Digital Design COEN 312 ***Sample A*** **Midterm exam, Feb. 15th, 2006**
Answer all questions. Time allowed: 1hr **Lecturer: A.J. Al-Khalili**

Question 1--- ***Use Boolean algebra for Question 1.***

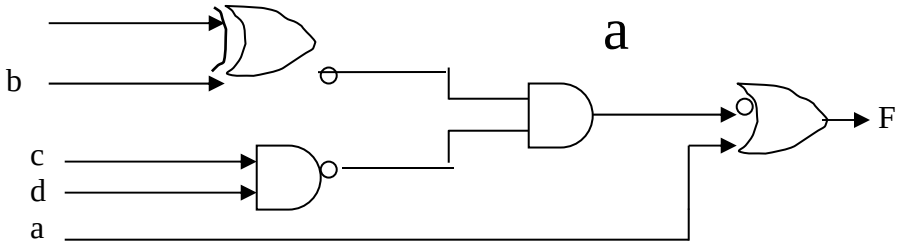
1.a Simplify to obtain minimum SOP (2 marks)

F(W,X,Y,Z) = [(X+Y') ⊕ W][(XY Θ (W+Y)]

1.b Simplify to obtain minimum SOP (3marks)

F(a,b,c,d)= a'b'(c+d')(c'+d) + ab(c'd+cd') + (a'b+1)cd

1.c Minimize the following circuit, draw final minimized circuit with minimum number of packages. You have single rails available to you. (3 marks)



Question 2---

2.a Give **minimal SOP** for F(a,b,c,d) given by the following K-map (3 marks)
Identify the Prime Implicants clearly. Which ones are essential? (1 mark)

2.b Give minimal POS for F(a,b,c,d). (2 marks)

a b / cd	00	01	11	10
00	1	X	X	X
01		1	1	
11	1			1
10	X	X	X	X

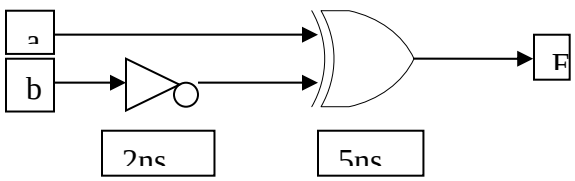
2.c Give the **POS** of F(A,B,C,D)= (A+BCD) (B+CD) (2 marks)

2.d Give the **minterm list** of F(A,B,C,D)=AB (1 marks)

2.e Give the **NOR-NOR** implementation of F(A,B,C,D)= ABC+CD+CB (3 marks)

Question 3

Draw the **timing diagram** for F(a,b) = b' ⊕ a for the following consecutive inputs:
ab= 00, 10, 01, 11 (**follow these vectors in order given. Start from AB=00 **). Assume the following gate delays, INV.= 2ns , XOR gate = 5ns (5 marks)



SOLUTION Midterm exam, Feb.15th, 2006

Question 1

a)

$$F(w,x,y,z)=[(x+\bar{y})\oplus w]*[(xy)\square\ (w+x)]=[\overline{(x+\bar{y})}w+\bar{w}(x+\bar{y})]*[xy(w+y)+\bar{xy}\,\overline{(w+y)}]=$$

$$=[\bar{x}yw+\bar{w}x+\bar{w}\bar{y}]*[\cancel{wxy}+xy+(\bar{x}+\bar{y})\bar{w}\bar{y}]=[\bar{x}yw+\bar{w}x+\bar{w}\bar{y}]*[xy+\cancel{\bar{x}\bar{w}y}+\bar{w}\bar{y}]=$$

$$=\underbrace{\bar{x}ywxy}_{0}+\underbrace{\bar{x}yw\bar{w}\bar{y}}_{0}+\underbrace{\bar{w}xxy}_{\bar{w}xy}+\underbrace{\bar{w}x\bar{w}\bar{y}}_{\bar{w}x\bar{y}}+\underbrace{\bar{w}\bar{y}xy}_{0}+\underbrace{\bar{w}y\bar{w}\bar{y}}_{\bar{w}\bar{y}}=\bar{w}x(y+\bar{y})+\bar{w}\bar{y}=\bar{w}x+\bar{w}\bar{y}$$

b)

$$F(a,b,c,d)=\bar{a}\bar{b}(c+\bar{d})(\bar{c}+d)+ab(\bar{c}d+c\bar{d})+(\bar{a}\bar{b}+1)cd=$$

$$=(\bar{a}\bar{b}c+\bar{a}\bar{b}\bar{d})(\bar{c}+d)+ab\bar{c}d+abc\bar{d}+cd=$$

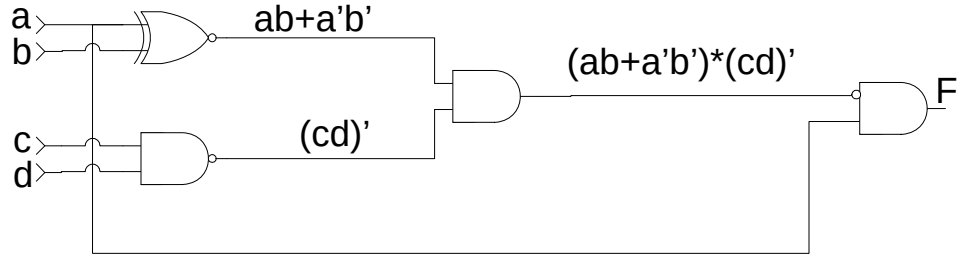
$$=\underbrace{\bar{a}\bar{b}c\bar{c}}_{0}+\cancel{\bar{a}\bar{b}cd}+\bar{a}\bar{b}cd+\underbrace{\bar{a}\bar{b}\bar{d}d}_{0}+ab\bar{c}d+abc\bar{d}+cd=$$

$$=\bar{a}\bar{b}cd+ab\bar{c}d+abc\bar{d}+cd=\bar{a}\bar{b}cd+ab\cancel{\bar{d}}+c(ab\cancel{\bar{d}}+d)=$$

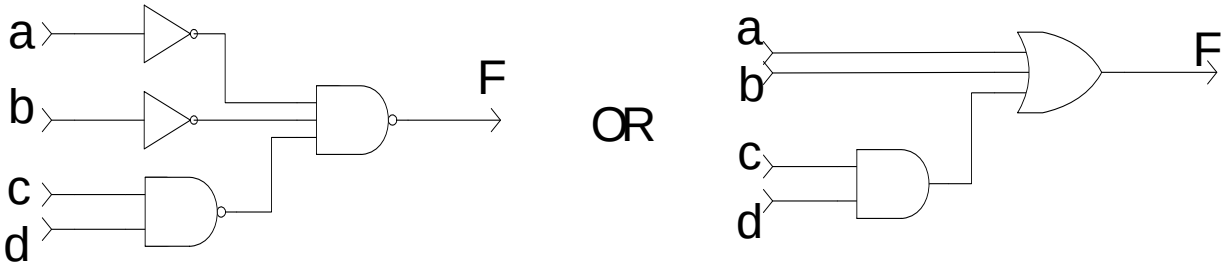
$$=\bar{a}\bar{b}cd+ab\bar{c}d+abc+cd=\bar{a}\bar{b}cd+ab(\cancel{\bar{c}}d+c)+cd=$$

$$=\bar{a}\bar{b}cd+abd+abc+cd$$

c)

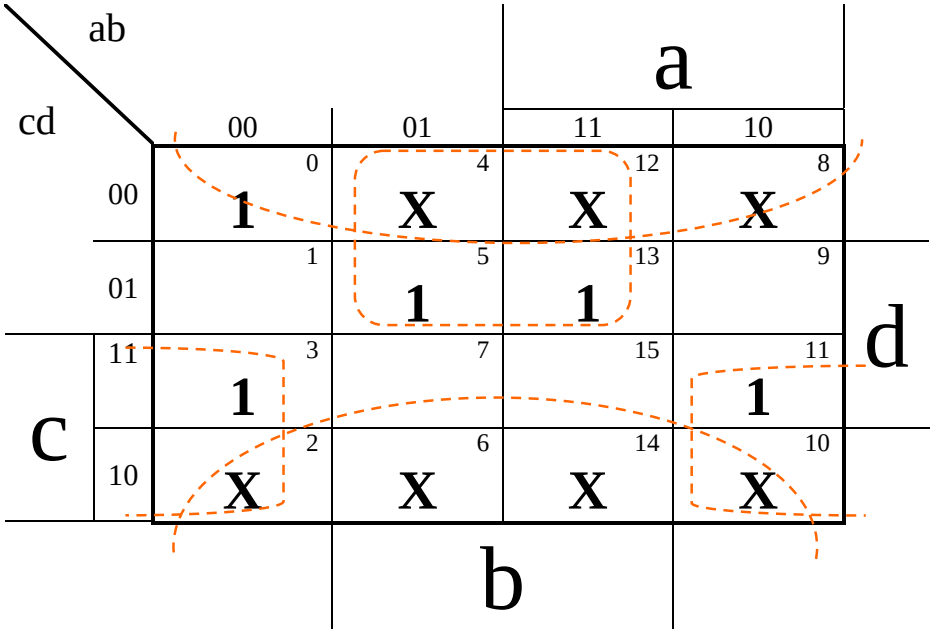


$$F=\overline{(ab+\bar{a}\bar{b})\bar{c}\bar{d}}+a=(\bar{a}b+a\bar{b})+\bar{c}\bar{d}+a=\bar{a}b+\cancel{a\bar{b}}+cd+a=\bar{a}b+cd+a=a+b+cd$$



Question 2

a)



$PI_1 = \bar{d} = \sum(0,2,4,6,8,10,12,14)$

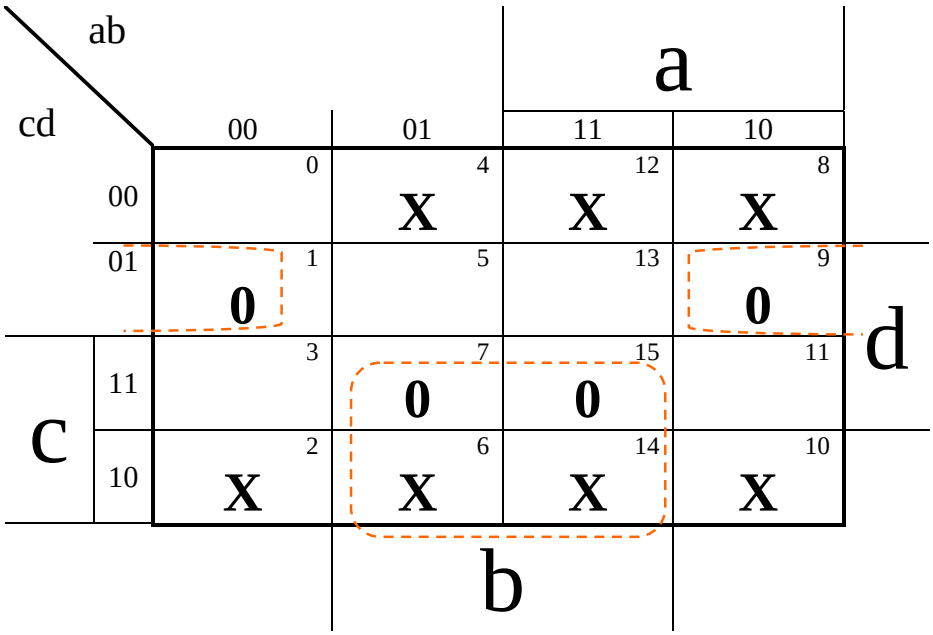
$PI_2 = b\bar{c} = \sum(4,5,12,13)$

$PI_3 = \bar{b}c = \sum(2,3,10,11)$

All three PI are essential.

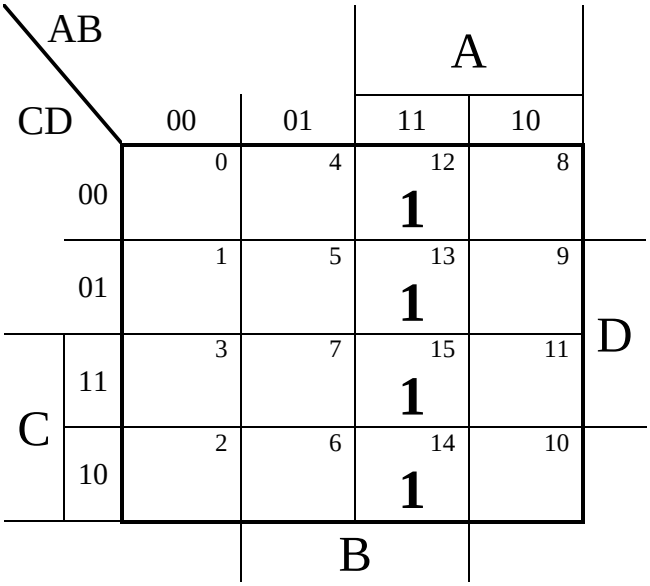
$F = \bar{d} + b\bar{c} + \bar{b}c$

b)



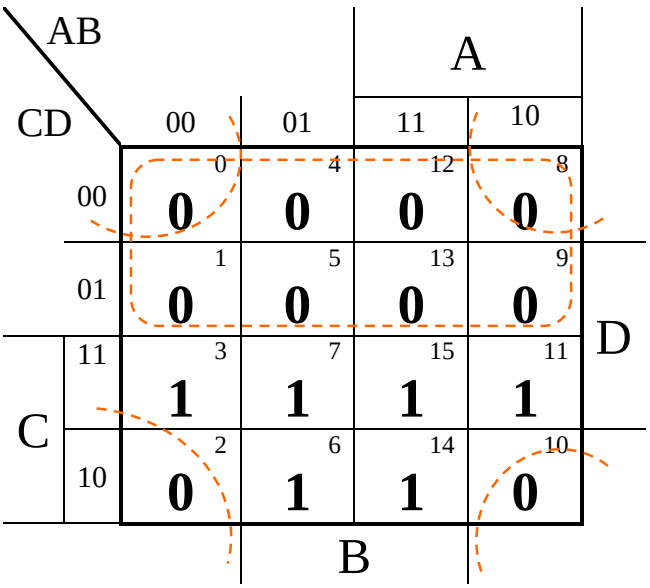
$F = (b + c + \bar{d})(\bar{b} + \bar{c})$

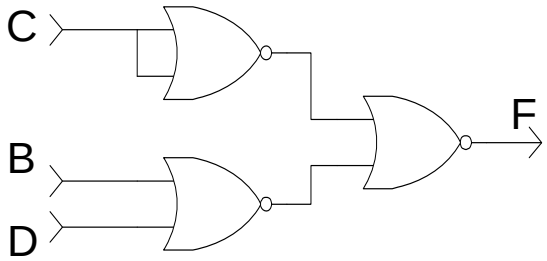
c) Applying distributive law:
F(A,B,C,D)=(A+BCD)(B+CD)=(A+B)(A+C)(A+D)(B+C)(B+D)
d) minterms of F(A,B,C,D)=AB:



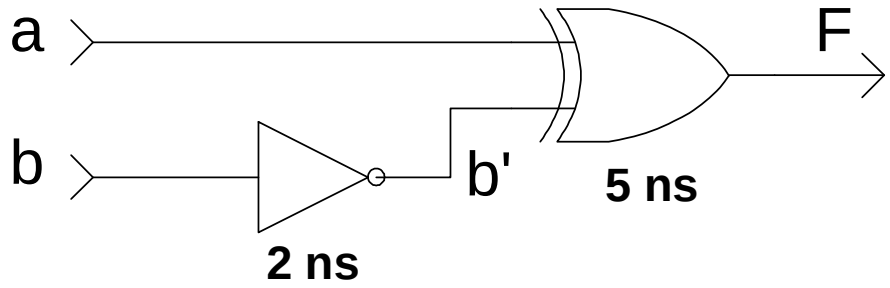
$F = \sum(12,13,14,15)$

e) NOR-NOR of F(A,B,C,D)=ABC+CD+CB=C(AB+D+B)=C(B+D)= $\overline{\overline{C} + \overline{(B+D)}}$

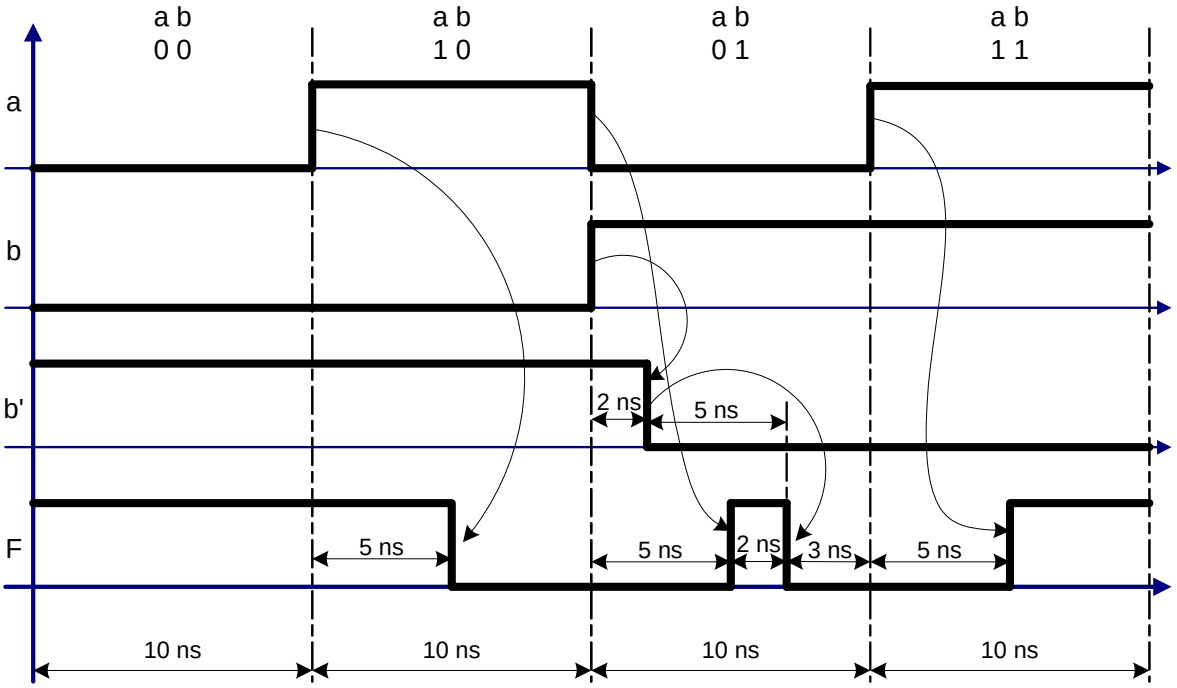




Question 3



Total delay is 7 ns => take period of 10 ns.



Edges that cause change in the output are shown with an arrow after delay of going through the gate.