010	
Answer All Questions	Time Allowed 3:00 hrs.
No calculators, notes or books are allowed	

--Question 1

a) Using Boolean Algebra, minimize the following function (step by step):

i) **F (A,B,C) = A' + AB + (A' + B)(A +B');**

ii) **iii) F =BC + AC + A'B'C** (4marks)

b) List all possible **prime implicants** and **essential prime implicants** for Function F:

F = ∑(0, 2, 6, 7, 10, 13, 14, 15,) with don't care conditions **d = ∑(1, 3, 12).**

Show groupings on a **Karnaugh map** accompanied with their algebraic forms. (3 marks)

c) A circuit of Fig. 1 has been designed by a inexperienced engineer. Re-design the circuit, by Express P and Q in Fig. 1 as function of A, B and C and then produce an optimized circuit. (3 marks)

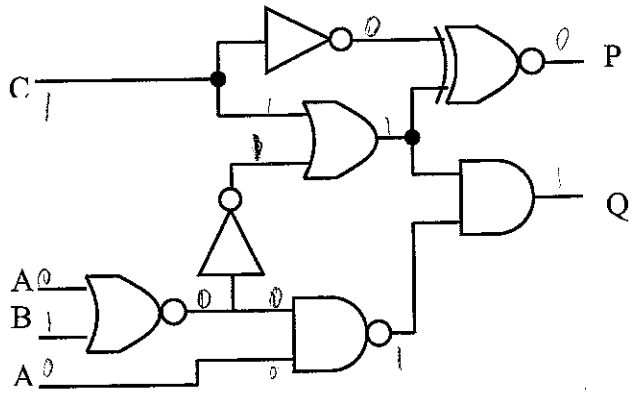

Fig. 1

--Question 2

2.1 Express function F

F= AB + A'B' with:

i) NAND-NAND ii) NOR-NOR iii) AND-OR-INV , iv) OR-AND-INV.

Show the different steps and the logic diagram. (6 marks)

2.2 Consider the circuit in Fig. 2. Draw the **timing diagram** of F when input A, goes from 0 to 1→ 0 → 1→0. Indicate all timings on the diagram. (4 Marks)

Assume delay of Inverter = 3 ns and delay of NAND =5n

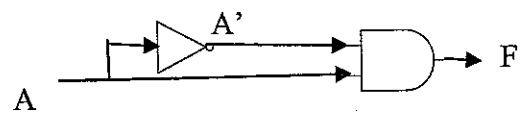

Fig. 2

--Question 3

a) Design a Circuit to give $F = A^3$.

A is a 2-bit unsigned binary number. **A=a_1a_0.** (6 marks)

b) Use 2 to 4 decoder plus OR gates to implement F. (4 marks)

--Question 4

a) What is the difference between ROM, PROM, PAL, and the PLA. (2 marks)

b) A designed sequential circuit is to be implemented using **2 JK Flip Flops and a PLA**. The excitation equations are:

$J_A = Bx'$ $K_A = Bx$

$J_B = x$ $K_B = (A \oplus x)$

Where A and B are the present states and x is an external input. Implement the circuit, with a PLA. Give, personality matrix and showing your programming of the PLA by placing an • on the programmable nodes. (5 marks)

c) Implement the following function F, using only 2 to 1 multiplexers:

$F = A + B + ACD$ (3marks)

Question 5

Design a synchronous counter that generates the odd numbers between 0 and 10. (1,3,5,7,9) repeatedly and to generate an output of "1" corresponding to the number 9.

1) Draw the State Diagram.
2) Use binary state assignment, derive the State Transition Table.
3) Using D flip flops, derive the excitation equations.
4) Draw the sequential circuit diagram. (10 marks)

--Question 6

The circuit in Figure 3 has been designed to perform certain function. Analyze the circuit. Derive the state transition table and state diagram including all inputs and outputs. (10 marks)

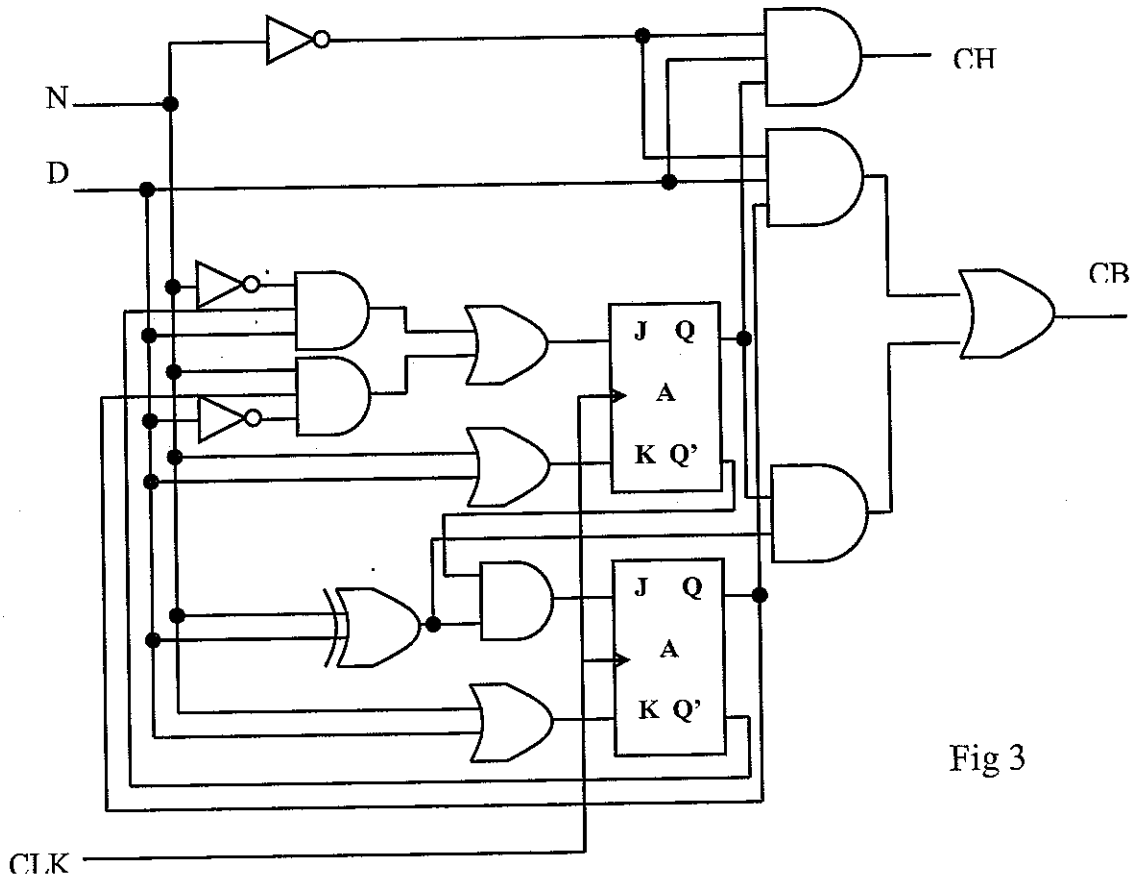

Fig 3

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P1 **Question 1:**

a) i) $F(A,B,C) = \bar{A} + AB + (\bar{A}+B)(A+\bar{B})$

$= \bar{A} + AB + \bar{A}\bar{B} + AB$

$= \bar{A}(1+\bar{B}) + AB$

$= \bar{A} + AB$

$= (\bar{A}+A)(\bar{A}+B)$

$F(A,B,C) = \bar{A} + B$ 2/2

ii) $F = BC + AC + \bar{A}\bar{B}C$

$= (A+\bar{A})BC + (B+\bar{B})AC + \bar{A}\bar{B}C$

$= ABC + \bar{A}BC + ABC + A\bar{B}C + \bar{A}\bar{B}C$

$= ABC + \bar{A}C(B+\bar{B}) + AC(B+\bar{B})$

$= ABC + \bar{A}C + AC$

$= AC(B+1) + \bar{A}C$

$= AC + \bar{A}C$

$= C(A+\bar{A})$

$F = C$ 2/2

b)

CD \ AB	00	01	11	10
00	1 0	4	X 12	8
01	X 1	5	1 13	9
11	X 3	1 7	1 15	11
10	1 2	1 6	1 14	1 10

$EPI_1 : \Sigma(0,1,2,3) = \bar{A}$

$EPI_2 : \Sigma(12,13,14,15) =$

$EPI_3 : \Sigma(2,6,10,14) =$

$PI_4 : \Sigma(6,7,14,15) = B$

$PI_5 : \Sigma(2,3,6,7) = \bar{A}$

$F = \bar{A}\bar{B} + AB + C\bar{D} + \begin{cases} BC \\ \bar{A}C \end{cases}$ 3/3

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c) $P = \bar{C} \odot (C + \overline{(\overline{A+B})})$

$Q = (C + \overline{(\overline{A+B})})\,\overline{(A\,(\overline{A+B}))}$

$P = \bar{C} \odot (A+B+C)$

$= \bar{C}(A+B+C) + C\,\overline{(A+B+C)}$

$= A\bar{C} + B\bar{C} + C(\bar{A}\bar{B}\bar{C})$

$P = A\bar{C} + B\bar{C}$

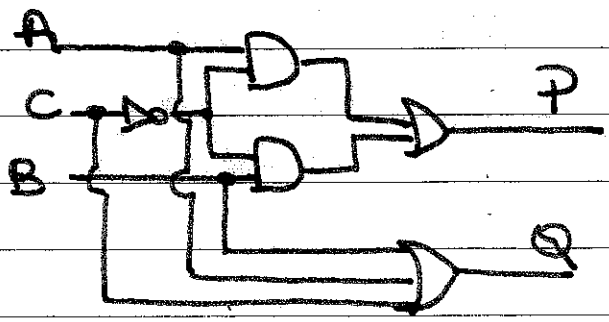
$Q = (C+A+B)\,\overline{(A\,(\bar{A}\bar{B}))}$

$= (C+A+B)(\bar{A} + (\overline{A\bar{B}}))$ 3/3

$= (A+B+C)(\bar{A}+A+B)$

$= (A+B+C)(1+B)$

$Q = A+B+C$



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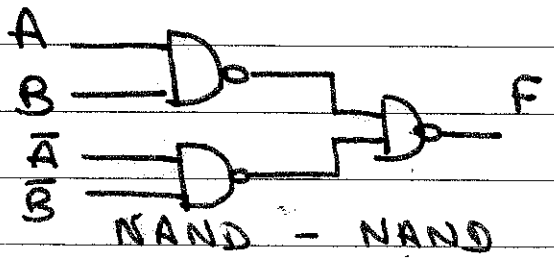
Question 2:

2.1) $F = AB + \bar{A}\bar{B}$

i) $F = \overline{\overline{AB + \bar{A}\bar{B}}}$

$F = \overline{(\overline{AB})(\overline{\bar{A}\bar{B}})}$



NAND - NAND

ii) $F = \overline{\overline{AB + \bar{A}\bar{B}}}$

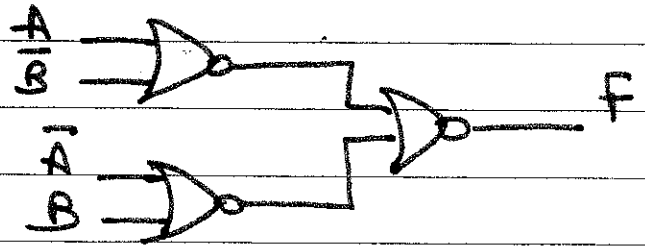
$= \overline{(\overline{AB})(\overline{\bar{A}\bar{B}})}$

$= \overline{(\bar{A}+\bar{B})(A+B)}$

$= \overline{\bar{A}B + A\bar{B}}$

$F = \overline{(\overline{A+\bar{B}}) + (\overline{\bar{A}+B})}$

NOR - NOR



iii) $F = \overline{(\overline{\bar{A}+\bar{B}}) + (\overline{A+B})}$

$= \overline{(\bar{A}+\bar{B})(A+B)}$

$F = \bar{A}B + A\bar{B}$

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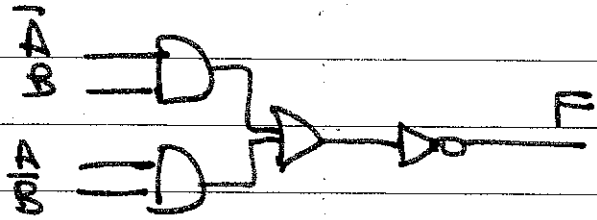
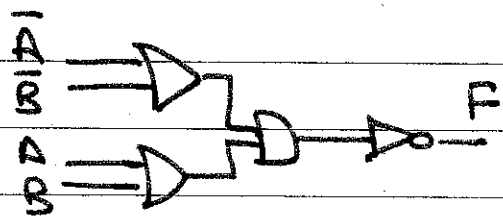
Q2

AND - OR - INV



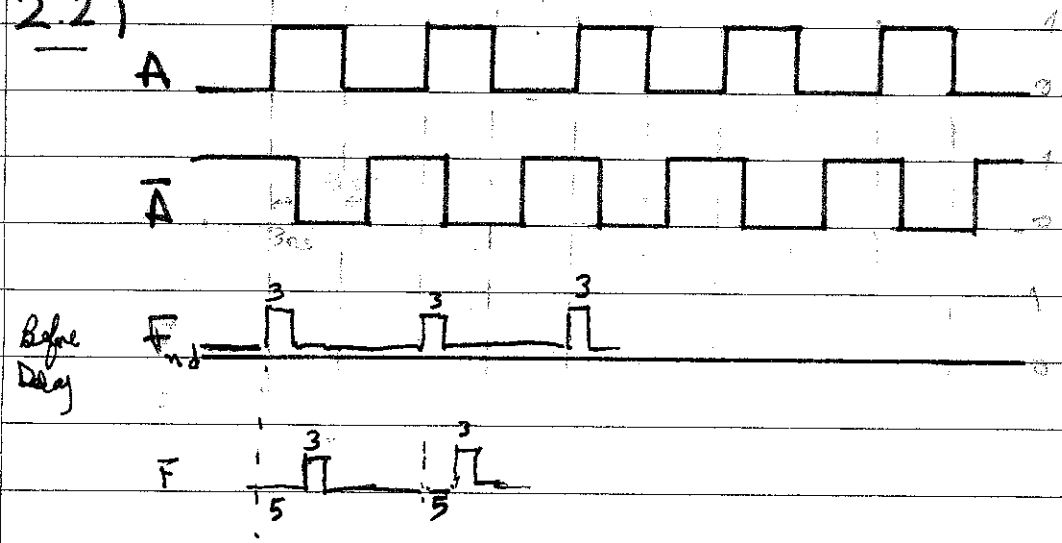
iiii) $F = (\overline{\bar{A}+\bar{B}}) + (\overline{A+B})$

$F = \overline{(\bar{A}+\bar{B})(A+B)}$



OR - AND - INV

2.2)



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Question 3:

a) $F = A^3$

A	F
$a_1 a_0$	$b_4 b_3 b_2 b_1 b_0$
0 0	0 0 0 0 0
0 1	0 0 0 0 1
1 0	0 1 0 0 0
1 1	1 1 0 1 1

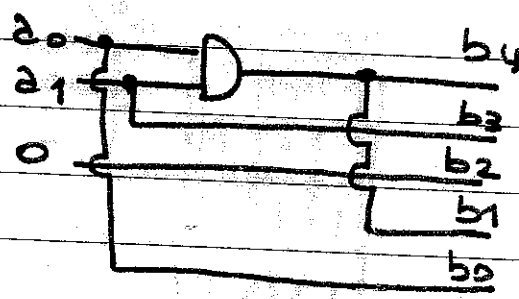
$b_4 = a_1 a_0$

$b_3 = a_1$

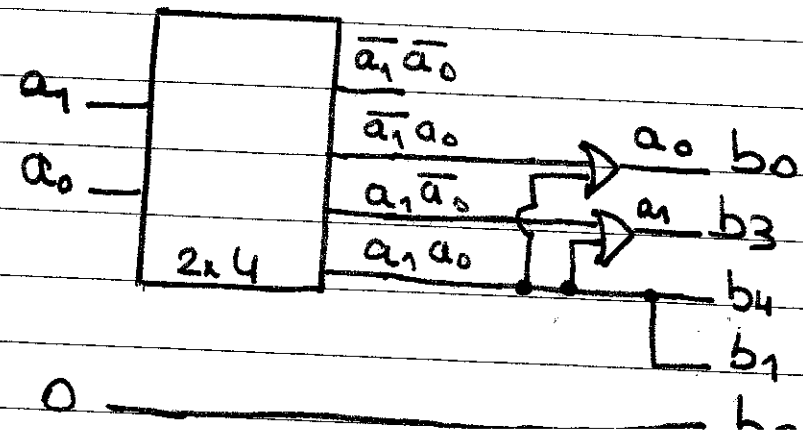
$b_2 = 0$

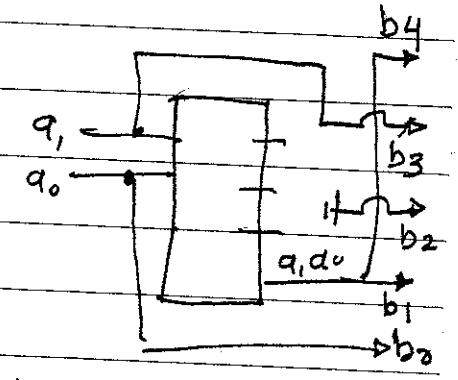
$b_1 = b_4 = a_1 a_0$

$b_0 = a_0$



b)





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Question 4

a) ROM : AND fixed OR fixed

PROM : AND fixed OR programmable

PAL : AND programmable OR fixed

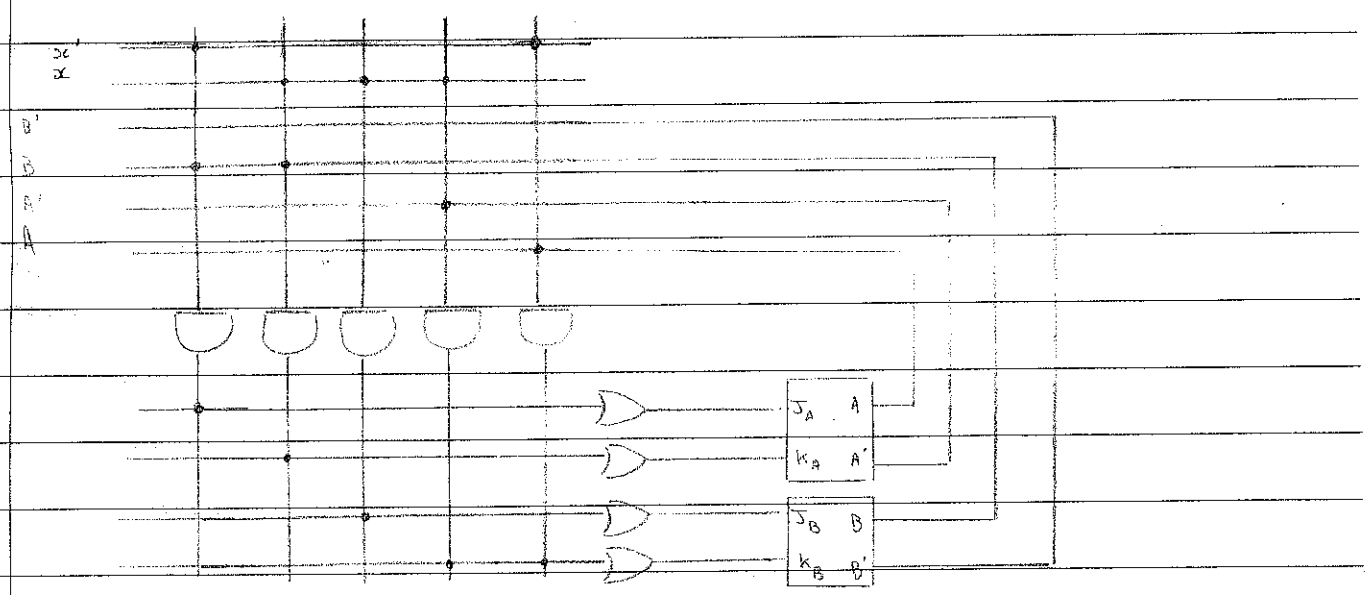
PLA : AND programmable OR programmable

b) $J_A = Bx'$ $\quad K_A = Bx$

$J_B = x$ $\quad K_B = A \oplus x = A'x + Ax'$

A	B	x
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

	AND A	B	x		OR J_A	J_B	K_A	K_B
1	-	1	1	Bx	-	-	1	-
2	0	-	1	$\bar{A}x$	-	-	-	1
3	-	1	0	$B\bar{x}$	1	-	-	-
4	-	-	1	x	-	1	-	-
5	1	-	0	$A\bar{x}$	1	-	-	1
					Bx	x	Bx	$A \oplus x$



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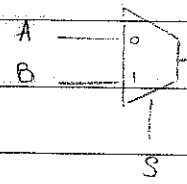
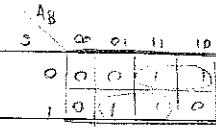
c) $F = A + B + ACD = B + A(1 + CD) = A + B$



A	B	S	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



$F = S'A + SB$

If $S = 0$ $F = A$

If $S = 1$ $F = B$

$F = A + B$

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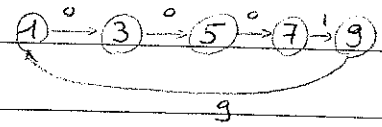
Question 5

1)



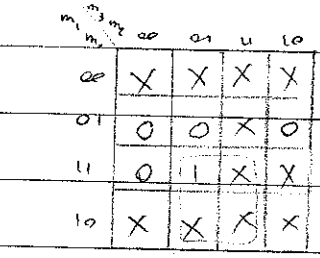
2)

	Present state m_3 m_2 m_1 m_0	Next state m_3^+ m_2^+ m_1^+ m_0^+	output
0,2,4,6,8	X X X X	X X X X	X
1	0 0 0 1	0 0 1 1	0
2	X X X X	X X X X	X
3	0 0 1 1	0 1 0 1	0
3	X X X X	X X X X	X
5	0 1 0 1	0 1 1 1	0
	X X X X	X X X X	X
9	0 1 1 1	1 0 0 1	0
	X X X X	X X X X	X
9	1 0 0 1	0 0 0 1	1

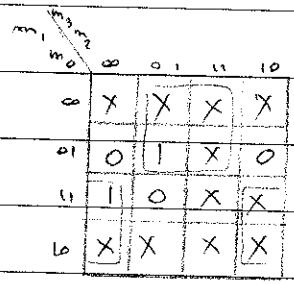
(representing even numbers & numbers ≥ 10)

others don't care

3)



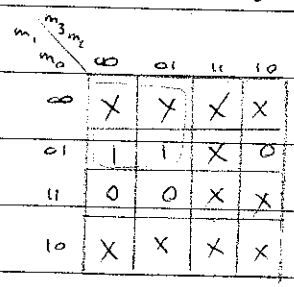
$m_3^+ = D_3$



m_2^+

For a D flip flop

$Q^+ = D$



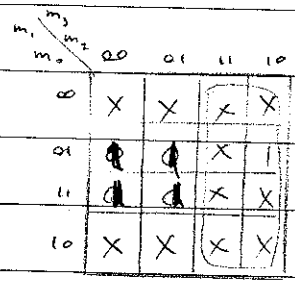
m_1^+

$m_3^+ = m_2 m_1 = D_3$

$m_2^+ = m_2 m_1' + m_2' m_1 = m_2 \oplus m_1 = D_2$

$m_1^+ = m_3' m_1'$

$m_0^+ = 1$



m_0

$x = m_3$

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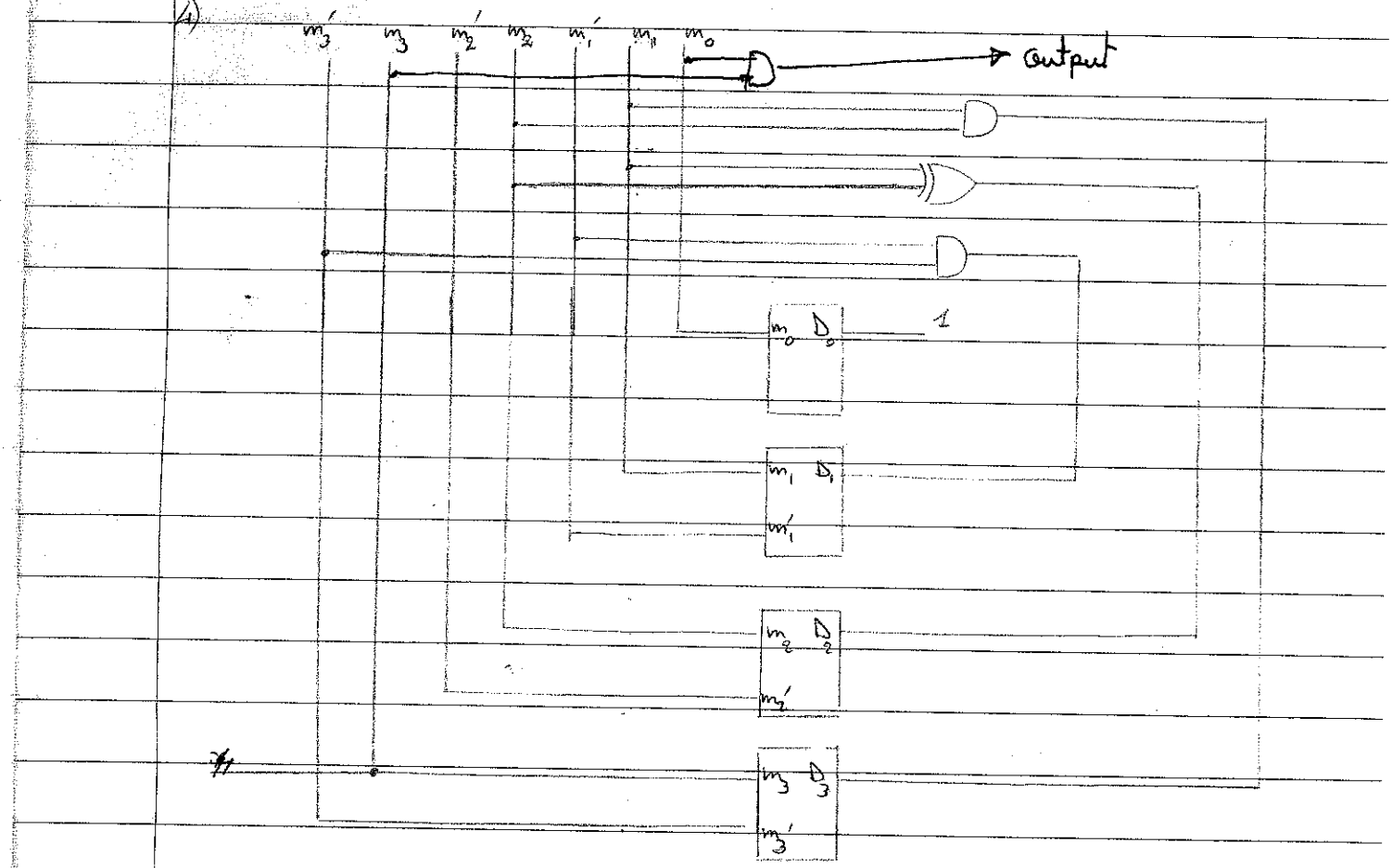

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Question 6:

$$J_{A1} = \overline{N}\,\overline{Q_{A2}}\,D + N\,Q_{A2}\,\overline{D}$$

$$K_{A1} = N + D \qquad \overline{K_A} = \overline{(N+D)} = \overline{N}\,\overline{D}$$

$$J_{A2} = \overline{Q_{A1}}\,(N \oplus D)$$

$$K_{A2} = N + D$$

$$Q^+ = J\overline{Q} + \overline{K}Q$$

$$Q_{A1}^+ = J_{A1}\,\overline{Q_{A1}} + \overline{K_{A1}}\,Q_{A1}$$

$$\underline{Q_{A1}^+ = \left(\overline{N}\,\overline{Q_{A2}}\,D + N\,Q_{A2}\,\overline{D}\right)\overline{Q_{A1}} + \left(\overline{N}\,\overline{D}\right)Q_{A1}}$$

$$Q_{A2}^+ = J_{A2}\,\overline{Q_{A2}} + \overline{K_{A2}}\,Q_{A2}$$

$$\underline{Q_{A2}^+ = \left(\overline{Q_{A1}}\,(N \oplus D)\right)\overline{Q_{A2}} + \left(\overline{N}\,\overline{D}\right)Q_{A2}}$$

$$\underline{CH = Q_{A1}^+\,D\,\overline{N}}$$

$$\underline{CB = \overline{N}\,D\,Q_{A2}^+ + Q_{A1}^+\,(N \oplus D)}$$

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Inputs		Present State		Next State		Outputs	
N	D	Q_{A_1}	Q_{A_2}	$Q_{A_1}^+$	$Q_{A_2}^+$	CH	CB
0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0
0	0	1	0	1	0	0	0
0	0	1	1	1	1	0	0
0	1	0	0	1	1	1	1
0	1	0	1	0	0	0	0
0	1	1	0	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	1	0	1
1	0	0	1	1	0	0	1
1	0	1	0	0	0	0	0
1	0	1	1	0	0	0	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	1	1	0	0	0	0