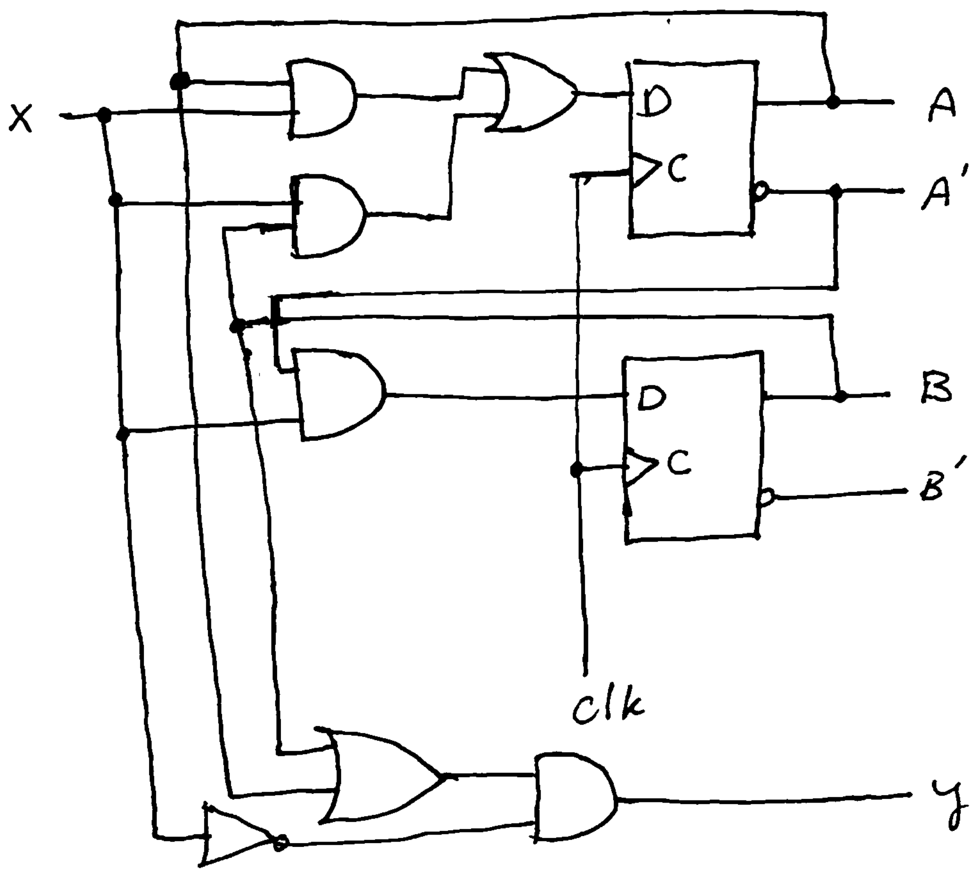
Lecture 15, March 7, 2007

Analysis of the sequential circuits

The analysis means to describe how a circuit works. That is, how its outputs and its state changes as a function of its inputs and its present state.

A <u>state table</u> is usually used to relate the inputs and the present state of a sequential circuit to its outputs and its next state.

<u>Example</u>: Consider the following sequential circuit:



State Equations for this circuit are

$$A(t+1) = A(t)\,x(t) + B(t)\,x(t)$$

and

$$B(t+1) = A'(t)\,x(t)$$

and the output equation is

$$y(t) = [A(t) + B(t)]\,x'(t)$$

The state-table or state-transition table will have 8 entries, since there are 2 flip-flops resulting in $2^2 = 4$ states and one input resulting in two possibilities:

Present State		Input	Next State		Output
$A(t)$	$B(t)$	$x(t)$	$A(t+1)$	$B(t+1)$	$y(t)$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Another format for deriving state-tables is to enumerate the state possibilities, thus having less rows in the table and list the outputs for different inputs in different columns:

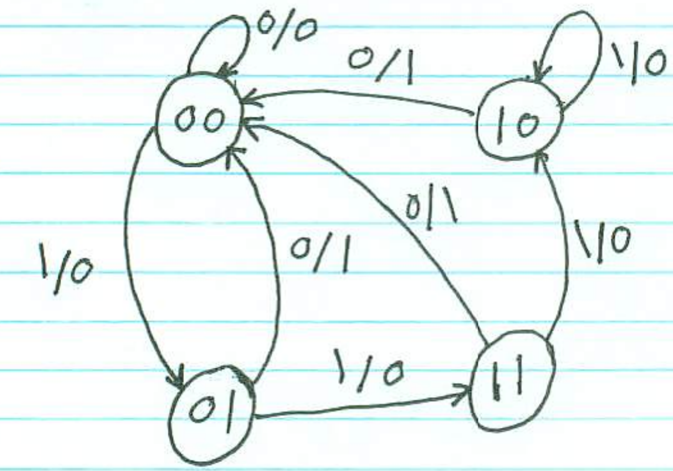
Present State	Next State		Output	
	$x=0$	$x=1$	$x=0$	$x=1$
AB	AB	AB	y	y
0 0	0 0	0 1	0	0
0 1	0 0	1 1	1	0
1 0	0 0	1 0	1	0
1 1	0 0	1 0	1	0

State Diagram

Another way to depict the operation of a sequential circuit is to use a state diagram (or a bubble diagram).

In a state diagram each state, i.e., each combination of values of the flip-flops, is represented by a circle (a bubble). Transition between states is shown by directed cords (arrows) connecting the bubbles.

As an example, consider the sequential circuit we discussed above. It has two flip-flops, so, it can be in one of the four states 00, 01, 10, 11. We index the circles (the states) with these numbers. Since there is only one input, there are two arrows leaving each state. We mark these arrows with in/out where <u>in</u> before / is the input and <u>out</u> after / is the output.



<u>output equations</u> and <u>flip-flop input equations</u>

Any sequential circuit consists of several gates and several flip-flops. The logic gates form the combinational part of the circuit that form either the outputs (as a function of the inputs and the outputs of the flip-flops) or form the next state of the circuit, i.e., the inputs to the flip-flops.

The first set of equations are called the <u>output equation</u> while the second set are called <u>state</u> <u>equations</u> or <u>flip-flop input</u> equations.

As an example, in the sequential circuit that we discussed, we have two D flip-flops. so, we have two <u>flip-flop</u> <u>input equations</u>

$$D_A = Ax + Bx$$

and

$$D_B = A'x$$

and one <u>output equation</u>

$$y = (A+B)x'.$$

Here, we have dropped the time, (t), in order to simplify the presentation.

Now, let's start talking about analysis using the definitions we presented in the previous paragraphs.

Analysis with D-flip-flops

Take the example of a sequential circuit with one D flip-flop with the FF input equation

$$D_A = A \oplus x \oplus y$$

and output equation

$$z = (x + y) A$$

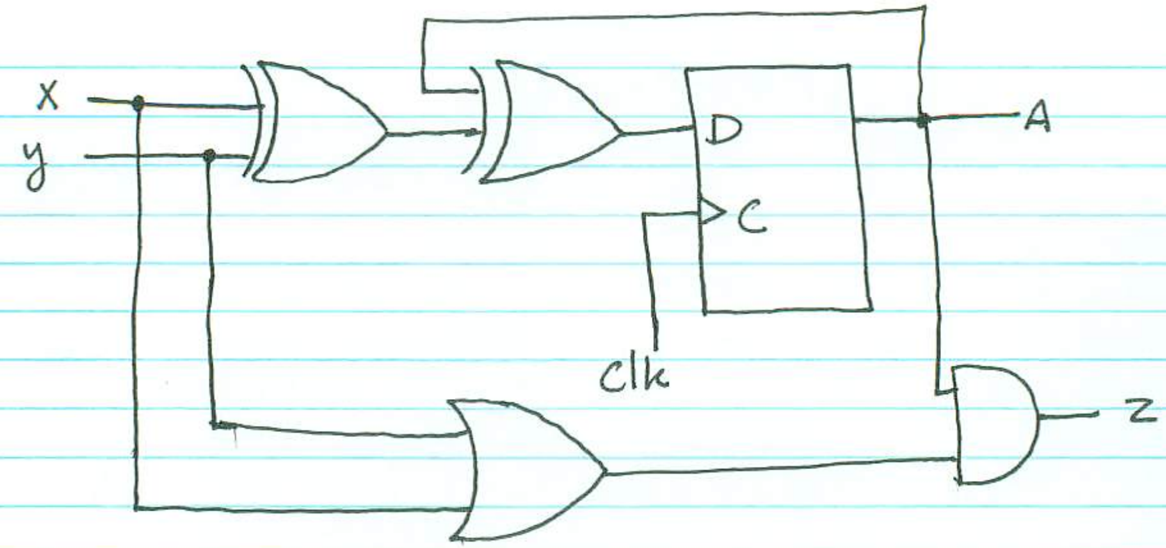
Note that the FF input equation is the same as the state equation

$$A(t+1) = A(t) \oplus x(t) \oplus y(t).$$

and the output equation, including the time is:

$$z(t) = (x(t) + y(t)) A(t).$$

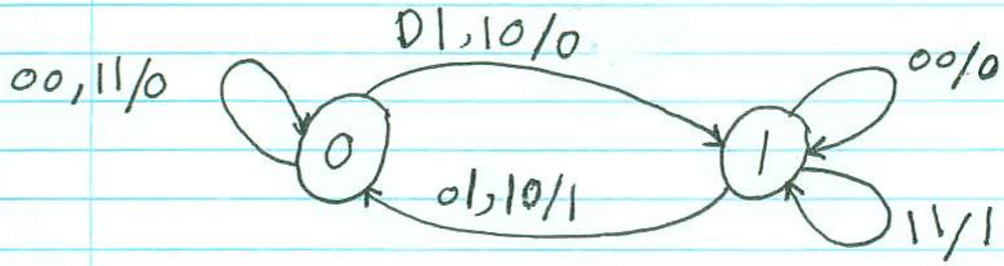
The state-table for this circuit is given on the next page, together with the circuit diagram (the schematic)



Present State	Inputs		Next State	output
$A(t)$	$x(t)$	$y(t)$	$A(t+1)$	$Z(t)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	0
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

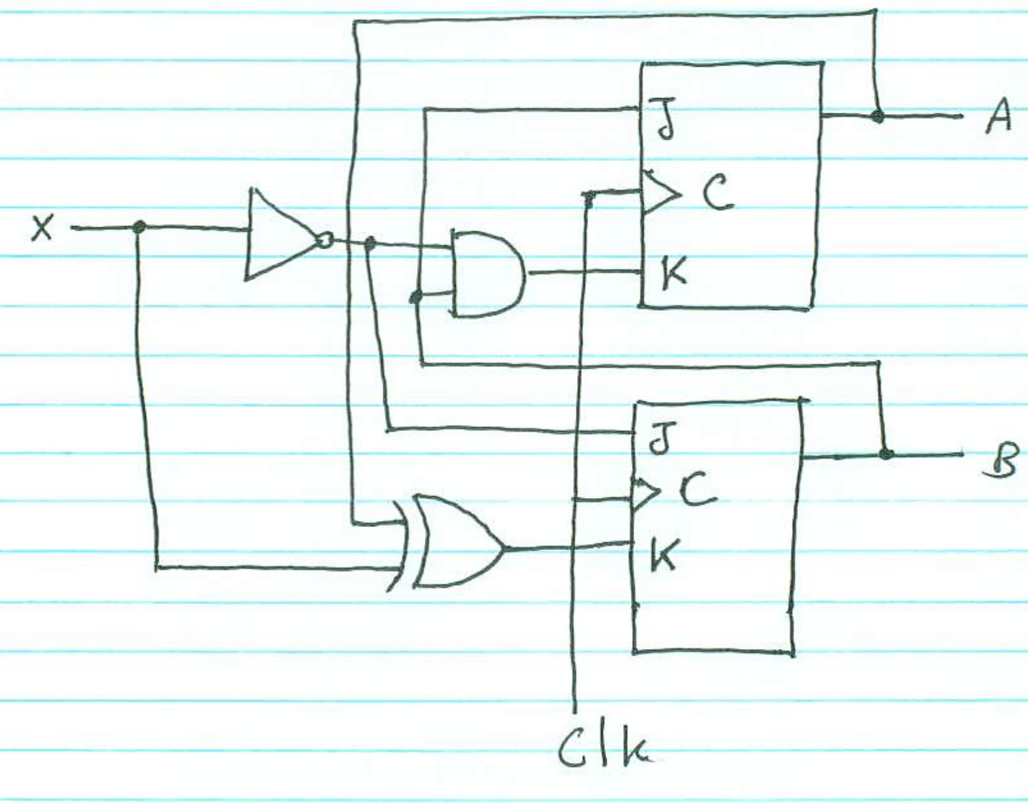
Since, the circuit has only one FF, it has two states. Since there are two inputs, there are four arrows leaving each state.

However, in this special case, the arrows corresponding to inputs 00 and 11 often coincide and so do those for 01 and 10. So, we only show two transitions. Each of these two arrows are the combination of two overlapping arrows.



Analysis with JK - flip-flops

Take the following example



1) Write the FF input equations

$J_A = B$, $\quad K_A = Bx'$

$J_B = x'$, $\quad K_B = A \oplus x = A'x + Ax'$

2) Find the binary values of each input equation

3) Use the characteristic table or characteristic equation of the JK FF, i.e.,

$Q(t+1) = JQ' + K'Q$ to find the next state.

Using the above procedure, we derive the following state transition table.

Current State		Input	FF inputs				Next State	
$A(t)$	$B(t)$	$x(t)$	J_A	K_A	J_B	K_B	$A(t+1)$	$B(t+1)$
0	0	0	0	0	1	0	0	1
0	0	1	0	0	0	1	0	0
0	1	0	1	1	1	0	1	1
0	1	1	1	0	0	1	1	0
1	0	0	0	0	1	1	1	1
1	0	1	0	0	0	0	1	0
1	1	0	1	1	1	1	0	0
1	1	1	1	0	0	0	1	1

where, we have used

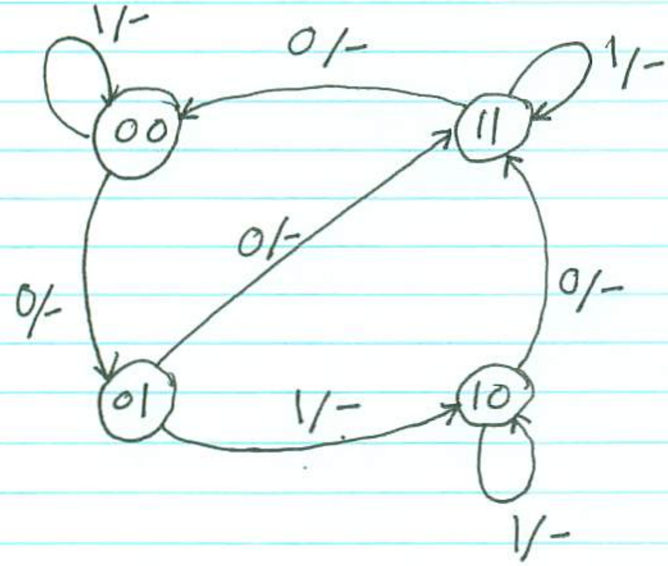
$$A(t+1) = J_A A' + K_A' A$$

and

$$B(t+1) = J_B B' + K_B' B$$

to get $A(t+1)$ and $B(t+1)$ from J_A, K_A, J_B, K_B.

The state diagram for this circuit is:



The sign - after / signifies the fact that this circuit does not produce any output.

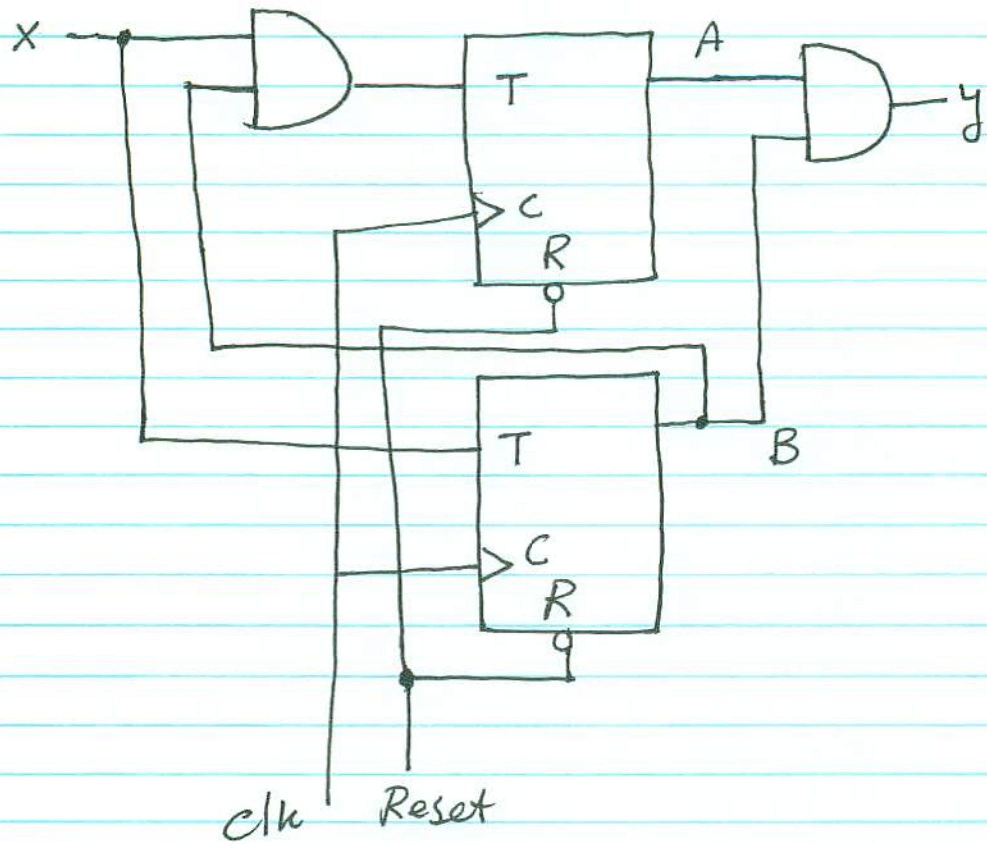
Analysis with T flip-flops

Everything is the same as the case of JK flip-flops, except that:

- You need to generate one FF input equation per flip-flop, and:
- You use the characteristic equation

$$Q(t+1) = T \oplus Q.$$

Example: Consider the circuit with two T-flip-flops shown on the next page.



1) $T_A = Bx$

$T_B = x$

$y = A + B$

2) Find T_A, T_B, for different combinations of $A(t)$, $B(t)$, $x(t)$.

3) $A(t+1) = T_A \oplus A = T_A' A + T_A A'$

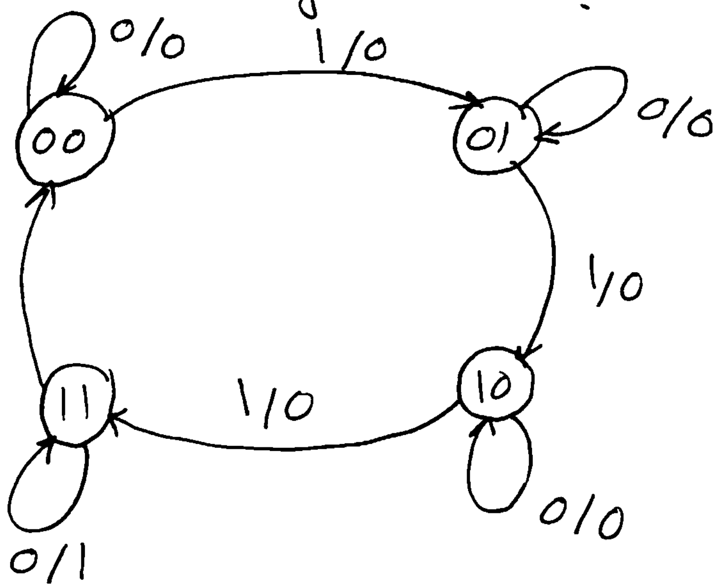
$$= (Bx)'A + (Bx)A' = AB' + Ax' + A'Bx$$

$B(t+1) = x \oplus B$

The state transition table is shown on the next page.

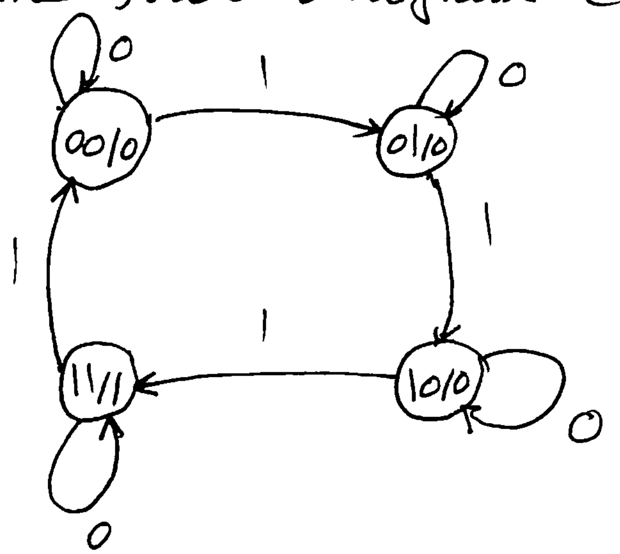
Present State		Input	Next State		output
A(t)	B(t)	x(t)	A(t+1)	B(t+1)	y(t)
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

The state diagram is :



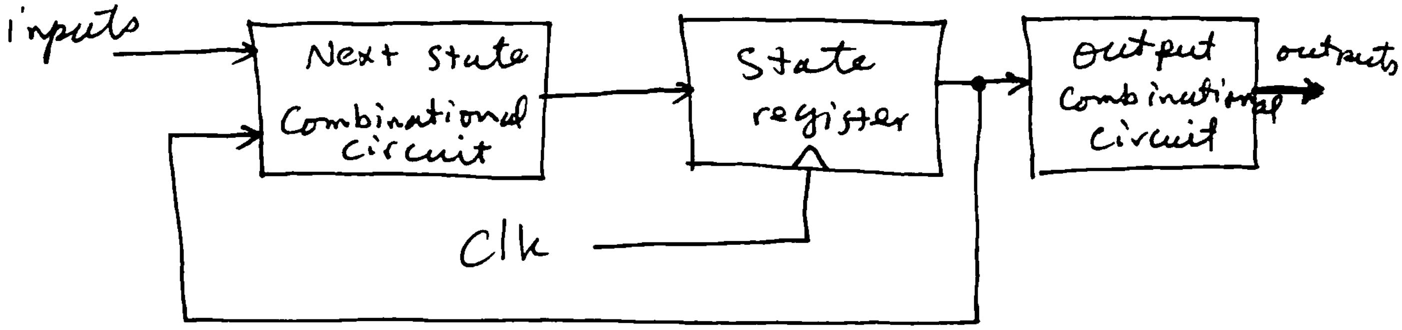
Note that in this state diagram, the output to an input is the same whether this input is 1 or 0. It only depends on the state where the FSM has been

at the time the input was applied.
Any input value (0 or 1) occuring in states 00, 01 and 10 results in the output value of 0. And in state 11 whether the input is 0 or 1 the output is 1.
In circuits like this, called as <u>Moore Machine</u>, we can put the outputs in the circles with the state index (instead of putting them on the arrows).
So, the state diagram can be like this:

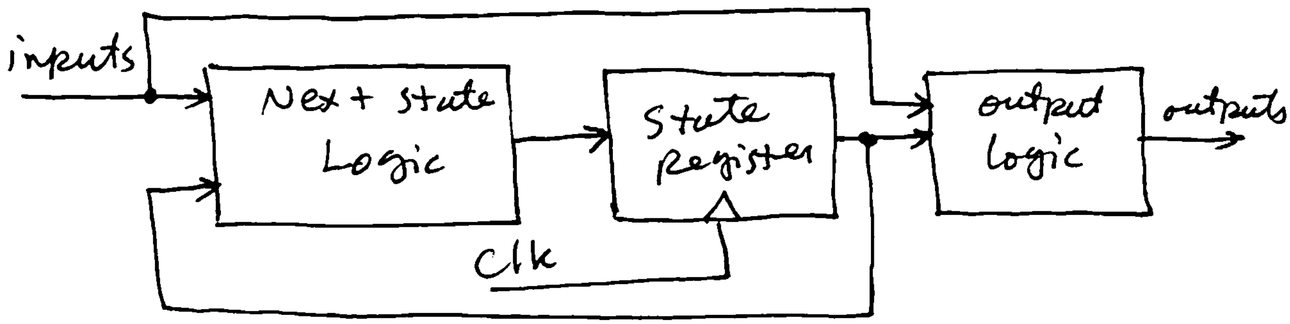


The reason for this can be observed from the circuit diagram. In the circuit, we observe that the output is only connected to the states (the outputs of flip-flops) and the input is not directly connected to the output.

A Moore Machine or a Moore FSM, in general, looks like this:



Another type of sequential circuit is a Mealy Machine. In a Mealy Machine, the output is formed by combining the present state and the present input:



A Mealy FSM