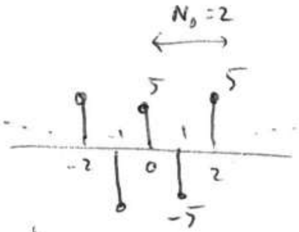
1.6

(i) $x_1[k] = 5\times(-1)^k$



Periodic ✓

$$x_1[k+2] = 5.(-1)^{k+2} = 5\times(-1)^k\cdot(-1)^2 = 5(-1)^k = x_1[k] \qquad N_0 = 2$$

(iii) $$x_3[k] = \underbrace{e^{j\frac{7\pi}{4}k}}_{x_{31}[k]} + \underbrace{e^{j\frac{3\pi}{4}k}}_{x_{32}[k]}$$

$N_{31} = \frac{2\pi}{7\pi/4}m_1 = \frac{8}{7}m_1 \underset{\overset{\uparrow}{m=7}}{=} 8 \in \mathbb{N}$: fundamental Period of $x_{31}[k]$

$N_{32} = \frac{2\pi}{3\pi/4}m_2 = \frac{8}{3}m_2 \underset{m_2=3}{=} 8 \in \mathbb{N}$: Fundumental Period of $x_{32}[k]$

$N_0 = L.C.M(N_{31}, N_{32}) = 8$ fundumental period of summation of two periodic signals is the least common mulltiple of the periods.

(iv) $$x_4[k] = \sin\left(\frac{3\pi k}{8}\right) + \cos\left(\frac{63\pi k}{64}\right)$$

$N_{41} = \frac{2\pi}{3\pi/8}m_1 = \frac{16}{3}m_1 = 16$

$N_{42} = \frac{2\pi}{63\pi/64}m_2 = \frac{128}{63}m_2 = 128$

$N_0 = LCM(16, 128) = 128$

1.8 Power or Energy signal

(i) $x_1[k] = \cos\left(\frac{\pi}{4}k\right)\sin\left(\frac{3\pi}{8}k\right)$

$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

$$= \frac{1}{2}\left[\sin\left(\frac{5\pi}{8}k\right)+\sin\left(\frac{\pi}{8}k\right)\right]$$

it's a periodic signal and it is a power signal

$N_1 = \frac{2\pi}{5\pi/8} m_1 = 16 \quad N_2 = \frac{2\pi}{\pi/8} m_2 = 16 \quad \rightarrow \boxed{N_0 = 16}$

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha-\beta)\right]$

$$|x_1[k]|^2 = \frac{1}{4}\left[\sin^2\left(\frac{5\pi}{8}k\right)+\sin^2\left(\frac{\pi}{8}k\right)+2\sin\left(\frac{5\pi}{8}k\right)\sin\left(\frac{\pi}{8}k\right)\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}-\frac{1}{2}\cos\frac{10\pi}{8}k+\frac{1}{2}-\frac{1}{2}\cos\frac{2\pi}{8}k+\cos\frac{4\pi}{8}k-\cos\frac{6\pi}{8}k\right]$$

$$\frac{1}{16}\sum_{k=0}^{15}|x_1[k]|^2 = \frac{1}{16}\sum_{k=0}^{15}\frac{1}{4}+\frac{1}{16}\left(\sum_{k=0}^{15}-\frac{1}{2}\cos\frac{10\pi}{8}k+\sum_{k=0}^{15}-\frac{1}{2}\cos\frac{2\pi}{8}k+\sum_{k=0}^{15}\cos\frac{4\pi}{8}k-\sum_{k=0}^{15}\cos\frac{6\pi}{8}k\right)$$

0 0 0 0

$$\Rightarrow \quad P_{x_1} = \frac{1}{4} \quad , \quad E_{x_1} = \infty$$

1.8 (ii)

$$x_2[k] = \begin{cases}\cos\left(\frac{3\pi k}{16}\right) & -10 \leq k \leq 0 \\ 0 & \text{o.w}\end{cases}$$

time limited signal and amplitude is limited as well. So the signal is an energy signal.

$E =$

$$\sum_{k=-\infty}^{\infty}|x_2[k]|^2 = \sum_{k=-10}^{0}\cos^2\left(\frac{3\pi}{16}k\right) = \sum_{k=-10}^{0}\frac{\left(e^{j\frac{3\pi}{16}k}+e^{-j\frac{3\pi}{16}k}\right)^2}{4}$$

$$= \frac{1}{4}\sum_{k=-10}^{0}\left[e^{j\frac{3\pi}{8}k}+e^{-j\frac{3\pi}{8}k}+2\right] \underset{\substack{\uparrow\\ \text{Geometric}\\ \text{series}}}{=} \frac{1}{4}\left[\frac{e^{j\frac{3\pi}{8}}-e^{-j\frac{3\pi}{8}10}}{e^{j\frac{3\pi}{8}}-1}+\frac{e^{-j\frac{3\pi}{8}}-e^{j\frac{3\pi}{8}10}}{e^{-j\frac{3\pi}{8}}-1}+22\right]$$

$$= \frac{1}{4}\left[\frac{e^{j\frac{3\pi}{8}}-e^{-j\frac{3\pi}{8}10}}{e^{j\frac{3\pi}{8}}-1}+\frac{1-e^{+j\frac{3\pi}{8}11}}{1-e^{j\frac{3\pi}{8}}}\right]+5.5 = \frac{\exp(j3\pi/8) - \exp(j\pi/4) -1 + \exp(j\pi/8)}{4[\exp(j3\pi/8)-1]}+5.5 = 5.6622$$

1.8 (iii) $x_3[k] = (-1)^k$: periodic signal $N_0 = 2$

$$P_{x_3} = \frac{1}{2}\sum_{k=0}^{1} |x_3[k]|^2 = \frac{1}{2}(1+1) = 1$$

1.15

(i) $x_1[k] = \sin(4k) + \cos(2\pi k/3)$

$x_1[-k] = -\sin(4k) + \cos(2\pi k/3) \neq x_1[k]$ and $x_1[-k] \neq -x[k]$

It is neither even nor odd

we know that $\sin(4k)$ is odd an $\cos(2\pi k/3)$ is an even signal so

$x_{1,even}[k] = \cos\left(\frac{2\pi k}{3}\right)$ $x_{1,odd}[k] = \sin(4k)$

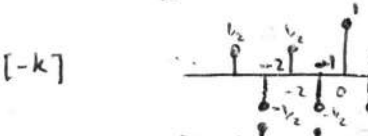
(ii) $x_2[k] = \sin\left(\frac{\pi}{3000}k\right) + \cos\left(\frac{2\pi}{3}k\right)$

$x_2[-k] = -\sin\left(\frac{\pi}{3000}k\right) + \cos\left(\frac{2\pi}{3}k\right)$

it is neither odd nor even.

$x_{2,odd}[k] = \sin\left(\frac{\pi}{3000}k\right)$, $x_{2,even}[k] = \cos\left(\frac{2\pi}{3}k\right)$

(v) $x_5[k] = \begin{cases} (-1)^k & k \geq 0 \\ 0 & k < 0 \end{cases}$

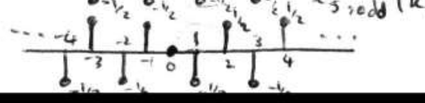


$\Rightarrow x_5[k] = (-1)^k U[k]$

it is neither odd nor even

$$x_{5,even}[k] = \frac{x_5[k]+x_5[-k]}{2} = \frac{(-1)^k U[k] + (-1)^{-k} U[-k]}{2} = \frac{1}{2}(-1)^k U[k] + \frac{1}{2}(-1)^k U[-k]$$

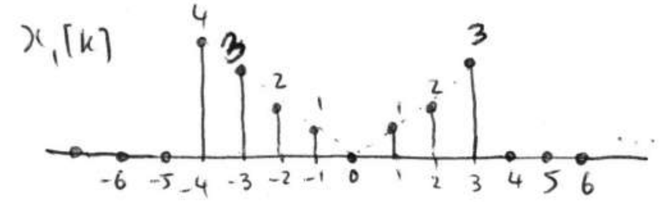
$$x_{5,odd}[k] = \frac{x_5[k]-x[-k]}{2} = \frac{1}{2}(-1)^k U[k] - \frac{1}{2}(-1)^k U[-k]$$



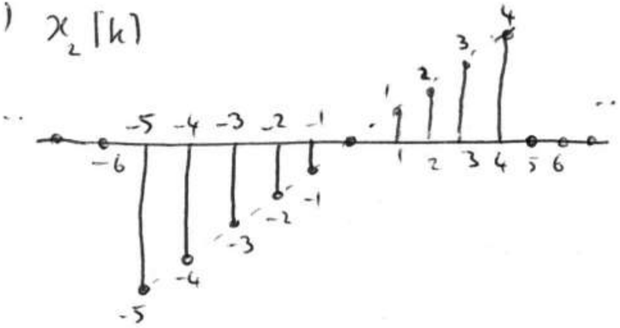
1.28

$x_1[k] = |k|\left(u[k+4] - u[k-4]\right)$, $x_2[k] = k\left(u[k+5] - u[k-5]\right)$

(i)

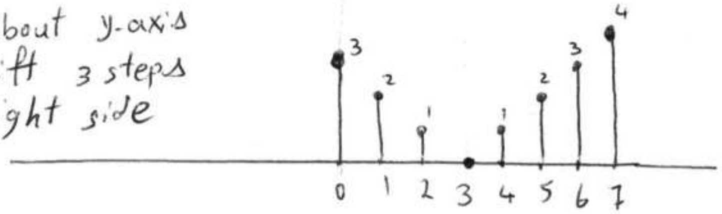


(ii)



(iii) $x_1[3-k]$

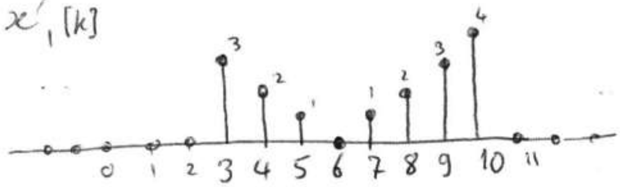
flip about y-axis
and sift 3 steps
to right side

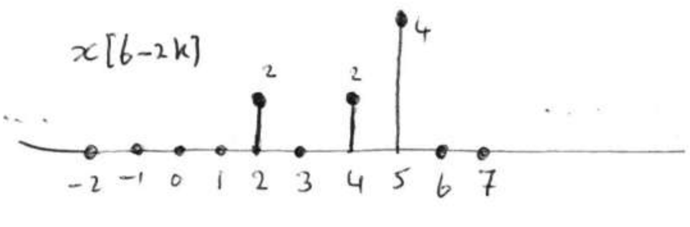


(iv) $x_1[6-2k]$

$x'_1[k] = x_1[6-k]$: flip + shift 6 steps +

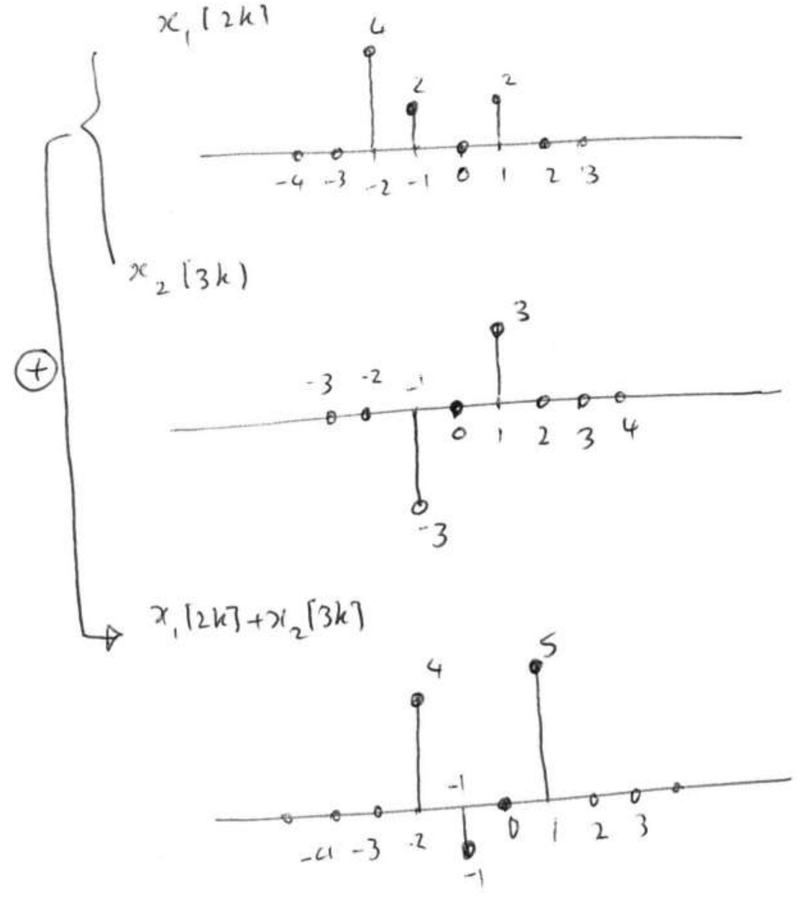
$x[6-2k] = x'_1[2k]$: Compaction with factor 2





1.28

(viii) $x_1[2k] + x_2[3k]$



1.29

- Continuous : It is defined for all $t \in R$ $0 < t < 80$ years
- Analog : Its amplitude can be any Real number
- aperiodic : It seems to be periodic but by looking at the signal it can be seen that the peeks are not the same
- Energy : the duration of the signal is limited to 80 years and the amplitude is limited as well
- Random : this signal can not be modeled deterministic for every body
- neither even nor odd $\begin{cases} x(t) \neq x(-t) \\ x(t) \neq -x(-t) \end{cases}$

2.10

(i) $y[k] = a x[k] + b$

- $x_1[k] = \alpha x[k] \longrightarrow \boxed{} \longrightarrow y_1[k] = a\alpha x[k] + b \neq \alpha y[k] = \alpha a x[k] + \alpha b$

  ⇒ not linear

- $x_2[k] = x[k-k_0] \longrightarrow \boxed{} \longrightarrow y_2[k] = a x_2[k] + b = a x[k-k_0] + b$

  $= y[k-k_0]$

  ⇒ time-invariant ✓

- if $|x[k]| < B \Rightarrow |y[k]| < |a|\cdot|x[k]| + |b| < |a|\cdot B + |b| = B_2$

  ⇒ It is BIBO stable ✓

- Output at time k is dependent on input at time k, so the system is causal ✓

- $x_1 \longrightarrow \boxed{} \longrightarrow y_1$ $\quad \Delta x = x_1 - x_2$

  $x_2 \longrightarrow \boxed{} \longrightarrow y_2$ $\quad \Delta y = y_1 - y_2$

$\Delta y = a x_1[k] + b - a x_2[k] - b = a(x_1[k] - x_2[k]) = a\Delta x$

⇒ $\Delta y = a\Delta x$ ⇒ It is a linear system ↓

$\Delta x \longrightarrow \boxed{\times a} \longrightarrow \Delta y = a\Delta x$

⇒ the system is icrementally linear

2.10

(iii) $y[k] = 2^{x[k]}$

- $x_1[k] = \alpha x[k] \longrightarrow \boxed{} \longrightarrow y_1[k] = 2^{x_1[k]} = 2^{(\alpha x[k])} = 2^{\alpha} \cdot 2^{x[k]}$

  $\neq \alpha y_1[k]$ **not** linear

$\Delta x = x_1[k] - x_2[k]$

$\Delta y = y_1[k] - y_2[k] = 2^{x_1[k]} - 2^{x_2[k]}$

there is no linear relation between Δx and $\Delta y \Rightarrow$ **not** incrementally linear

- $x[k-k_0] \longrightarrow \boxed{} \longrightarrow 2^{x[k-k_0]} = y[k-k_0]$

  $\Rightarrow$ It is time-invariant ✓

- $|x[k]| < B \Rightarrow |y[k]| = |2^{x[k]}| < 2^{|x[k]|} < 2^{B}$

  $\Rightarrow$ It is BIBO stable

- $y[k]$ is only function of $x[k] \Rightarrow$ It is causal

2.10

(iv)

$$y[k] = \sum_{m=-\infty}^{k} x[m]$$

- $x_3[k] = \alpha x_1[k] + \beta x_2[k] \longrightarrow \boxed{\quad} \longrightarrow y_3[k] = \sum_{m=-\infty}^{k} x_3[m]$

$$= \sum_{m=-\infty}^{k} \alpha x_1[m] + \beta x_2[m] = \alpha \sum_{m=-\infty}^{k} x_1[m] + \beta \sum_{m=-\infty}^{k} x_2[m]$$

$\Rightarrow y_3[k] = \alpha y_1[k] + \beta y_2[k] \Rightarrow$ It is linear ✓

- $x_1[k] = x[k-k_0] \longrightarrow \boxed{\quad} \longrightarrow y_1[k] = \sum_{m=-\infty}^{k} x_1[m] = \sum_{m=-\infty}^{k} x[m-k_0]$

$$m' \triangleq m-k_0$$

$\Rightarrow y_1[k] = \sum_{m'=-\infty}^{k-k_0} x[m'] = y[k-k_0] \Rightarrow$ It is time-invariant ✓

- If input is $x[k] = u[k] \Rightarrow y[k] = \begin{cases} \sum_{m=0}^{k} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$

$\Rightarrow y[k] = (k+1)\,u[k]$: It is not a bounded signal because by increasing k, $y[k]$ increases without any bound.

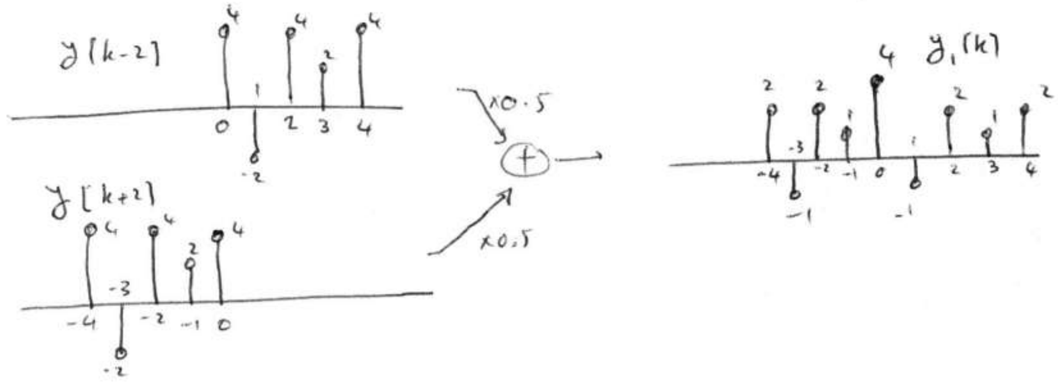
$\Rightarrow$ The system is not BIBO stable

- The system is not causal, becase $y[k]$ is dependent on the past of Input, i.e. $x[-\infty], \dots, x[k-1], x[k]$

2.12

Discrete_time LTI sys.

(ii) $x_1[k] = 0.5\,x[k-2] + 0.5\,x[k+2] \rightarrow \boxed{LTI} \rightarrow y_1[k] = 0.5\,y[k-2] + 0.5\,y[k+2]$



(iv) It cannot be determined, $x[-k]$ is not a linear combination of shifted version of $x[k]$, so, output cannot be written a function of $x[k]$ or $x[k-k_0]$.

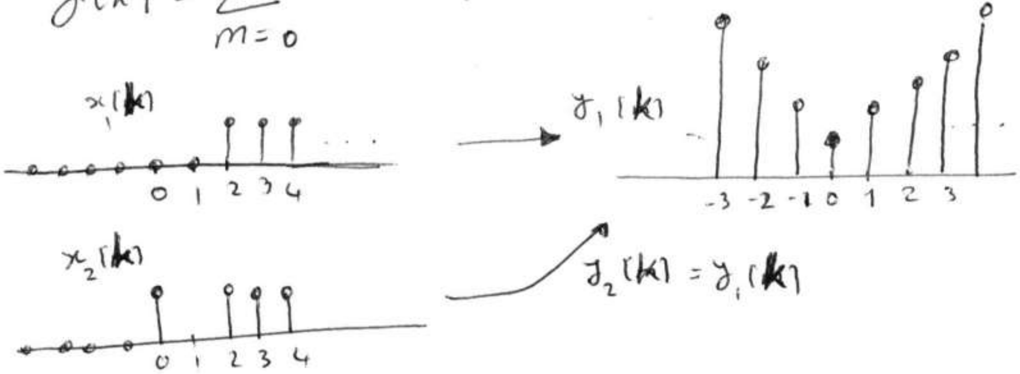
2.14

(i) $y[k] = (k+1)\, x[k+2]$

if $k \neq -1 \Rightarrow x[k+2] = \frac{1}{k+1}\, y[k] \Rightarrow x[k] = \frac{1}{k-1}\, y[k-2] : \forall k \neq 1$

Since this inverse relation is valid for $k \neq 1$, $x[k]$ cannot be found uniquely from $y[k]$, ⇒ The system is not invertable

(ii) $y[k] = \sum_{m=0}^{|k|} x[m+2]$



two inputs $x_1[k]$ and $x_2[k]$ make the same output

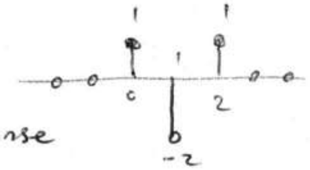
⇒ the system is not invertable

2.19 $y[k] = x[k] - 2x[k-1] + x[k-2]$

(i) $x[k] = \delta[k] \longrightarrow \boxed{\text{LTI sys.}} \longrightarrow y[k] = ?$

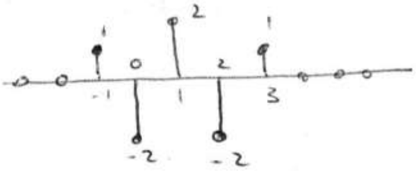
$\Rightarrow y[k] = \delta[k] - 2\delta[k-1] + \delta[k-2] = h[k]$

impulse response



(ii) $x[k] = \delta[k-1] + \delta[k+1]$

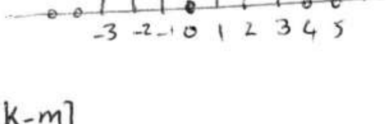
$$\Rightarrow y[k] \underset{\underset{\text{LTI}}{\uparrow}}{=} h[k-1] + h[k+1] = \delta[k-1] - 2\delta[k-2] + \delta[k-3]$$
$$\delta[k+1] - 2\delta[k] + \delta[k-1]$$
$$= \delta[k+1] - 2\delta[k] + 2\delta[k-1] - 2\delta[k-2] + \delta[k-3]$$



(iii) $x[k] = \begin{cases} |k| & |k| \le 3 \\ 0 & \text{otherwise} \end{cases}$

$$y[k] = x[k] * h[k] = \sum_{m=-\infty}^{\infty} x[m]\, h[k-m]$$

x[m]



h[k-m]

$$\left.\begin{array}{l}
\text{if } k < -3 \Rightarrow y[k] = 0 \\
k = -3 \Rightarrow y[k] = 3 \\
k = -2 \Rightarrow y[k] = 2-6 = -4 \\
k = -1 \Rightarrow y[k] = 0 \\
k = 0 \Rightarrow y[k] = 0 \\
k = 1 \Rightarrow y[k] = 2 \\
k = 2 \Rightarrow y[k] = 0 \\
k = 3 \Rightarrow y[k] = 0 \\
k = 4 \Rightarrow y[k] = -4 \\
k = 5 \Rightarrow y[k] = 3 \\
k \geq 6 \Rightarrow y[k] = 0
\end{array}\right\} \Rightarrow y[k] = 3\delta[k+3] - 4\delta[k+2] + 2\delta[k-1] - 4\delta[k-4] + 3\delta[k-5]$$

2.21 $S_1: y[k] = x[k] - 2x[k-1] + x[k-2]$ $S_2: y[k] = x[k] + x[k-1] - 2x[k-2]$

(i)

S_1: $x_3[k] = \alpha x_1[k] + \beta x_2[k]$

$$\Rightarrow y_3[k] = \alpha x_1[k] + \beta x_2[k] - 2\alpha x_1[k-1] - 2\beta x_2[k-1] + \alpha x_1[k-2] + \beta x_2[k-2]$$

$$= \left[\alpha x_1[k] - 2\alpha x_1[k-1] + \alpha x_1[k-2]\right] + \left[\beta x_2[k] - 2\beta x_2[k-1] + \beta x_2[k-2]\right]$$

$$= \alpha y_1[k] + \beta y_2[k] \Rightarrow \text{linear} \checkmark$$

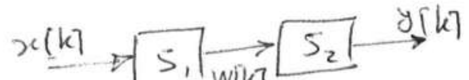
$$x_4[k] = x[k-k_0] \rightarrow y_4[k] = x[k-k_0] - 2x[k-k_0-1] + x[k-k_0-2]$$

$$= y[k-k_0] \Rightarrow \text{Time-invariant} \checkmark$$

S_2: As S_2 output is linear combination of time shifter inputs like S_1 it can be easily shown that S_2 is LTI in a similar way for S_1

(ii)

(a)



$W[k] = x[k] - 2x[k-1] + x[k-2]$

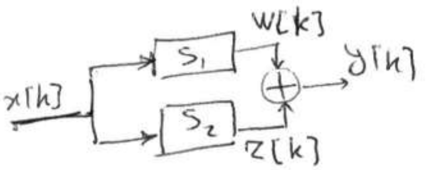
$y[k] = w[k] + w[k-1] - 2w[k-2]$

$= x[k] - 2x[k-1] + x[k-2]$
$+ x[k-1] - 2x[k-2] + x[k-3]$
$-2x[k-2] + 4x[k-3] - 2x[k-4]$

$= x[k] - x[k-1] - 3x[k-2]$
$+ 5x[k-3] - 2x[k-4]$

(iii)

b)



$y[k] = w[k] + z[k]$

$= x[k] - 2x[k-1] + x[k-2]$
$+ x[k] + x[k-1] - 2x[k-2]$

$= 2x[k] - x[k-1] - x[k-2]$

2.21

(iv)

(a) $y[k] = x[k] - x[k-1] - 3x[k-2] + 5x[k-3] - 2x[k-4]$

$x_3[k] = \alpha x_1[k] + \beta x_2[k] \longrightarrow y_3[k] = ?$

$$y_3[k] = \alpha x_1[k] + \beta x_2[k] - \alpha x_1[k-1] - \beta x_2[k-1] - 3\alpha x_1[k-2] - 3\beta x_2[k-2]$$
$$+ 5\alpha x_1[k-3] + 5\beta x_2[k-3] - 2\alpha x_1[k-4] - 2\beta x_2[k-4]$$

$$= \alpha\left[x_1[k] - x_1[k-1] - 3x_1[k-2] + 5x_1[k-3] - 2x_1[k-4]\right]$$
$$+ \beta\left[x_2[k] - x_2[k-1] - 3x_2[k-2] + 5x_2[k-3] - 2x_2[k-4]\right]$$

$$= \alpha y_1[k] + \beta y_2[k] \implies \text{Linear} \checkmark$$

$$x_4[k] = x[k-k_0] \longrightarrow \boxed{a} \longrightarrow y_4[k] = x_4[k] - x_4[k-1] - 3x_4[k-2] + 5x_4[k-3] - 2x_4[k-4]$$

$$\implies y_4[k] = x[k-k_0] - x[k-k_0-1] - 3x_4[k-k_0-2] + 5x[k-k_0-3] - 2x[k-k_0-4)$$
$$= y[k-k_0]$$

$\implies$ Time-invariant ✓

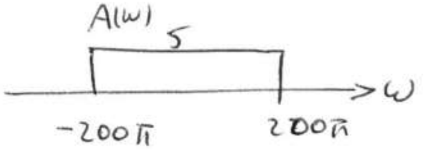
It can be shown that (b) is also LTI in a similar way.

9.1 (b)

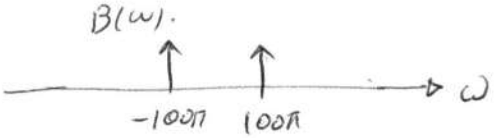
$$x_2(t) = \overbrace{5\,\mathrm{sinc}(200t)}^{a(t)} + \overbrace{8\sin(100\pi t)}^{b(t)}$$

$$H(\omega) = \begin{cases} 1 & |\omega| \leq \omega_0 \\ 0 & \text{o.w.} \end{cases} \underset{F}{\overset{F^{-1}}{\longleftrightarrow}} h(t) = \mathrm{sinc}\left(\frac{t\omega_0}{\pi}\right)$$

$$\Rightarrow 5\,\mathrm{sinc}(200t) \xrightarrow{F}$$



$$8\sin(100\pi t) \xrightarrow{F}$$



$$\rightarrow \omega_M = 200\pi \Rightarrow \text{max freq} = 100 \rightarrow T_s < \frac{1}{200}\text{ sec}$$

$$f_s > 200\text{ Hz}$$

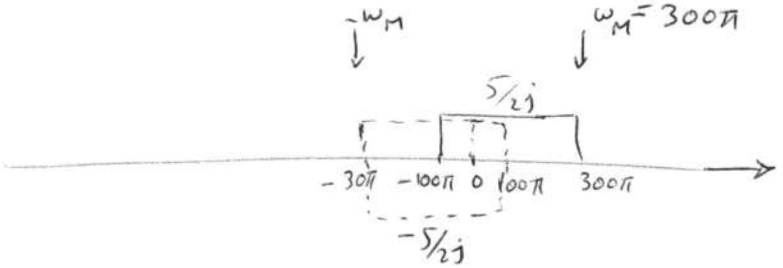
9.1 (c)

$$x_3(t) = \overbrace{5\,\mathrm{Sinc}(200t)}^{a(t)}\ \overbrace{\sin(100\pi t)}^{b(t)}$$

$$\Rightarrow X_3(\omega) = \frac{1}{2\pi} A(\omega) * B(\omega)$$

$$= \frac{1}{\cancel{2\pi}} A(\omega) * \left[\frac{1}{2j} \cancel{2\pi}\delta(\omega - 100\pi) - \frac{1}{2j} \cancel{2\pi}\delta(\omega + 100\pi)\right]$$

$$= \frac{1}{2j} A(\omega - 100\pi) - \frac{1}{2j} A(\omega + 100\pi) = \frac{1}{2j}\left[A(\omega - 100\pi) - A(\omega + 100\pi)\right]$$

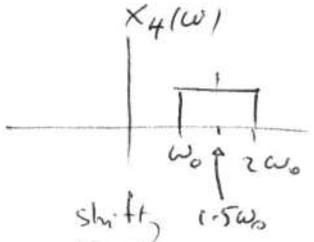


$$\omega_s \geq 2\omega_M = 600\pi$$

$$f_s \geq 300\text{ Hz} \Rightarrow T_s < \frac{1}{300}\text{ sec}$$

9.2

(d) $$X_4(\omega) = u(\omega - \omega_0) - u(\omega - 2\omega_0)$$



$$\Rightarrow X_4(\omega) = \text{rect}\left(\frac{\overbrace{\omega - 1.5\omega_0}^{\text{shift}}}{\omega_0}\right) = \text{rect}\left(\frac{\overbrace{\omega - \frac{3}{2}\omega_0}^{\text{shift}}}{\underbrace{\omega_0}_{\tau\ 2W}}\right)$$

$\frac{W}{\pi}\text{sinc}\left(\frac{Wt}{\pi}\right) \xleftrightarrow{F} \text{rect}\left(\frac{\omega}{2W}\right)$ ⇐ from table 5.2 (Page 204)

$W = \omega_0/2 \quad \Rightarrow \quad x_4(t) = \frac{\omega_0}{2\pi}\,\text{sinc}\left(\frac{\omega_0}{2\pi}t\right) e^{j\frac{3}{2}\omega_0 t}$

$x_4(t)$: is not time limited signal because $X_4(\omega)$ is a band limited signal

9.3

(b) $$x_2(t) = \overbrace{\text{rect}(t/\tau)}^{a(t)} * \overbrace{\text{rect}(t/\tau)}^{b(t)}$$

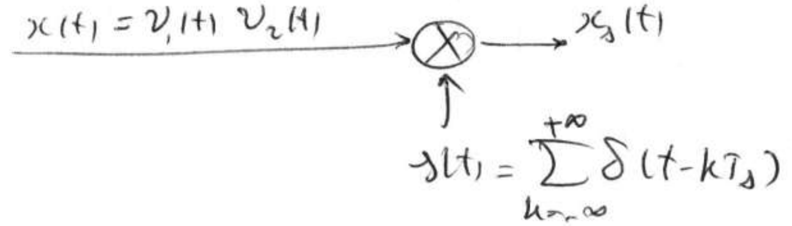
$\boxed{\frac{\text{table}}{5.2}} \rightarrow \text{rect}(t/\tau) \xleftrightarrow{F} \tau\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)$

$x_2(t) = a(t) * b(t) \Rightarrow X_2(\omega) = A(\omega)\,B(\omega)$
$\boxed{\frac{\text{table}}{5.4}}$

$$\Rightarrow X_2(\omega) = \left(\tau\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right)\right)^2$$

⇒ $X_2(\omega)$ is not band limited, because $x_2(t)$ is a time-limited signal

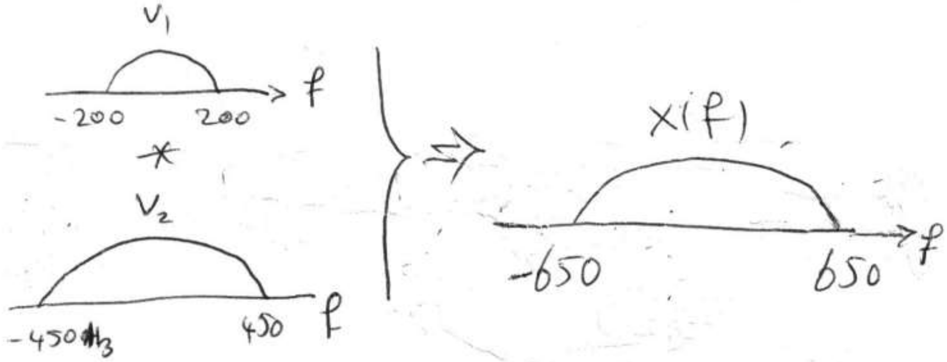
9.4



a)

$v_1(t)$, $v_2(t)$: baseband 200 Hz , 450 Hz

$X(\omega) = \frac{1}{2\pi} V_1(\omega) * V_2(\omega)$; $X(f) = V_1(f) * V_2(f)$



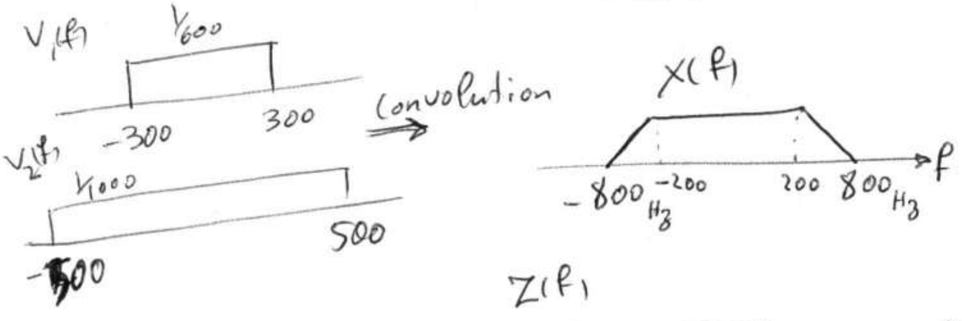
$f_M = 650$ Hz

$f_s > 2 f_M = 1300$ Hz

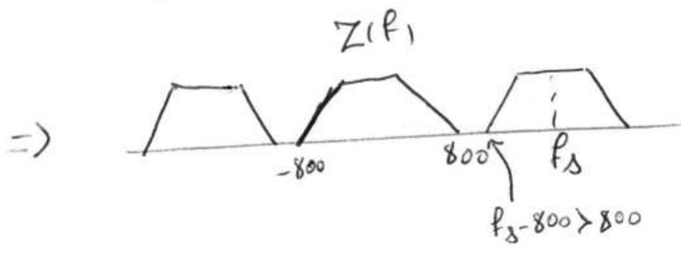
b) $v_1(t) = \text{sinc}(600t) \xrightarrow[5.2]{\text{table}} V_1(f) = \frac{1}{600}\, \text{rect}\left(\frac{f}{600}\right)$

$v_2(t) = \text{sinc}(1000t) \longrightarrow V_2(f) = \frac{1}{1000}\, \text{rect}\left(\frac{f}{1000}\right)$

$\text{sinc}(t) \xleftrightarrow{F} \text{rect}(f)$



$\rightarrow f_s > 2 \times 800$ Hz $= 1600$ Hz



there is no overlap between the replicas, X(f) is reconstructable from Z(f)

9.7

$$r(t) = \sum_{k=-\infty}^{\infty} p_1(t-kT_s) = p_1(t) * \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$$

$$p_1(t) = \text{rect}(t/\tau) \xrightarrow{F} \tau\,\text{sinc}\left(\frac{\omega\tau}{2\pi}\right) \quad Ⓐ$$

$s(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$: Periodic with period of T_s → foureir seris coef: 1

$$\Longrightarrow \text{Fourier Series}: s(t) = \sum_{k=-\infty}^{\infty} e^{j\omega_s k t} = \sum_{k=-\infty}^{\infty} e^{jk\frac{2\pi}{T_s}t} \xrightarrow[\text{shift prop.}]{F} S(\omega) = \sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T_s}\right) \quad Ⓑ$$

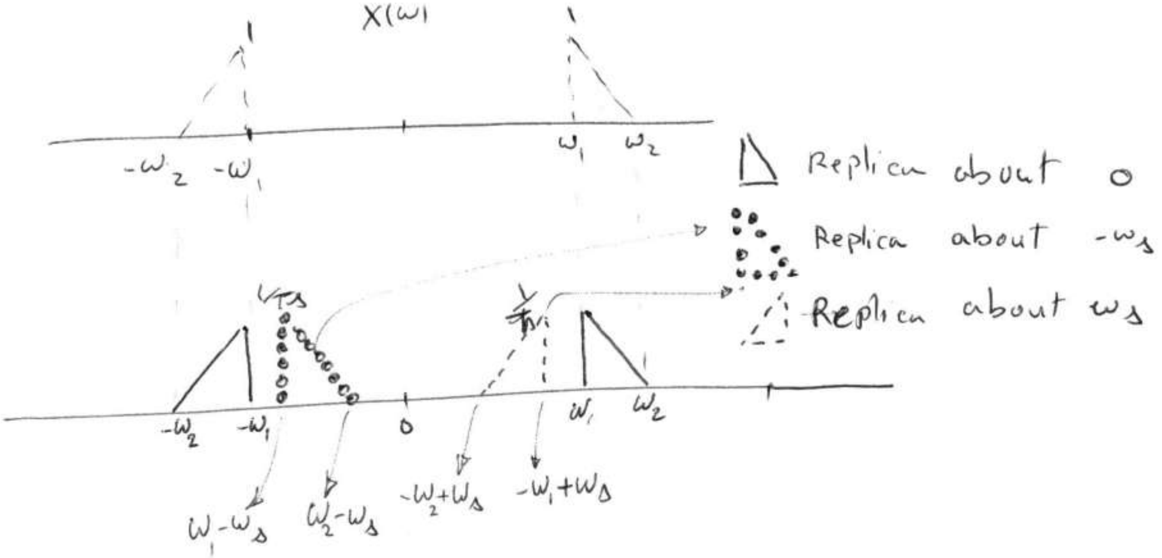
$$R(\omega) = P_1(\omega) \times S(\omega) = \sum_{k=-\infty}^{\infty} \tau \sin\left(\frac{\omega\tau}{2\pi}\right)\delta(\omega - k\omega_s)$$

$$= \sum_{k=-\infty}^{\infty} \tau \sin\left(\frac{k\omega_s\tau}{2\pi}\right)\delta(\omega - k\omega_s)$$

9.8

$X(\omega) = 0$: $|\omega| < \omega_1$ $|\omega| > \omega_2$ $\omega_2 > \omega_1 > 0$

$$X_s(\omega) = \frac{1}{T_s}\sum_{m=-\infty}^{\infty} X\left(\omega - \frac{2m\pi}{T_s}\right)$$



9.8

Continue:

a) to have a replica between 0 and ω_1

$\omega_2-\omega_1$ should be less than ω_1, i.e.

$$\omega_2-\omega_1 < \omega_1 \Rightarrow \omega_2 < 2\omega_1$$

b)

$$-\omega_1+\omega_s < \omega_1 \Rightarrow \omega_s < 2\omega_1$$

$$-\omega_2+\omega_s > \omega_2-\omega_s \Rightarrow \omega_s > \omega_2$$

$$\Rightarrow \boxed{\omega_2 < \omega_s < 2\omega_1}$$

c)

reconstruction filter $H(\omega) = \begin{cases} T_s & \omega_l \leq |\omega| \leq \omega_{\omega_h} \\ 0 & \text{o.w.} \end{cases}$

where

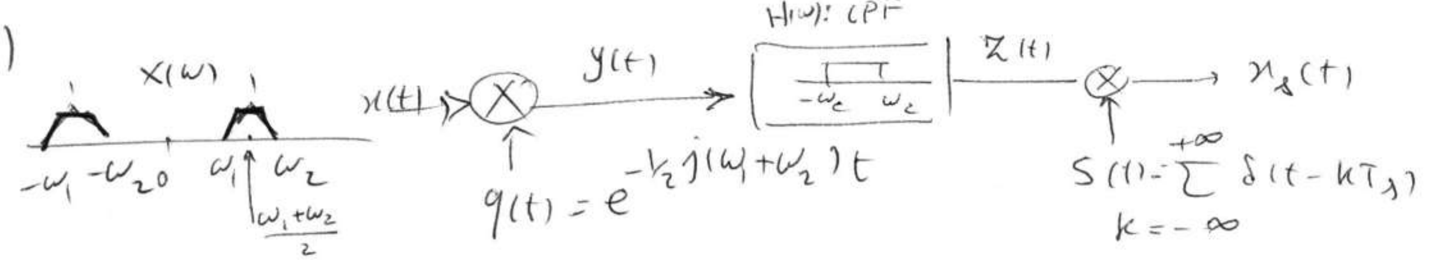
$$\left.\begin{aligned} -\omega_1+\omega_s \leq\ & \omega_l \leq \omega_1 \\ \omega_2 \leq\ & \omega_h \leq 2\omega_s+\omega_2 \end{aligned}\right\} \xrightarrow{\text{or}} \begin{aligned} \omega_l &= \omega_1 \\ \omega_h &= \omega_2 \end{aligned}$$

to keep just the replica about 0

9.9

$\omega_c = \frac{1}{2}(\omega_2 - \omega_1)$

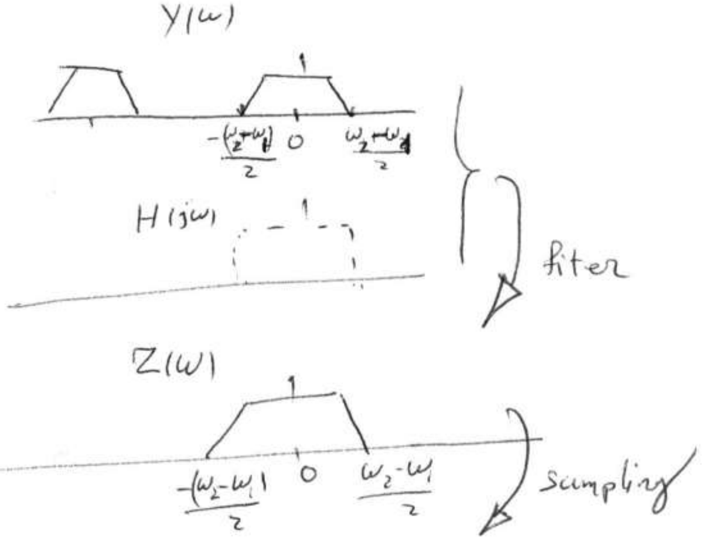
a)



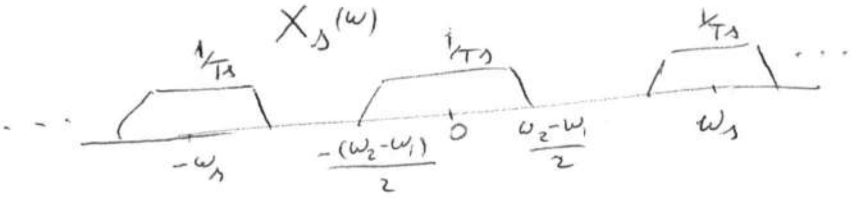
$x(t) \times e^{j\omega_0 t} \longrightarrow X(\omega - \omega_0)$

table 5.4

$\Rightarrow \quad Y(\omega) = X\left(\omega + \frac{\omega_1 + \omega_2}{2}\right) \xrightarrow{\text{left shift}}$



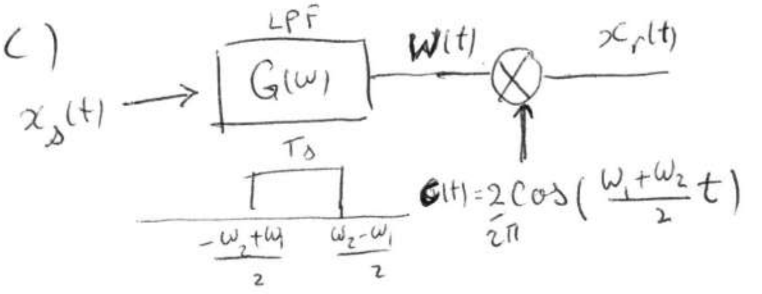
$$X_s(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} Z(\omega - k\omega_s)$$

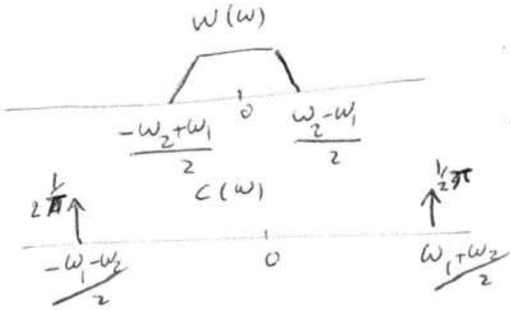


b) from Z(ω) spectrum:

$\omega_M = \frac{\omega_2 - \omega_1}{2} \Rightarrow \omega_s > 2\omega_M = \omega_2 - \omega_1 \Rightarrow T_s < \frac{2\pi}{\omega_2 - \omega_1}$

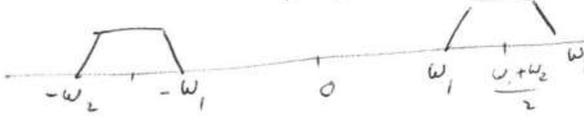
c)



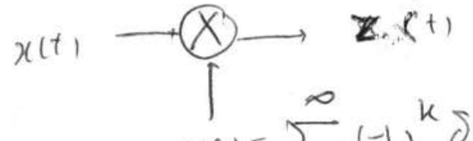


* Convolution
$\times 2\pi$

Reconstructed signal = original ⇐

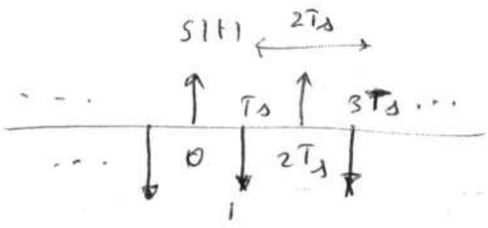


9.11



$$s(t)=\sum_{k=-\infty}^{\infty}(-1)^k\delta(t-kT_s)$$

$z(t)=x(t)\,s(t)$



a) $Z(\omega)=?$

$s(t)$: Periodic with period of $2T_s$

Fourier series coef. : $a_k=\frac{1}{2T_s}\int_{\langle 2T_s\rangle} s(t)\,e^{-j\frac{\cancel{2}\pi}{\cancel{2}T_s}kt}\,dt$

$$=\frac{1}{2T_s}\int\left[\delta(t)-\delta(t-T_s)\right]e^{-j\frac{\pi}{T_s}kt}\,dt$$

$$=\frac{1}{2T_s}\left[1-e^{-jk\pi}\right]=\frac{1-(-1)^k}{2T_s}$$

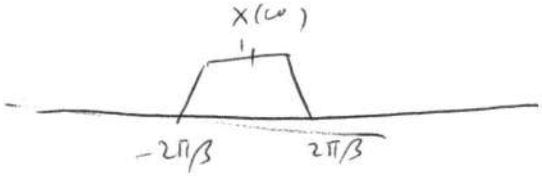
$a_0=0$
$a_1=1/T_s$
$a_2=0$
⋮
or : $a_k=\begin{cases}0 & k: \text{even}\\ 1 & k: \text{odd}\end{cases}$

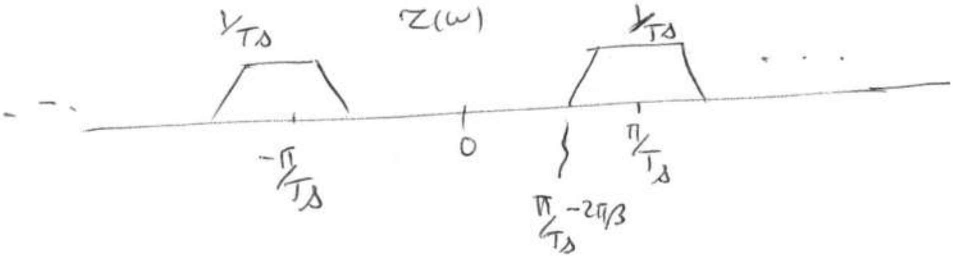
$$s(t)=\sum_{k=-\infty}^{\infty}\frac{1}{T_s}e^{j(2k+1)\frac{2\pi}{2T_s}t}\xrightarrow{\mathcal{F}}S(\omega)=\frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{(2k+1)\pi}{T_s}\right)$$

$$Z(\omega)=\frac{1}{2\pi}X(\omega)*S(\omega)$$

$$=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}X\left(\omega-\frac{(2k+1)\pi}{T_s}\right)$$

b)





C) no aliasing condition

$\frac{\pi}{T_s}-2\pi\beta>0$

$\Rightarrow T_s<\frac{1}{2\beta}$

$f_s>2\beta$

$\omega_s>4\pi\beta$

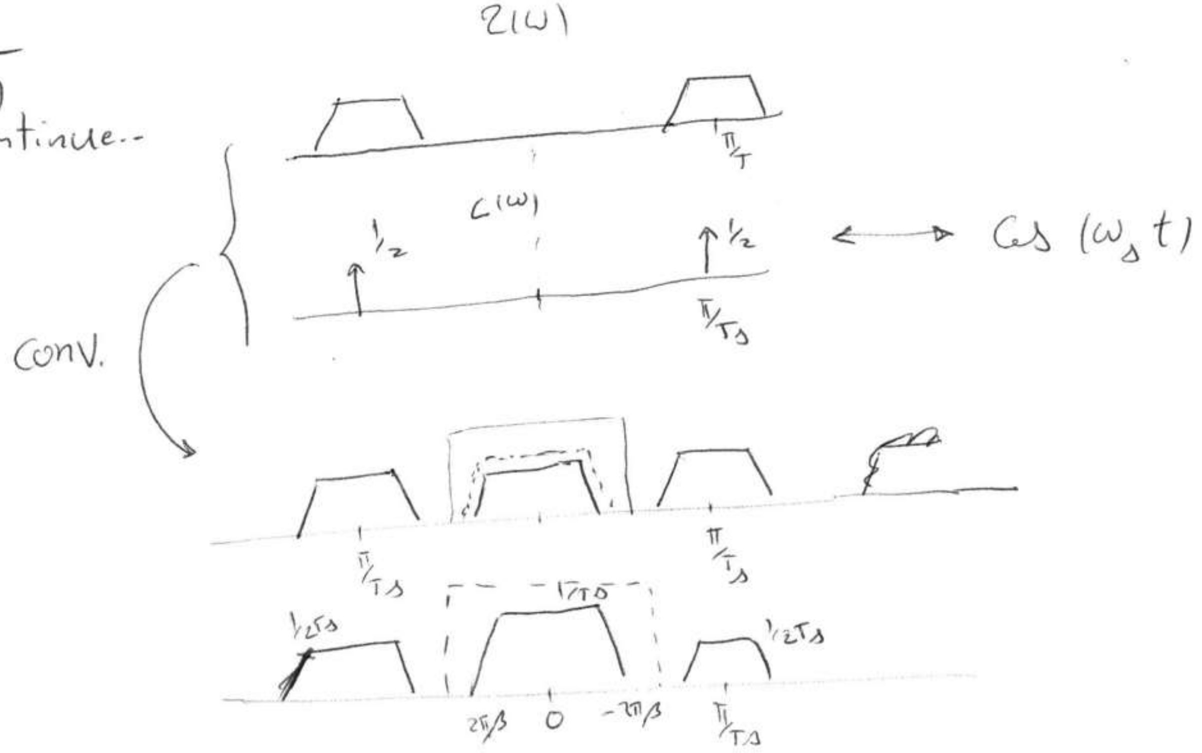
$\omega_s\triangleq\frac{2\pi}{T_s}$

p. 9.8

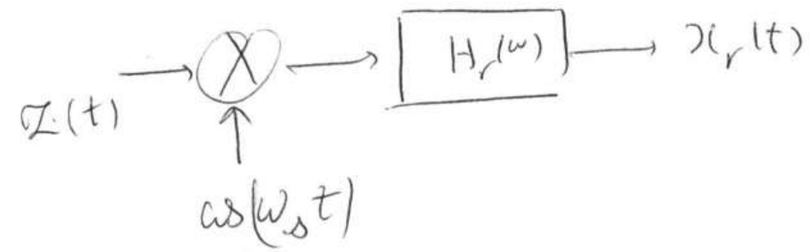
㉓

9.11

(C) continue..



$$\Rightarrow H_r(\omega) = \begin{cases} T_s & 0 \leqslant |\omega| \leqslant \frac{\pi}{2T_s} \\ 0 & o.w \end{cases}$$



9.14

$$\frac{\Delta}{2} = 0.01p \times V_{pp} \Rightarrow \Delta = 0.02p \cdot V_{pp}$$

$$\text{Number of levels} = \frac{V_{pp}}{\Delta} + 1 = \frac{1}{0.02p} + 1 = \frac{50}{p} + 1$$

$$N = \text{Number of bits per level} \geq \log_2 \left[\frac{50}{p} + 1\right] = \frac{\log[50/p + 1]}{\log 2}$$

$$\text{if } p \ll 1 \Rightarrow \log\left[\frac{50}{p} + 1\right] \simeq \log(50/p)$$

$$\Rightarrow N \geq 3.32 \log_{10}\left(\frac{50}{p}\right)$$

9.15 Bandwidth = 4 KHz PCM

$-20 <$ amplitude < 20

a) $f_s > 2 \times 4 \text{ kHz} = 8 \text{ kHz } \frac{\text{sample}}{\text{sec}} \Rightarrow T_s < \frac{1}{8000}$ sec

b) $\xrightarrow{\text{Problem 14}} p = 5\%$

$$\Rightarrow N \geq 3.32 \log_{10}\left(\frac{50}{p}\right) = 3.32 \log_{10}(10) = 3.32$$

$$\Rightarrow N = 4 \text{ bits/sample}$$

c) data rate =?

$$\text{data rate} = N f_s = 32\,000 \; \frac{\text{sample}}{\text{sec}} \; \frac{\text{bits}}{\text{sample}} = 32 \; \text{kbits/sec}$$

9.9.10

9.18 $y[k] = Q\{x(k)\} = \frac{1}{2}[d_m + d_{m+1}]$: $d_m \leq x[k] < d_{m+1}$

$0 \leq m < L$

i) nonlinear ?:

if $x_1[k] = 2$, $x_2[k] = 4$ $\Big\} \to x_1[k] + x_2[k] = 6 \to \boxed{Q} \to 5.5$

$L = 3$

$x[k] \to \boxed{Q} \to y[k]$

$y[k] = 0$: $d_1 = -1 \leq x[k] < 1 = d_2$

$y[k] = 2.5$: $d_2 = 1 \leq x[k] < d_3 = 4$

$y[k] = 5.5$ $d_3 = 4 \leq x[k] < d_4 = 7$

$x_1[k] \to \boxed{Q} \to y_1[k] = 2.5$

$x_2[k] \to \boxed{Q} \to y_2[k] = 5.5$

$y_1[k] + y_2[k] = 8 \neq 5.5$ X nonlinear

ii) if $x_1[k] = x[k-n] \to \boxed{Q}$ $y_1[k] = y[k-n]$ Time invariant ✓

output at time k is only dependent on the input at time k.

iii) memory?

Since output at time k is only dependent on the input at time k it is memoryless ✓

iv) causal?

memoryless ⇒ Causal ✓

v) stable?

since reconstruction levels are bounded the system is stable

vi) invertible?

Example (i)

$x_1[k] = 1 \to y_1[k] = 2.5$

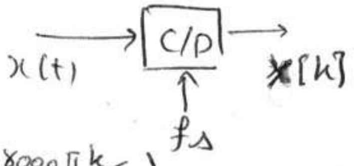
$x_2[k] = 2 \to y_2[k] = 2.5$ $\Big\}$ $y_1[k] = y_2[k]$

There is not a one-to-one relation between output and input → it is not invertible X

11.21 $\quad f_s = 22\ kHz \longrightarrow T_s = \frac{1}{22000}$

(i)

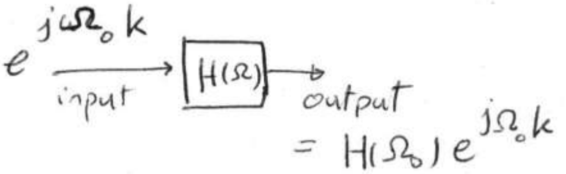
$$x_1(t) = 2 + 3\cos(8000\pi t) + 7\cos(18000\pi t)$$



$$x_1[k] = 2 + 3\cos\left(8000\pi k \frac{1}{22000}\right) + 7\cos\left(\frac{18000\pi k}{22000}\right)$$

$$= \underset{\substack{\downarrow \\ \Omega_1 = 0}}{2} + \underbrace{3\cos\left(\frac{4k\pi}{11}\right)}_{\Omega_2 = \frac{4\pi}{11}} + \underbrace{7\cos\left(\frac{9\pi k}{11}\right)}_{\Omega_3 = \frac{9\pi}{11}}$$

: each term is a mnotone signal



$$\begin{cases} H_1(\Omega_1) = \dfrac{2}{1-\frac{3}{4}+\frac{1}{8}} = 16/3 \\ H_1(\Omega_2) = \dfrac{2}{1-\frac{3}{4}e^{-j\frac{4\pi}{11}}+\frac{1}{8}e^{-j\frac{8\pi}{11}}} = 1.7005 - j1.6478 = 2.3679 \angle -0.7696 \text{ rad} \\ H_1(\Omega_3) = \dfrac{2}{1-\frac{3}{4}e^{-j\frac{9\pi}{11}}+\frac{1}{8}e^{-j\frac{18\pi}{11}}} = 1.0852 - j0.3348 = 1.1356 \angle -0.2992 \text{ rad} \end{cases}$$

linearity

$$\Rightarrow y_{|H_1}[k] = 2H_1(\Omega_1) + 3\cos\left(\frac{4k\pi}{11} - 0.7696\right) \times 2.3679$$

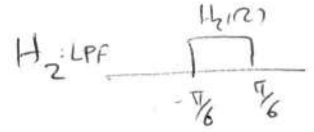
$$+ 7\cos\left(\frac{9\pi k}{11} - 0.2992\right) \times 1.1356$$

$$= \frac{32}{3} + 7.103\cos\left(\frac{4k\pi}{11} - 0.7696\right) + 7.949\cos\left(\frac{9\pi k}{11} - 0.2992\right)$$

$\frac{11.21}{(i)}$ continue...

$$\Omega_1 = 0 \quad \Omega_2 = \frac{4\pi}{11} > \frac{\pi}{6} \quad \Omega_3 = \frac{9\pi}{11} > \frac{\pi}{6}$$

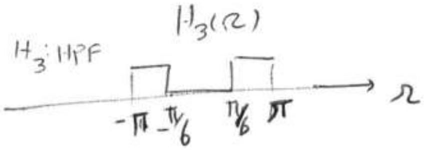
$$H_2(\Omega_1) = 1$$



$$H_2(\Omega_2) = 0$$

$$H_2(\Omega_3) = 0$$

$$y_{1,H_2}[k] = 2 + 0 + 0$$



$$H_3(\Omega_1) = 0$$

$$H_3(\Omega_2) = 1$$

$$H_3(\Omega_3) = 1$$

$$\rightarrow y_{1,H_3}[k] = 0 + 3\cos\left(\frac{4k\pi}{11}\right) + 7\cos\left(\frac{9\pi k}{11}\right)$$

$\frac{11.21}{(iii)}$ $$x_3(t) = 5\cos(600\pi t) + 9\cos(900\pi t) + 2\cos(3000\pi t)$$

$$\Rightarrow x_3[k] = x_3(kT_s) = \underbrace{5\cos\left(\frac{3\pi}{110}k\right)}_{\Omega_0 = \frac{3\pi}{110} < \pi/6} + \underbrace{9\cos\left(\frac{9\pi k}{220}\right)}_{\Omega_1 = \frac{9\pi}{220} < \frac{\pi}{6}} + \underbrace{2\cos\left(\frac{3\pi}{22}k\right)}_{\Omega_2 = \frac{3\pi}{22} < \pi/6}$$

$$H_1\left(\Omega_0 = \frac{3\pi}{110}\right) = \frac{2}{1 - \frac{3}{4}e^{-j3\pi/110} + \frac{1}{8}e^{-j\frac{3\pi}{55}}} = 5.252 - j0.5989 = 5.286 \measuredangle -0.1135$$

$$H_1\left(\Omega_1 = \frac{9\pi}{220}\right) = \frac{2}{1 - \frac{3}{4}e^{-j\frac{9\pi}{220}} + \frac{1}{8}e^{-j\frac{9\pi}{110}}} = 5.1538 - j0.8795 = 5.2284 \measuredangle -1.690$$

$$H_1\left(\Omega_2 = 3\pi/22\right) = \frac{2}{1 - \frac{3}{4}e^{j3\pi/22} + \frac{1}{8}e^{-j\frac{3\pi}{11}}} = 3.8643 - j2.0992 = 4.397 \measuredangle -0.4976$$

$$\Rightarrow y_{3H_1}[k] = 5 \times 5.2284\cos\left(\frac{3\pi}{110}k - 0.1135\right) + 9 \times 5.2284\cos\left(\frac{9\pi k}{220} - 1.69\right) + 2 \times 4.397\cos\left(\frac{3\pi k}{22} - \overset{-0.4976}{\cancel{\;}}\right)$$

11.21
(iii)

Continue...

$\Omega_0, \Omega_1, \Omega_2 < \pi/6$

$$H_2(\Omega_0) = H_2(\Omega_1) = H_2(\Omega_3) = 1$$

$$\Rightarrow y_{3,H_2}[k] = y_1[k] \quad ✓$$

$$H_2(\Omega_0) = H_2(\Omega_1) = H_2(\Omega_3) = 0 \Rightarrow$$

$$\Rightarrow y_{3,H_3}[k] = 0 \quad ✓$$

10.2 input $x[k] = \left(\frac{1}{2}\right)^{k} u[k]$

$$y[k+2] - y[k+1] + \frac{1}{2} y[k] = x[k]$$

initial condition: $y[-1] = 0 \quad y[-2] = 1$

a)

Ⓐ $y[k+2] = y[k+1] - \frac{1}{2} y[k] + x[k]$; $k+2 \rightarrow k$

$$\Rightarrow y[k] = y[k-1] - \frac{1}{2} y[k-2] + \underbrace{x[k-2]}$$

; $x[k-2] = \left(\frac{1}{2}\right)^{k-2} u[k-2]$

$$\Rightarrow y[k] = y[k-1] - \frac{1}{2} y[k-2] + \left(\frac{1}{2}\right)^{k-2} u[k-2]$$

$$\Rightarrow y[k] = \begin{cases} y[k-1] - \frac{1}{2} y[k-2] + \left(\frac{1}{2}\right)^{k-2} & k \geqslant 2 \\ y[k-1] - \frac{1}{2} y[k-2] & k < 2 \end{cases}$$

$$\Rightarrow \begin{cases} y[0] = \overset{\nearrow 0}{y[-1]} - \frac{1}{2} y[-2] = -\frac{1}{2} \\ y[1] = y[0] - \frac{1}{2} y[-1] = -\frac{1}{2} \\ y[2] = y[1] - \frac{1}{2} y[0] + \left(\frac{1}{2}\right)^{2-2} = -\frac{1}{2} + \frac{1}{4} + 1 = \frac{3}{4} \\ y[3] = y[2] - \frac{1}{2} y[1] + \left(\frac{1}{2}\right)^{3-2} = \frac{3}{4} + \frac{1}{4} + \frac{1}{2} = \frac{3}{2} \\ y[4] = y[3] - \frac{1}{2} y[2] + \left(\frac{1}{2}\right)^{4-2} = \frac{3}{2} - \frac{3}{8} + \frac{1}{4} = \frac{11}{8} \\ y[5] = y[4] - \frac{1}{2} y[4] + \left(\frac{1}{2}\right)^{5-2} = \frac{11}{8} - \frac{11}{16} + \frac{1}{8} = \frac{13}{16} \end{cases}$$

b) zero-input response : $x[k] = 0$

Ⓐ

$\Rightarrow$ $\boxed{y_{zi}[k] = y[k-1] - \frac{1}{2} y[k-2]}$

$y_{zi}[0] = y[-1] - \frac{1}{2} y[-2] = -\frac{1}{2}$

$y_{zi}[1] = y[0] - \frac{1}{2} y[-1] = -\frac{1}{2}$

$y_{zi}[2] = y[1] - \frac{1}{2} y[0] = -\frac{1}{2} + \frac{1}{4} = -\frac{1}{4}$

$y_{zi}[3] = y[2] - \frac{1}{2} y[1] = -\frac{1}{4} + \frac{1}{4} = 0$

$y_{zi}[4] = y[3] - \frac{1}{2} y[2] = 0 + \frac{1}{8} = \frac{1}{8}$

$y_{zi}[5] = y[4] - \frac{1}{2} y[3] = \frac{1}{8} + 0 = \frac{1}{8}$

c) Zero-state response : $\begin{cases} y[-1] = y[-2] = 0 \\ x[k] = (\frac{1}{2})^k u[k] \end{cases}$

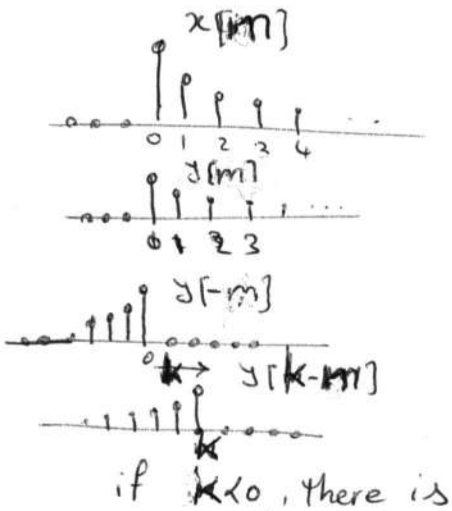
$$\Rightarrow \; y_{zs}[0] = 0$$
$$y_{zs}[1] = 0$$
$$y_{zs}[2] = y[1] - \tfrac{1}{2} y[0] + 1 \quad = 1$$
$$y_{zs}[3] = y[2] - \tfrac{1}{2} y[1] + \tfrac{1}{2} = \tfrac{3}{2}$$
$$y_{zs}[4] = y[3] - \tfrac{1}{2} y[2] + \tfrac{1}{4} = \tfrac{3}{2} - \tfrac{1}{2} + \tfrac{1}{4} = \tfrac{5}{4}$$
$$y_{zs}[5] = y[4] - \tfrac{1}{2} y[3] + \tfrac{1}{8} = \tfrac{5}{4} - \tfrac{3}{4} + \tfrac{1}{8} = \tfrac{5}{8}$$

d) By adding $y_{zs}[k]$ and $y_{zi}[k]$ from parts (c) and (b), it can be seen that the result is as same as the result from part (a), i.e.

$$\boxed{y[k] = y_{zs}[k] + y_{zi}[k]} \quad \checkmark$$

10.4

$x[k] = a^k u[k] \quad y[k] = b^k u[k]$



if k<0, there is no overlap between x[k] and y[m-k]:

$\Rightarrow x[k] \cdot y(m-k] = 0 : k<0$

$$x[k] * y[k] = \sum_{m=-\infty}^{+\infty} a^m u[m]\, b^{k-m} u[k-m]$$
$$= b^k \sum_{m=-\infty}^{\infty} (a/b)^m u[m]\, u[k-m]$$
$$= b^k \sum_{m=0}^{\infty} \left(\frac{a}{b}\right)^m u[k-m] = \begin{cases} b^k \sum_{m=0}^{k} \left(\frac{a}{b}\right)^m & k \geq 0 \\ 0 & k < 0 \end{cases}$$

Geometric $\Longrightarrow$ series

$$= \begin{cases} b^k \dfrac{(a/b)^{k+1} - 1}{a/b - 1} & a \neq b \\ (k+1)\, b^k & a = b \end{cases}$$

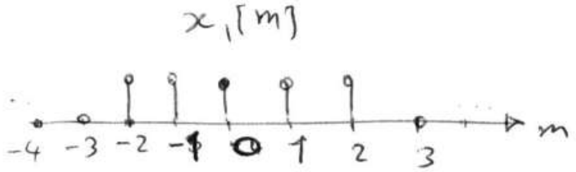
$$\sum_{n=0}^{n_2} r^n = \frac{r^{n_2+1} - 1}{r - 1} \quad : r \neq 1$$

10.5

(a)

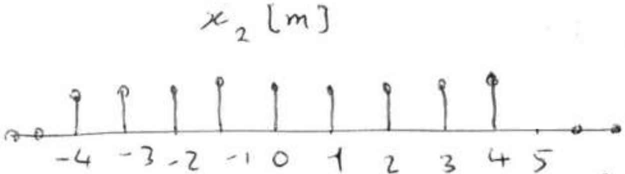
$$x_1[k] = u[k+2] - u[k-3]$$

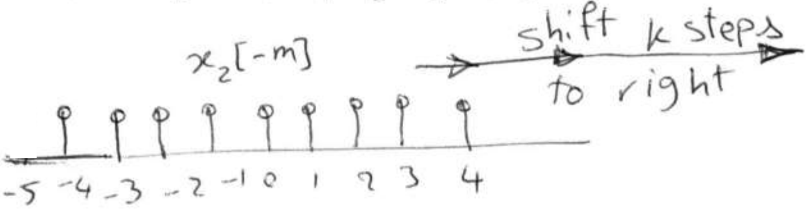
$$x_2[k] = u[k+4] - u[k-5]$$

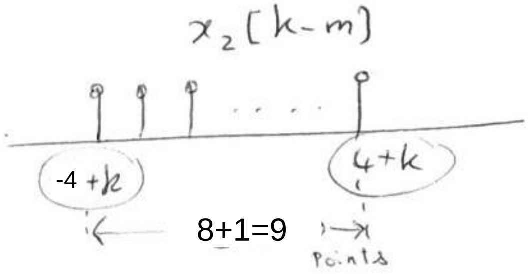


$$y[k] = x_1[k] * x_2[k] = \sum_{m=-\infty}^{\infty} x_1[m]\, x_2[k-m]$$

× and sum.







if $k+4 < -2 \Rightarrow$ no overlap : $y[k]=0$

if $-2 \leq k+4 < 3 \Rightarrow y[k] = \sum_{m=-2}^{k+4} 1 = k+7$

if $\begin{cases} 3 \leq k+4 \\ \text{and} \\ -4+k<-1 \end{cases} \Rightarrow y[k] = \sum_{-2}^{2} 1 = 5$

if $-1 \leq -4+k < 3 \Rightarrow y[k] = \sum_{-4+k}^{2} 1 = 7-k$

if $3 \leq -4+k \Rightarrow y[k]=0$

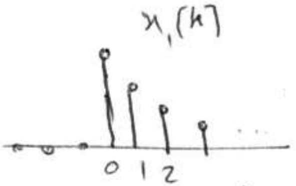
$$\Rightarrow y[k] = \begin{cases} 0 & k<-6 \\ k+7 & -6 \leq k < -1 \\ 5 & -1 \leq k < 3 \\ 7-k & 3 \leq k < 7 \\ 0 & 7 \leq k \end{cases}$$

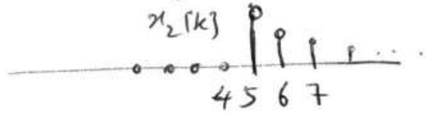
$\frac{10.5}{(b)}$

$$y[k] = x_1[k] * x_2[k]$$

$$x_1[k] = \left(\tfrac{1}{2}\right)^k U[k]$$

$$x_2[k] = (0.8)^k U[k-5]$$





$$y[k] = x_1[k] * x_2[k] = \sum_{m=-\infty}^{\infty} x_1[k-m]\, x_2[m] = \sum_{m=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{k-m} U[k-m]\,(0.8)^m\, U[m-5]$$

$$= \left(\tfrac{1}{2}\right)^k \sum_{m=-\infty}^{\infty} (1.6)^m\, U[k-m]\, U[m-5]$$

$$= \left(\tfrac{1}{2}\right)^k \sum_{m=5}^{\infty} (1.6)^m\, U[k-m] = \begin{cases} \left(\tfrac{1}{2}\right)^k \sum_{m=5}^{k} (1.6)^m & k \geq 5 \\ 0 & k < 5 \end{cases}$$

useing Geometric series $\sum_{n=0}^{N} r^n = \frac{r^{N+1}-1}{r-1} : r \neq 1$, $\sum_{n=N_1}^{N_2} r^n = \frac{r^{N_2+1} - r^{N_1}}{r-1}$

$$\Rightarrow y[k] = \begin{cases} \left(\tfrac{1}{2}\right)^k \frac{(1.6)^{k+1} - (1.6)^{5}}{1.6-1} = \frac{1}{0.6}\left[(0.8)^k \times 1.6 - (1.6)^{5}\left(\tfrac{1}{2}\right)^k\right] \\ 0 \end{cases}$$

$$\Rightarrow y[k] = \begin{cases} 2.667\,(0.8)^k - 0.159\,(0.5)^k & : k \geq 5 \\ 0 & : k < 5 \end{cases}$$

or $y[k] = \left[2.667\,(0.8)^k - 0.159\,(0.5)^k\right] U[k-5]$

10.7 $x[k] = \begin{cases} 2 & 0 \leq k \leq 2 \\ 0 & O.W \end{cases}$ $h[k] = \begin{cases} k+1 & 0 \leq k \leq 4 \\ 0 & O.W. \end{cases}$

The sliding method is shown in table 10.1 (Page 437)
The other method ~~is~~ convolution of delta funtions

$$\delta[k-N] * f[k] = f[k-N] \quad \circledast$$

$$\left\{ \begin{array}{l} x[k] = 2\delta[k] + 2\delta[k-1] + 2\delta[k-2] \\ h[k] = \delta[k] + 2\delta[k-1] + 3\delta[k-2] + 4\delta[k-3] + 5\delta[k-4] \end{array} \right.$$

$\circledast$

$$x[k] * h[k] = 2\delta[k] + 4\delta[k-1] + 6\delta[k-2] + 8\delta[k-3] + 10\delta[k-4]$$
$$+ 2\delta[k-1] + 4\delta[k-2] + 6\delta[k-3] + 8\delta[k-4] + 10\delta[k-5]$$
$$+ 2\delta[k-2] + 4\delta[k-3] + 6\delta[k-4] + 8\delta[k-5] + 10\delta[k-6]$$

$$= 2\delta[k] + 6\delta[k-1] + 12\delta[k-2] + 18\delta[k-3] + 24\delta[k-4] + 18\delta[k-5] + 10\delta[k-6]$$

Comparing this with table 10.1 ⇒ they are the same

$\frac{10.9}{1}$ $K_0 = 10$ Peridic Convolution

	m:	0	1	2	3	4	5	6	7	8	9	k	$y_p[k] = \sum_{m=\langle K_0\rangle} h_p[m]x_p[k-m]$
one period	$h_p[m]$:	1	2	3	4	5	0	0	0	0	0		
one period	$x_p[m]$:	2	2	2	0	0	0	0	0	0	0		
circular reflection	$x_p[-m]$:	2	0	0	0	0	0	0	0	2	2	0	2
Circular Shift	$x_p[1-m]$:	2	2	0	0	0	0	0	0	0	2	1	6
	$x_p[2-m]$:	2	2	2	0	0	0	0	0	0	0	2	12
	$x_p[3-m]$	0	2	2	2	0	0	0	0	0	0	3	18
	$x_p[4-m]$	0	0	2	2	2	0	0	0	0	0	4	24
	$x_p[5-m]$	0	0	0	2	2	2	0	0	0	0	5	18
	$x_p[6-m]$	0	0	0	0	2	2	2	0	0	0	6	10
	$x_p[7-m]$	0	0	0	0	0	2	2	2	0	0	7	0
	$x[8-m]$	0	0	0	0	0	0	2	2	2	0	8	0
	$x[9-m]$	0	0	0	0	0	0	0	2	2	2	9	0

The other way: find the linear convolution between h[k] and x[k] and make it periodic using $y_p[k]=\sum_{\ell=-\infty}^{\infty} y[k-\ell K_0]$

$\frac{10.9}{2}$

$K_0=13$

m: 0 1 2 3 4 5 6 7 8 9 10 11 12

$h_p[m]$ 1 2 3 4 5 0 0 0 0 0 0 0 0

$x_p[m]$ 2 2 2 0 0 0 0 0 0 0 0 0 0

$x_p[-m]$ 2 0 0 0 0 0 0 0 0 0 0 2 2

$x_p[1-m]$ 2 2 0 ———————— 0 2

$x_p[2-m]$ 2 2 2 0 0 - - - - - - - - 0 0

$x_p[3-m]$ 0 2 2 2 0 . . . . 0 0

$x_p[4-m]$ 0 0 2 2 2 0 - - - - 0 0

$x_p[5-m]$ 0 0 0 2 2 2 0 - - - 0 0

$x_p[6-m]$ 0 0 0 0 2 2 2 0 . . . . . 0 0

$x_p[7-m]$ 0 0 0 0 0 2 2 2 0 0 . . . .

no overlap {
⋮
. . . . 0 0 2 2 2
$x_p[12-m]$ 0 - - - - - - - - -

k	$y_p[k]$
0	2
1	6
2	12
3	18
4	24
5	18
6	10
7	0
8	0
⋮	⋮
12	0

if $K_0 = 13$

$K_1=5$: length of $h[k]$

$K_2 = 3$: length of $x[k]$

} $K_0=13 \geq K_1+K_2-1 = 7$

$\Rightarrow$ linear conv. = One period of periodic conv.
for both cases
$K_0=13$ and $K_0=10$

if $K_0=10$ → $10 \geq 7$

10.12

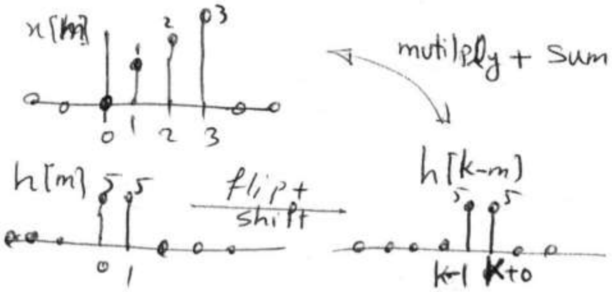
$K_0 = 8$

$$x_p[k] = \begin{cases} k & 0 \le k \le 3 \\ 0 & 4 \le k \le 7 \end{cases} \qquad h_p[k] = \begin{cases} 5 & k=0,1 \\ 0 & k=2,3,4,\cdots 7 \end{cases}$$

way.1

$$x[k] = \begin{cases} k & 0 \le k \le 3 \\ 0 & \text{o.w} \end{cases} \qquad h[k] = \begin{cases} 5 & k=0,1 \\ 0 & \text{o.w.} \end{cases}$$

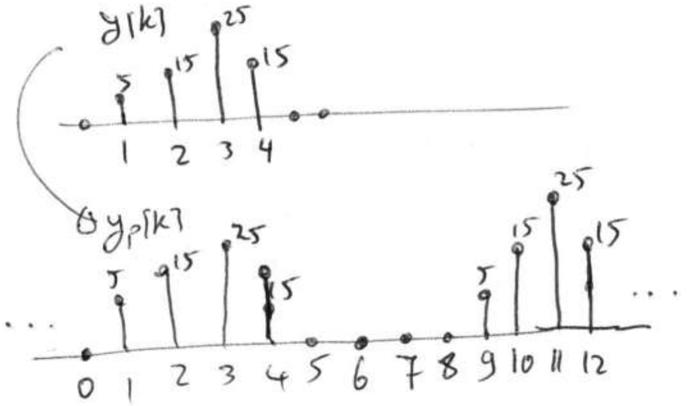
$$y[k] = x[k] * h[k] = \begin{cases} 0 & k<1 \\ \sum_{1}^{k} 5m & 1 \le k \le 2 \\ \sum_{k-1}^{3} 5m & 3 \le k < 5 \\ 0 & 5 \le k \end{cases}$$



$$= \begin{cases} 0 & k<1 \\ 5 & k=1 \\ 15 & k=2 \\ 25 & k=3 \\ 15 & k=4 \\ 0 & 5 \le k \end{cases}$$

$$y_p[k] = \sum_{l=-\infty}^{\infty} y[k - K_0 l]$$

$$= \sum_{l=-\infty}^{\infty} y[k - 8l]$$



way 2

m:	0	1	2	3	4	5	6	7	k	$y_p[k] = \sum_{\langle K_0 \rangle} h_p[m]\, x_p[k-m]$
$h_p[m]$:	5	5	0	0	0	0	0	0		
$x_p[m]$:	0	1	2	3	0	0	0	0		
$x_p[-m]$:	0	0	0	0	0	3	2	1	0	0
$x_p[1-m]$:	1	0	0	0	0	0	3	2	1	5
$x_p[2-m]$	2	1	0	0	0	0	0	3	2	15
$x_p[3-m]$	3	2	1	0	0	0	0	0	3	25
$x_p[4-m]$	0	3	2	1	0	0	0	0	4	15
$x_p[5-m]$	0	0	3	2	1	0	0	0	5	0
$x_p[6-m]$	0	0	0	3	2	1	0	0	6	0
$x_p[7-m]$	0	0	0	0	3	2	1	0	7	0

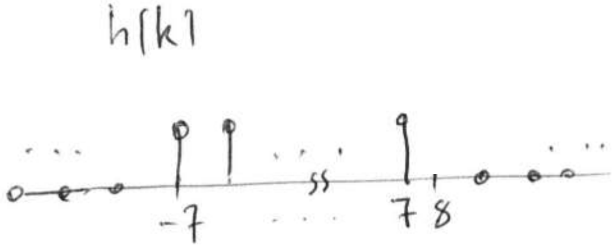
10.13 (a)

Unit step response ?

$$h[k] = U[k+7] - U[k-8]$$

$$x[k] = U[k]$$

Unit step respons$t \longrightarrow S[k] = x[k] * h[k] = \sum_{m=-\infty}^{\infty} h[m]\, U[k-m] = \sum_{m=-\infty}^{k} h[m]$



if $k < -7 \rightarrow S[k] = 0$

if $-7 \leq k \leq 7 \rightarrow S[k] = \sum_{-7}^{k} 1 = k+7+1 = k+8$

if $8 \leq k \rightarrow S[k] = \sum_{-7}^{7} 1 = 15$

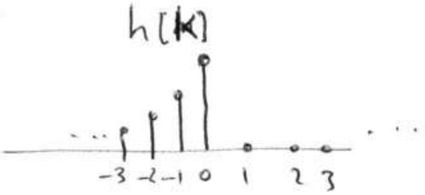
$$\therefore\ S[k] = \begin{cases} 0 & k < -7 \\ k+8 & -7 \leq k \leq 7 \\ 15 & 8 \leq k \end{cases}$$

10.13 (b)

$$S[k] = \begin{cases} \sum_{m=0}^{k} (0.4)^{m} & k \geq 0 \\ 0 & k < 0 \end{cases} = \begin{cases} \dfrac{(0.4)^{k+1} - 1}{0.4 - 1} & k \geq 0 \\ 0 & k < 0 \end{cases}$$

10.13 (c)

$$h[k] = 2^{k} U[-k]$$



$$S[k] = \begin{cases} \sum_{m=-\infty}^{k} 2^{m} & k \leq 0 \\ \sum_{m=-\infty}^{0} 2^{m} & k > 0 \end{cases}$$

$$\sum_{n=N_1}^{N_2} r^{n} = \frac{r^{N_2+1} - r^{N_1}}{r-1} \quad r \neq 1$$

$$\Rightarrow S[k] = \begin{cases} \dfrac{2^{k+1} - 0}{2-1} = 2^{k+1} \\ \dfrac{2^{1} - 2^{-\infty}}{2-1} = 2 \end{cases} \Rightarrow S[k] = \begin{cases} 2^{k+1} & k \leq 0 \\ 2 & k > 0 \end{cases}$$

10.14

$$\boxed{x[k] * \delta[k-k_0] = x[k-k_0]}$$

(b) $(x[k]+2\delta[k-1]) * (\delta[k+1]+\delta[k-2])$ distributive prop.

$$= x[k] * \delta[k+1] + 2\delta[k-1] * \delta[k+1] + x[k] * \delta[k-2] + 2\delta[k-1] * \delta[k-2]$$

$$= x[k+1] + 2\delta[k] + x[k-2] + 2\delta[k-3]$$

10.14

(d) $(x[k]-x[k-1]) * U[k] = x[k] * U[k] - x[k-1] * U[k]$

$$= \sum_{m=-\infty}^{k} x[m] - \sum_{m=-\infty}^{k} x[\overbrace{m-1}^{m-1=n}]$$

$$= \sum_{m=-\infty}^{k} x[m] - \sum_{n=-\infty}^{k-1} x[n] \underset{\uparrow}{=} \sum_{m=-\infty}^{k} x[m] - \sum_{m=-\infty}^{k-1} x[m] = \underset{\uparrow}{x[k]}$$

combine two sumations

10.16 Shift property $x_1[k] * x_2[k] = g[k]$

$$x_1[k-k_1] * x_2[k-k_2] \overset{?}{=} g[k-k_1-k_2]$$

$$g[k] = \sum_{m=-\infty}^{\infty} x_1[m]\, x_2[k-m] \quad Ⓐ$$

$$x_1[k-k_1] * x_2[k-k_2] = \sum_{m=-\infty}^{\infty} x_1[\overbrace{m-k_1}^{n=m-k_1 \Rightarrow n+k_1=m}]\, x_2[k-m-k_2] =$$

$$= \sum_{n=-\infty}^{\infty} x_1[n]\, x_2[k-n-k_1-k_2]$$

$$= \sum_{n=-\infty}^{\infty} x_1[n]\, x_2[(k-k_1-k_2)-n]$$

Use the deffinition of Convolution Ⓐ $= g(k-k_1-k_2)$ ✓

10.18 i) memoryless? ii) causal? iii) stable?

a) $h[k] = U[k+7] - U[k-8] = \begin{cases} 1 & -7 \leq k \leq 7 \\ 0 & \text{o.w.} \end{cases}$

i) $h[k] \neq 0 \ : k \neq 0 \rightarrow$ memoryless X

ii) $h[k] \neq 0 \ : k < 0 \rightarrow$ Causal X

iii) $\sum |h[k]| = 15 < \infty \rightarrow$ BIBO stable ✓

b) $h[k] = \sin\left(\frac{k\pi}{8}\right) U[k]$

i) $h[k] \neq 0 \quad k \neq 0 \rightarrow$ memoryless X

ii) $h[k] = 0 \quad k < 0 \rightarrow$ causal ✓

iii) $\sum_{k=0}^{\infty} \left|\sin \frac{k\pi}{8}\right| = \infty \rightarrow$ stable X

c) $h[k] = 6^k U[-k]$

i) $h[k] \neq 0 \quad k \neq 0 \rightarrow$ memoryless X

ii) $h[k] \neq 0 \quad k < 0 \rightarrow$ Causal X

iii) $\sum_{-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{0} 6^k = \sum_{0}^{\infty} \left(\frac{1}{6}\right)^k = \frac{1}{1-\frac{1}{6}} < \infty$ stable ✓

d) $h[k] = (0.9)^{|k|}$

i) $h[k] \neq 0 \quad k \neq 0$ memoryless X

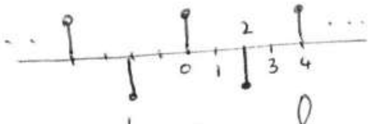
ii) $h[k] \neq 0 \quad k < 0$, for example $h[-1] = 0.9 \rightarrow$ causal X

iii) $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{-\infty}^{-1} (0.9)^{-k} + \sum_{0}^{\infty} (0.9)^k = \sum_{1}^{\infty} (0.9)^k + \sum_{0}^{\infty} (0.9)^k \underset{\uparrow}{=} \frac{0.9}{1-0.9} - \frac{1}{1-0.9} < \infty$

Geometric series

$\Rightarrow$ stable ✓

e) $h[k] = \sum_{m=-\infty}^{\infty} (-1)^m \delta[k-2m]$



i), ii) $h[k] \neq 0 \quad k < 0 \Rightarrow$ Not causal
Not memoryless

iii) $\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} 1 = \infty$ Not stable

10.19 if $h_1[k]$ and $h_2[k]$ are impulse rsponses of two inverse systems $\Rightarrow h_1[k] * h_2[k] = \delta[k]$

(b) $h_1[k] = (0.5)^k u[k] \qquad h_2[k] = \delta[k] - 0.5\,\delta[k-1]$

$$h_1[k] * h_2[k] \underset{\substack{\uparrow \\ \text{dist. Prop.}}}{=} (0.5)^k u[k] - 0.5\,(0.5)^{k-1} u[k-1]$$

$$= (0.5)^k u[k] - (0.5)^k u[k-1] = (0.5)^k \left[u[k] - u[k-1]\right]$$

$$= (0.5)^k \delta[k] = (0.5)^0 \delta[k] = \delta[k] \quad ✓$$

$\therefore$ $h_2[k]$ is inverse systen of $h_1[k]$

(d) $h_1[k] = k\,u[k] \qquad h_2[k] = \delta[k+1] - 2\delta[k] + \delta[k-1]$

$$h_1[k] * h_2[k] = (k+1)\,u[k+1] - 2k\,u[k] + (k-1)\,u[k-1]$$

$$= (1+k)\,u[k+1] + (k-1)\,u[k-1] - 2k\,u[k]$$

$$\left.\begin{array}{ll} k > 1 \Rightarrow & h_1[k] * h_2[k] = (1+k) + (k-1) - 2k = 0 \\ k = 1 \Rightarrow & h_1[k] * h_2[k] = 1 + k - 2k = 2 - 2 = 0 \\ k = 0 \Rightarrow & h_1[k] * h_2[k] = 1 + k = 1 \\ k \leq -1 \Rightarrow & h_1[k] * h_2[k] = 0 + 0 + 0 = 0 \end{array}\right\} \Rightarrow h_1[k] * h_2[k] = \delta[k]$$

$\therefore$ $h_2[k]$ is imp. response of inverse of systen $h_1[k]$

11.1

(iii) $x[k] = 3\sin\left(\frac{2\pi}{7}k + \frac{\pi}{4}\right)$ $k_0 = 7$ $\Omega_0 = \frac{2\pi}{7}$

Fourier Series Def. ⇒ $x[k] = \sum_{n=\langle k_0\rangle} D_n e^{jn\Omega_0 k}$ Ⓐ

Reconstruction

$$\to x[k] = 3\left(\frac{e^{j(\frac{2\pi}{7}k+\pi/4)} - e^{-j(\frac{2\pi}{7}k+\pi/4)}}{2j}\right)$$

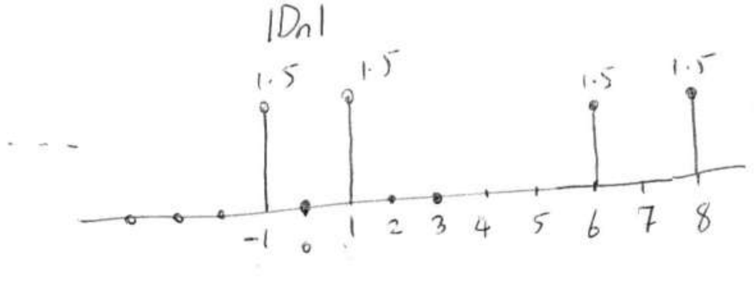
$$= \frac{3}{2j}e^{j\pi/4}e^{j\frac{2\pi}{7}k} - \frac{3}{2j}e^{-j\pi/4}e^{-j\frac{2\pi}{7}k}$$

$$= \frac{3}{2}e^{-j\frac{\pi}{4}}e^{j\frac{2\pi}{7}k} + \frac{3}{2}e^{j\pi/4}e^{-j\frac{2\pi}{7}k}$$

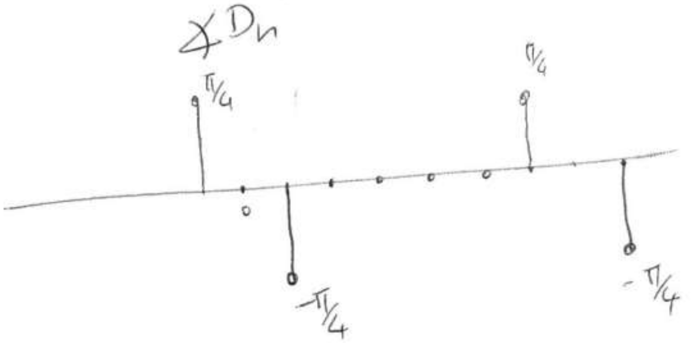
Ⓐ $\to$ $x[k] = D_{+1}e^{-j\frac{2\pi}{7}k} + D_{-1}e^{-j\frac{2\pi}{7}k}$

$$\Rightarrow D_n = \begin{cases} 1.5e^{-j\pi/4} & n=1 \\ 1.5e^{j\pi/4} & n=-1 \\ 0 & \text{otherwise} \end{cases} \quad \text{for } -3 \leq n \leq 3$$

and $D_n = D_{n+7}$



Periodic with period of 7



11.1)

(ii) $x[k] = \begin{cases} 1 & 0 \leq k \leq 2 \\ 0.5 & 3 \leq k \leq 5 \\ 0 & 6 \leq k \leq 8 \end{cases}$ $\qquad x[k+9] = x[k] \quad \forall k.$

$$\Downarrow$$

$$k_0 = 9 \quad \Omega_0 = \frac{2\pi}{9}$$

Definition $\Rightarrow$ F.S coeff : $D_n = \frac{1}{K_0} \sum_{\langle K_0 \rangle} x[k] e^{-j n \Omega_0 k}$

$$\Rightarrow D_n = \frac{1}{9} \sum_{k=0}^{2} e^{-jn\Omega_0} + \frac{1}{18} \sum_{k=3}^{5} e^{-jn\Omega_0}$$

$$= \frac{1}{9} \left(1 + e^{-jn\frac{2\pi}{9}} + e^{-j\frac{4\pi}{9}n} + 0.5 e^{-j\frac{6\pi}{9}n} + 0.5 e^{-j\frac{8\pi}{9}n} + 0.5 e^{-j\frac{10\pi}{9}n} \right]$$

$$e^{-jn\frac{2\pi}{9}} = a \rightarrow D_n = \frac{1}{9}\left[1 + a + a^2 + 0.5a^3 + 0.5a^4 + 0.5a^5\right]$$

$$= \frac{1}{9}\left[1 + a + a^2 + \tfrac{1}{2}a^3(1 + a + a^2)\right] =$$

$$= \frac{1}{9}\left[1 + a + a^2\right]\left[1 + \tfrac{1}{2}a^3\right]$$

$$= \frac{1}{9}\left(1 + e^{-jn\frac{2\pi}{9}} + e^{-jn\frac{4\pi}{9}}\right)\left(1 + \tfrac{1}{2}e^{-jn\frac{6\pi}{9}}\right)$$

$D_0 = \frac{1}{9}(1+1+1)(1+0.5) = 0.5 \angle 0$

$D_1 = 0.2437 \angle -70^\circ$

$D_2 = 0.1296 \angle -50^\circ$

$D_3 = 0$

$D_4 = 0.0846 \angle -10^\circ$

$D_5 = 0.0846 \angle -10^\circ$

$D_6 = 0$

$D_7 = 0.1296 \angle 50^\circ$

$D_8 = 0.2437 \angle 70^\circ$

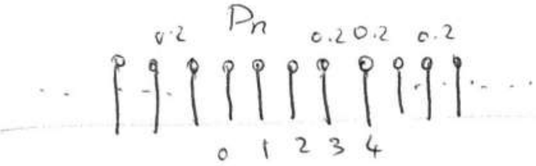
11.1

(v)

$$x[k] = \sum_{m=-\infty}^{\infty} \delta[k-5m] \qquad \Omega_0 = \frac{2\pi}{K_0} = \frac{2\pi}{5}$$

$$D_n = \frac{1}{K_0} \sum_{k=\langle K_0 \rangle} x[k] e^{-jn\Omega_0 k} = \frac{1}{5} \sum_{k=0}^{4} \delta[k] e^{-jn\Omega_0 k}$$

$$= \frac{1}{5} \; \forall \; 0$$



11.2

(iv)

$$D_n = (-1)^n \quad 0 \leqslant n \leqslant 7 \quad D_{n+8} = D_n \qquad \Omega_0 = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$x[k] = \sum_{n=0}^{7} D_n e^{jn\Omega_0 k} = \sum_{n=0}^{7} (-1)^n e^{jn\Omega_0 k} = \sum_{n=0}^{7} \left(-e^{j\pi/4 k}\right)^n$$

Geometric series $\overset{k \neq 4}{\Rightarrow}$ $x[k] = \dfrac{1 - \left(-e^{j\frac{\pi k}{4}}\right)^8}{1 + e^{j\frac{\pi k}{4}}} = \dfrac{1 - e^{j2\pi k}}{1 + e^{j\frac{\pi k}{4}}} = 0$ for $k \neq 4$

$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r} ; r \neq 1$$

if $k=4$ $\quad x[4] = \sum_{n=0}^{7} (1)^n = 8$ $\qquad \Rightarrow x[k] = \begin{cases} 8 & k=4 \\ 0 & k \neq 4 \end{cases} \quad 0 \leqslant k \leqslant 7$

and $x[k] = x[k+8]$

$$\Rightarrow x[k] = \sum_{m=-\infty}^{\infty} 8\,\delta[k-4-8m]$$

shifted impulse train

11.2
(v)

$$D_n = e^{jn\pi/4} \quad 0 \leqslant n \leqslant 7 \quad , \quad D_{n+8} = D_n$$

$$\Omega_0 = 2\pi/8 = \pi/4$$

$$x[k] = \sum_{n=0}^{7} \underbrace{e^{jn\pi/4}}_{D_n} e^{jn\frac{\pi}{4}k} = \sum_{n=0}^{7} \left(e^{j\frac{\pi}{4}} \cdot e^{j\frac{\pi}{4}k}\right)^n$$

$$= \sum_{n=0}^{7} \left(e^{j\frac{\pi}{4}(k+1)}\right)^n \underset{r\neq 1}{=} \frac{1-e^{j\frac{\pi}{4}(k+1)8}}{1-e^{j\frac{\pi}{4}(k+1)}} = \frac{1-e^{j2\pi(1+k)}}{1-e^{-j\frac{\pi}{4}(k+1)}} = 0 \quad Ⓐ$$

$$r = e^{j\frac{\pi}{4}(k+1)}$$

$$\text{if } r=1 \Rightarrow e^{j\frac{\pi}{4}(k+1)} = e^{j2k'\pi} \Rightarrow \frac{k+1}{4} = 2k' \Rightarrow k = 8k'-1 \xrightarrow{k'=1} k=7$$

Ⓐ → if $k \neq 7 \Rightarrow x[k]=0$

if $k=7 \Rightarrow x[k] = \sum_{n=0}^{7} (1)^n = 8$

$$\Rightarrow x[k] = \begin{cases} 0 & 0 \leqslant k \leqslant 6 \\ 8 & k=7 \end{cases} \quad \text{and} \quad x[k+8] = x[k]$$

11.3

(ix) $x[k] = e^{j(0.2\pi k + \pi/4)}$

$\sum_{k=-\infty}^{\infty} |x[k]| = \sum_{-\infty}^{\infty} 1 = \infty$: it is not absolutly summable ⇒ DTFT does not exist

(x) $x[k] = k3^{-k}u[k] + e^{j(0.2\pi k + \pi/4)}$

$$|x[k]| = \Big| \underbrace{k3^{-k}u[k]}_{\text{this is a decaying Component and always positive}} + e^{\underbrace{j(0.2\pi k + \pi/4)}_{\text{oscilating Component}}} \Big|$$

$\sum_{k=-\infty}^{\infty} |x(k)| = \infty$ ⇒ DTFT does not exist

(vii) $x[k] = \sum_{m=-\infty}^{\infty} \delta[k-5m-3]$ → Periodic with period of $K_0 = 5$ DTFT does not exist

$\Rightarrow \sum_{\langle K_0 \rangle} |x[k]| = 5 \Rightarrow \sum_{k=-\infty}^{\infty} |x[k]| = \infty$: DTFT does not exist

(vi) $$\sum_{k=-\infty}^{\infty} |x[k]| = \underbrace{x[0]}_{\downarrow \text{limit } 1/35} + \sum_{k=-\infty}^{\infty} \Big| \underbrace{\frac{\sin(\pi k/5)\,\sin(\pi k/7)}{\pi^2 k^2}}_{\text{even function}} \Big| = 1/35 + 2\sum_{k=1}^{\infty} \frac{|\sin \frac{\pi k}{5}|\,|\sin \frac{\pi k}{7}|}{|\pi^2 k^2|}$$

$$\sum_{k=-\infty}^{\infty} |x[k]| \leq 1/35 + \frac{2}{\pi^2}\sum_{k=1}^{\infty} \frac{1}{k^2} < 1/35 + 4/\pi^2 < \infty \quad \checkmark$$

the DTFT exist

11.5

(ii)

$$x_1[k] \longleftrightarrow X_1(\Omega)$$
$$x_2[k] \longrightarrow X_2(\Omega)$$

$$X(\Omega) = \sum_{k=-\infty}^{\infty} x[k] e^{-jk\Omega}$$

$$x_{3i}[k] = (k-5)^2 x_2[k-4]$$
$$= (k^2 - 10k + 25) x_2[k-4]$$

$$h[k] = x_2[k-4] \longrightarrow e^{-j4\Omega} X_2(\Omega) = H(\Omega) \quad Ⓑ$$

$$\frac{dX(\Omega)}{d\Omega} = \sum -kjx[k]\, e^{-jk\Omega}$$
$$\Downarrow$$
$$\boxed{k\,x[k] \xleftrightarrow{DTFT} j\frac{dX}{d\Omega}} \quad Ⓐ$$

(A,B)

$$\Rightarrow X_{ii}(\Omega) = (j)^2 \frac{d^2 H(\Omega)}{d\Omega^2} - 10j\frac{dH}{d\Omega} + 25 H(\Omega)$$

$$\Rightarrow X_{ii}(\Omega) = -\frac{d^2}{d\Omega^2}\left(e^{-j4\Omega} X_2(\Omega)\right) - 10j\frac{d}{d\Omega}\left(e^{-j4\Omega} X_2(\Omega)\right) + 25 e^{-j4\Omega} X_2(\Omega)$$

$$= -\frac{d}{d\Omega}\left(-4je^{-j4\Omega} X_2(\Omega) + e^{-j4\Omega} X'(\Omega)\right) - 10j\left(-4je^{-j4\Omega} X_2(\Omega) + e^{-j4\Omega} X'(\Omega)\right) + 25e^{-j4\Omega} X_2(\Omega)$$

$$= -(4j)^2 e^{-j4\Omega} X_2(\Omega) + 4je^{-j4\Omega} X_2'(\Omega) + 4je^{-j4\Omega} X_2'(\Omega) - e^{j4\Omega} X_2''(\Omega)$$
$$+ 40(j)^2 e^{-j4\Omega} X_2(\Omega) - 10je^{-j4\Omega} X_2'(\Omega) + 25e^{-j4\Omega} X_2(\Omega)$$

$$= \left[-X_2''(\Omega) - 2j X_2'(\Omega) + 31\, X_2(\Omega)\right] e^{-j4\Omega}$$

11.5 / (iii)

$x_3[k] = k e^{-j4k} x_1[3-k]$ table 11.6 Page 505

$$h[k] = x_1[-k] \xleftrightarrow{F} H(\Omega) = X_1(-\Omega)$$

$$g[k] = h[k-3] = x_1[3-k] \xleftarrow{F} G(\Omega) = H(\Omega)e^{-j3\Omega} = X_1(-\Omega)e^{-j3\Omega}$$

$$p[k] = e^{-j4k} g[k] = e^{-j4k} x_1[3-k] \xrightarrow{F} P(\Omega) = G(\Omega+4)$$
$$= X_1(-\Omega-4)e^{-j3(\Omega+4)}$$

$$x_3[k] = k p[k] \xrightarrow{F} j\frac{dp(\Omega)}{d\Omega} = j\Big[-X_1'(-\Omega-4)e^{-j(\Omega+4)3} +$$
$$-3X_1(-\Omega-4)e^{-j3(\Omega+4)}\Big]$$

$$\Rightarrow X_3(\Omega) = \left[-j\frac{dX_1(-(\Omega+4))}{d\Omega} - 3X_1(-(\Omega+4))\right]e^{-j3(\Omega+4)}$$

11.5 / (iv)

$$x_4[k] = \sum_{m=-\infty}^{\infty} x_1[k-4m] + x_2[k-6m] \xrightarrow{F} ?$$

$(K_0 = 4)$ $(k_0 = 6)$ Ⓑ

$\delta(ax) = \frac{1}{|a|}\delta(x)$

$$y[k] \triangleq \sum_{m=-\infty}^{\infty} x[k-k_0 m] \xrightarrow{F} X(\Omega)\sum_{m=-\infty}^{\infty} e^{-jk_0 m\Omega} = X(\Omega)\cdot 2\pi\sum_{m=-\infty}^{\infty}\delta(k_0\Omega - 2\pi m)$$
$$= X(\Omega)\frac{2\pi}{k_0}\sum_{m=-\infty}^{\infty}\delta\left(\Omega - \frac{2\pi m}{k_0}\right)$$
Ⓐ

$$\sum_{m=-\infty}^{\infty} e^{-jk_0 m\Omega} = 2\pi\sum_{m=-\infty}^{\infty}\delta(k_0\Omega - 2\pi m)$$

Because: $F\{1\} = 2\pi\sum_{m=-\infty}^{\infty}\delta(\Omega - 2\pi m) = \sum_{k=-\infty}^{\infty} e^{-j\Omega k}$

Ⓐ,Ⓑ →
$$X_4(\Omega) = \frac{2\pi}{4}X_1(\Omega)\sum_{m=-\infty}^{\infty}\delta\left(\Omega - \frac{2\pi m}{4}\right) + \frac{2\pi}{6}X_2(\Omega)\sum_{m=-\infty}^{\infty}\delta\left(\Omega - \frac{2\pi m}{6}\right)$$

$$= \frac{\pi}{2}\sum_{m-\infty}^{\infty} X_1\left(\frac{m\pi}{2}\right)\delta\left(\Omega - \frac{m\pi}{2}\right) + \frac{\pi}{3}\sum_{m=-\infty}^{\infty} X_2\left(\frac{m\pi}{3}\right)\delta\left(\Omega - \frac{m\pi}{3}\right)$$

11.6 / (ii)

$$X(\Omega) = \frac{2e^{-j2\Omega}}{(1-4e^{-j\Omega})^2(1-2e^{-j\Omega})}$$

Partial fraction expansion

$$X(\Omega) = \frac{A}{(1-4e^{-j\Omega})^2} + \frac{B}{(1-4e^{-j\Omega})} + \frac{C}{1-2e^{-j\Omega}}$$

$$A = \left.\frac{2e^{-j2\Omega}}{1-2e^{-j\Omega}}\right|_{e^{-j\Omega}=1/4} = \frac{2\cdot(1/4)^2}{1-2/4} = 1/4$$

$$C = \left.\frac{2e^{-j2\Omega}}{(1-4e^{-j\Omega})^2}\right|_{e^{-j\Omega}=1/2} = \frac{2(1/2)^2}{(1-4/2)^2} = \frac{1/2}{1} = 1/2$$

$$B: \quad A(1-2e^{-j\Omega}) + B(1-4e^{-j\Omega}) + C(1-4e^{-j\Omega})^2 \equiv 2e^{-j2\Omega}$$

$$\Rightarrow A+B+C = 0 \Rightarrow B = 0 - A - C = 0 - 1/4 - 1/2 = -3/4$$

$$\Rightarrow X(\Omega) = \frac{1/4}{(1-4e^{-j\Omega})^2} - \frac{3/4}{(1-4e^{-j\Omega})^2} + \frac{1/2}{1-2e^{-j\Omega}} \qquad *$$

$$\begin{cases} -a^k u[-k-1] \xrightarrow[|a|>1]{F} \dfrac{1}{1-ae^{-j\Omega}} \xRightarrow{d/d\Omega} -k a^k u[-k-1] \xleftrightarrow{F} j\dfrac{-jae^{-j\Omega}}{(1-a^{-j\Omega})^2} \\ \overset{\text{time shift}}{\Rightarrow} \quad -(k+1)\dfrac{a^{k+1}}{a} u[-k-2] = -(k+1)a^k u[-k-2] \xleftrightarrow[|a|>1]{F} \dfrac{1}{(1-ae^{-j\Omega})^2} \end{cases}$$

$$\overset{*}{\Rightarrow} x(t) = -\frac{1}{4}(k+1)\,4^k u[-k-2] + \frac{3}{4}\,4^k u[-k-1] - \frac{1}{2}\,2^k u[-k-1]$$

$$= \left\{-\frac{1}{4}(k+1) + \frac{3}{4}\,4^k - \frac{1}{2}2^k\right\} u[-k-2] + \underbrace{\left(\frac{3}{16} - \frac{1}{4}\right)}_{-1/16} \delta[-k-1]$$

11.6 / (iii)

$$X(\Omega) = 8\sin(7\Omega)\cos(9\Omega)$$

$$\boxed{\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]}$$

$$\Rightarrow \quad X(\Omega) = 4\sin(16\Omega) + 4\sin(-2\Omega)$$

$$= -2j\,e^{j16\Omega} + 2j\,e^{-j16\Omega} + 2j\,e^{j2\Omega} - 2j\,e^{-j2\Omega}$$

$$\boxed{\delta[k] \xrightarrow{F} 1 \Rightarrow \delta[k-k_0] \xrightarrow{f} e^{-jk_0\Omega}}$$

$$\Rightarrow X(\Omega) = -j2\left\{\delta[k+16] - \delta[k-16] - \delta[k+2] + \delta[k-2]\right\}$$

11.12 $$y[k]+y[k-1]+\tfrac{1}{4}y[k-2]=x[k]-x[k-2]$$

i)

$$\xrightarrow[\text{from both sides}]{\text{Fourie}} \quad Y(\Omega)+e^{-j\Omega}Y(\Omega)+\tfrac{1}{4}e^{-j2\Omega}Y(\Omega)=X(\Omega)-e^{-j2\Omega}X(\Omega)$$

$$H(\Omega)=\frac{Y(\Omega)}{X(\Omega)}=\frac{1-e^{-j2\Omega}}{1+e^{-j\Omega}+\tfrac{1}{4}e^{-j2\Omega}}$$

ii) impulse response $H(\Omega)\xrightarrow{F^{-1}}h[k]=?$

$$1+e^{-j\Omega}+\tfrac{1}{4}e^{-j2\Omega}=0 \Rightarrow \text{roots}=\frac{-1\pm\sqrt{1-1}}{2}$$

$$\Rightarrow H(\Omega)=\frac{1-e^{-j2\Omega}}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}=\underbrace{\frac{1}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}}_{H_1(\Omega)}-\underbrace{\frac{e^{-j2\Omega}}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}}_{H_2(\Omega)}$$

$$a^k u[k]\xrightarrow[|a|<1]{F}\frac{1}{1-ae^{-j\Omega}} \quad \xrightarrow{d/d\Omega} \quad k\,a^k u[k]\xleftarrow{F} j\cdot\frac{-(-a(-j))e^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$

$$=\frac{a\,e^{-j\Omega}}{(1-ae^{-j\Omega})^2}$$

$$\overset{\text{time}}{\underset{\text{shift}}{\Rightarrow}} (k+1)a^k u[k+1]\xleftarrow{F}\frac{1}{(1-ae^{-j\Omega})^2} \quad \circledast$$

$$\Rightarrow\begin{cases} H_1(\Omega)=\dfrac{1}{1+\frac{1}{2}e^{-j\Omega}}\xrightarrow{F^{-1}}h_1[k]=(k+1)\left(-\tfrac{1}{2}\right)^k u[k+1] \\ H_2(\Omega)=\dfrac{-e^{-j2\Omega}}{1+\frac{1}{2}e^{-j\Omega}}\xrightarrow{F^{-1}}h_2[k]=-(k-1)\left(-\tfrac{1}{2}\right)^{k-2}u[k-1] \end{cases}$$

$$\Rightarrow h[k]=(k+1)\left(-\tfrac{1}{2}\right)^k u[k+1]-(k-1)\left(-\tfrac{1}{2}\right)^{k-2}u[k-1]$$

$$=(k+1)\left(-\tfrac{1}{2}\right)^k u[k]-(k-1)\left(-\tfrac{1}{2}\right)^{k-2}u[k-2]$$

$$=\delta[k]-\delta[k-1]+\left[(k+1)\left(-\tfrac{1}{2}\right)^k-(k-1)\left(-\tfrac{1}{2}\right)^{k-2}\right]u[k-2]$$

$$=\delta[k]-\delta[k-1]+\left[k+1-4k+4\right]\left(-\tfrac{1}{2}\right)^k u[k-2]$$

11.12

(iii) and (iv)

$$x[k] = \left(\tfrac{1}{2}\right)^k U[k] \xrightarrow{F} X(\Omega) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}}$$

$$H(\Omega) = \frac{1-e^{-j2\Omega}}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}$$

$$Y(\Omega) = X(\Omega)H(\Omega) = \frac{1-e^{-j2\Omega}}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2\left(1-\frac{1}{2}e^{-j\Omega}\right)}$$

Partial Fraction expansion $\Rightarrow Y(\Omega) = \dfrac{A}{1-\frac{1}{2}e^{-j\Omega}} + \dfrac{B}{1+\frac{1}{2}e^{-j\Omega}} + \dfrac{C}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}$

$$A = \frac{1-e^{-j2\Omega}}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}\Bigg|_{e^{-j\Omega}=\left(\frac{1}{2}\right)^{-1}=+2} = \frac{1-4}{4} = -3/4$$

$$C = \frac{1-e^{-j2\Omega}}{1+\frac{1}{2}e^{-j\Omega}}\Bigg|_{e^{-j\Omega}=-2} = \frac{1-4}{2} = -3/2$$

$$A\left(1+\tfrac{1}{2}e^{-j\Omega}\right)^2 + B\left(1+\tfrac{1}{2}e^{-j\Omega}\right) + C\left(1-\tfrac{1}{2}e^{-j\Omega}\right) \equiv 1-e^{-j2\Omega}$$

$$A+B+C=1 \Rightarrow B = 1-(A+C) = 1-\left(-3/4-3/2\right) = 1+9/4 = 13/4$$

$$\Rightarrow Y(\Omega) = \frac{-3/4}{1-\frac{1}{2}e^{-j\Omega}} + \frac{13/4}{1+\frac{1}{2}e^{-j\Omega}} + \frac{-3/2}{\left(1+\frac{1}{2}e^{-j\Omega}\right)^2}$$

$$\Rightarrow y[k] = -3/4\left(\tfrac{1}{2}\right)^k U[k] + 13/4\left(-\tfrac{1}{2}\right)^k U[k] - 3/2\,(k+1)\underbrace{\left(-\tfrac{1}{2}\right)^k U[k+1]}_{@\ k=-1 \Rightarrow 0}$$

$$\Rightarrow y[k] = \left[-\tfrac{3}{4}\left(\tfrac{1}{2}\right)^k + \tfrac{13}{4}\left(-\tfrac{1}{2}\right)^k - \tfrac{3}{2}(k+1)\left(-\tfrac{1}{2}\right)^k\right]U[k]$$

11.13

(iii) $x[k] = u[k] - u[k-9] \quad h[k] = 3^k u[-k+4]$

$$X(\Omega) = \sum_{k=0}^{8} e^{-j\Omega k} = \frac{1-e^{-j\Omega 9}}{1-e^{-j\Omega}} \quad ⊛$$

$$H(\Omega) = \sum_{k=-\infty}^{\infty} 3^k u[-k+4] e^{-j\Omega k} = \sum_{k=-\infty}^{4} 3^k e^{-j\Omega k} = \sum_{k=-4}^{\infty} \left(3^{-1} e^{+j\Omega}\right)^k$$

$$= \frac{\left(\tfrac{1}{3}e^{j\Omega}\right)^{-4} - \left(\tfrac{1}{3}e^{j\Omega}\right)^{\infty}}{1-\tfrac{1}{3}e^{j\Omega}} = 3^4 \frac{e^{-j4\Omega}}{1-\tfrac{1}{3}e^{j\Omega}} \qquad \sum_{n=N_1}^{N_2} r^n = \frac{r^{N_1} - r^{N_2+1}}{1-r}$$

$$= -3^4 \frac{e^{-j5\Omega}}{\tfrac{1}{3} - e^{-j\Omega}} = -3^5 \frac{e^{-j5\Omega}}{1-3e^{-j\Omega}}$$

$$Y(\Omega) = H(\Omega) X(\Omega)$$

$$= -3^5 \frac{\left(1-e^{-j\Omega 9}\right) e^{-j5\Omega}}{\left(1-e^{-j\Omega}\right)\left(1-3e^{-j\Omega}\right)}$$

$$= 3^5 \left(-e^{-j5\Omega} + e^{-j14\Omega}\right) \frac{1}{(1-e^{-j\Omega})(1-3e^{-j\Omega})} \qquad \text{Partial fraction}$$

$$= 3^5 \left(-e^{-j5\Omega} + e^{-j14\Omega}\right) \underbrace{\left[\frac{A}{1-e^{-j\Omega}} + \frac{B}{1-3e^{-j\Omega}}\right]} \qquad \left.\begin{array}{l} A+B=1 \\ -3A-B=0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} A=-1/2 \\ B=3/2 \end{array}\right.$$

$$= 3^5 e^{-j5\Omega} \underbrace{\left(\tfrac{1}{2}\right) \frac{-e^{-j9\Omega}+1}{1-e^{-j\Omega}}}_{\downarrow\, (F^{-1})\ ⊛} + \underbrace{3^5 \cdot \tfrac{3}{2} \frac{\left(-e^{-j5\Omega}+e^{-j14\Omega}\right)}{1-3e^{-j\Omega}}}_{\downarrow\, (F^{-1})\ \text{time shift Property}}$$

$$y[k] = 3^5\left(\tfrac{1}{2}\right) x(k-5) + 3^5\left(\tfrac{3}{2}\right)\left[-(3)^{k-5} u[k-5] + (3)^{k-14} u[k-14]\right]$$

11.12

(iii) $h[k] = \delta[k] - \delta[k-1] + (5-3k)\left(-\frac{1}{2}\right)^k u[k-2]$

$$= \delta[k] - \delta[k-1] + (5-3k)\left(-\tfrac{1}{2}\right)^k u[k] - 5\left(-\tfrac{1}{2}\right)^0 \delta[k] - [5-3]\left(-\tfrac{1}{2}\right)\delta[k-1]$$

$$\left.\begin{aligned} h[k] &= -4\,\delta[k] - (3k-5)(-0.5)^k u[k] \\ x[k] &= \left(\tfrac{1}{2}\right)^k u[k] \end{aligned}\right\} \quad y[k] = x[k] * h[k]$$

$$\Rightarrow y[k] = \underbrace{-4\left(\tfrac{1}{2}\right)^k u[k]}_{y_3[k]\ \checkmark} \underbrace{- 3k(-0.5)^k u[k] * \left(\tfrac{1}{2}\right)^k u[k]}_{y_1[k]} + \underbrace{5\,(-0.5)^k u[k] * \left(\tfrac{1}{2}\right)^k u[k]}_{y_2[k]}$$

$$\begin{cases} a^k u[k] * b^k u[k] = \begin{cases} (k+1)\,a^k u[k] & a \neq b \\ \frac{1}{a-b}\left(a^{k+1} - b^{k+1}\right) u[k] & a \neq b \end{cases} \\ k a^k u[k] * b^k u[k] = \begin{cases} \frac{k(k+1)}{2}\, a^k u[k] & a = b \\ \frac{a}{(a-b)^2}\left[k a^{k+1} - (k+1) a^k b + b^{k+1}\right] u[k] & a \neq b \end{cases} \end{cases}$$

$a = -0.5$
$b = 0.5$

$$y_2[k] = 5\,\frac{(0.5)^{k+1} - (-0.5)^{k+1}}{1}\, u[k] \quad \checkmark$$

$$y_1[k] = -3\,\frac{-0.5}{(-0.5-0.5)^2}\left[k(-0.5)^{k+1} - (k+1)(-0.5)^k(0.5) + (0.5)^{k+1}\right] u[k]$$

✓

$$y[k] = y_1[k] + y_2[k] + y_3[k] \quad \checkmark\checkmark$$

11.14

$$H(\Omega) = \frac{1}{1+3e^{-j\Omega}} = \frac{1}{1+3e^{-j\Omega}} \cdot \frac{1+3e^{j\Omega}}{1+3e^{j\Omega}}$$

$$= \frac{1+3e^{j\Omega}}{10+6\cos\Omega} = \frac{1+3\cos\Omega}{10+6\cos\Omega} + j\frac{3}{10+6\cos\Omega}\sin\Omega$$

$$\Rightarrow \operatorname{Re}\{H(\Omega)\} = \frac{1+3\cos\Omega}{10+6\cos\Omega} \qquad \operatorname{Im}\{H(\Omega)\} = \frac{3\sin\Omega}{10+6\cos\Omega}$$

$$|H(\Omega)| = \sqrt{\frac{1}{(1+3\cos\Omega)^2+(3\sin\Omega)^2}} = \frac{1}{\sqrt{10+6\cos\Omega}}$$

$$\measuredangle H(\Omega) = \tan^{-1}\left[\frac{3\sin\Omega/10+6\cos\Omega}{(1+3\cos\Omega)/10+6\cos\Omega}\right] = \tan^{-1}\left[\frac{3\sin\Omega}{1+3\cos\Omega}\right]$$

11.16

$$h[k] = 3\delta[k+3] - 2\delta[k+2] + \delta[k+1] + 5\delta[k]$$
$$- \delta[k-1] - 2\delta[k-2] - 3\delta[k-3] + 4\delta[k-4]$$

i) $H(\Omega)\Big|_{\Omega=0} = \sum_{-\infty}^{\infty} h[k] = 3-2+1+5-1-2-3+4 = 5$

ii) $H(\Omega)\Big|_{\Omega=\pi} = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\pi k} = \sum_{k=-\infty}^{\infty} (-1)^k h[k] = -3-2-1+5+1-2+3+4 = 5$

iv) $h[k] = \frac{1}{2\pi} \oint_{\Omega=-\pi}^{\pi} H(\Omega) e^{jk\Omega} d\Omega \Rightarrow 2\pi h[0] = \int_{-\pi}^{\pi} H(\Omega) d\Omega = 10\pi$

v) $H(\Omega) \xleftrightarrow{F^{-1}} h[k] \Rightarrow H(-\Omega) \xleftrightarrow{F^{-1}} h[-k] \Rightarrow h[-k] = 3\delta[-k+3] - 2\delta[-k+2] + \delta[-k+1] + 5\delta[k]$
$- \delta[-k-1] - 2\delta[-k-2] - 3\delta[-k-3] + 4\delta[-k-4]$

11.16 Continue..

$$\text{Re}\{H(\Omega)\} = \text{Re}\left\{\sum_{k=-\infty}^{\infty} h[k]e^{-j\Omega k}\right\} \underset{\substack{\uparrow \\ h[k]:\text{ is real}}}{=} \sum_{k=-\infty}^{\infty} h[k]\,\text{Re}\{e^{-j\Omega k}\}$$

$$= \sum_{k=-\infty}^{\infty} h[k]\cos(\Omega k) = \sum_{k=-\infty}^{\infty} h[k]\left(\frac{e^{j\Omega k}+e^{-j\Omega k}}{2}\right)$$

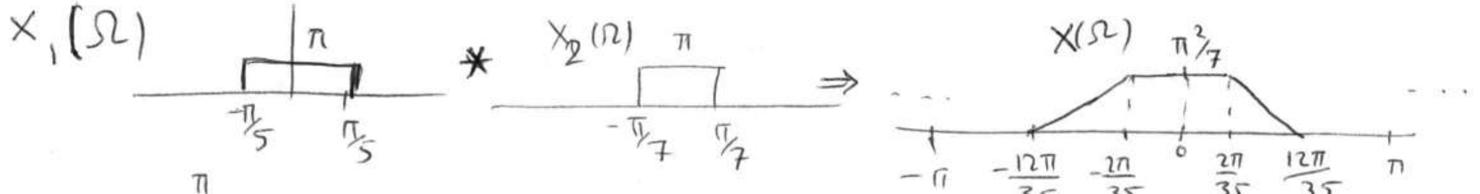
$$= \frac{1}{2}\sum_{-\infty}^{\infty} h[k]e^{j\Omega k} + \frac{1}{2}\sum_{k=-\infty} h[k]e^{-j\Omega k}$$

$$= \frac{1}{2}\left[H(\Omega) + H(-\Omega)\right] \xrightarrow{F^{-1}} \frac{1}{2}\left[h[k]+h[-k]\right]$$

11.17 ~~Parseval's theorem~~ typo mistake

$$\sum_{k=-\infty}^{\infty} \underbrace{\frac{\sin\left(\frac{k\pi}{5}\right)\sin\left(\frac{k\pi}{7}\right)}{k^2}}_{x[k]} = \sum_{k=-\infty}^{\infty} x[k] = X(\Omega)\Big|_{\Omega=0}$$

$$x[k] = x_1[k]\cdot x_2[k] = \frac{\sin\frac{k\pi}{5}}{k}\cdot\frac{\sin\frac{k\pi}{7}}{k} \xrightarrow{F} X(\Omega) = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X_1(\theta)\,X_2(\Omega-\theta)\,d\theta$$



$$X(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\theta)X_2(\theta)\,d\theta = \frac{\pi^2\times(\pi/7\times 2)}{2\pi} = \pi^2/7$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} \frac{\sin\left(\frac{k\pi}{5}\right)\sin\left(\frac{k\pi}{7}\right)}{k^2} = \pi^2/7$$

11.19

$$x[k] = 4^{-k}u[k] + 3^{-k}u[k] \longrightarrow \underset{H(\Omega)}{\boxed{LTID}} \longrightarrow y[k] = 2\left(\tfrac{1}{4}\right)^{k}u[k] - 4\left(\tfrac{3}{4}\right)^{k}u[k]$$

i)
$$X(\Omega) = \frac{1}{1-\frac{1}{4}e^{-j\Omega}} + \frac{1}{1-\frac{1}{3}e^{-j\Omega}} = \frac{2(1-\frac{7}{24}e^{-j\Omega})}{(1-\frac{1}{4}e^{-j\Omega})(1-\frac{1}{3}e^{-j\Omega})}$$

$$Y(\Omega) = 2\,\frac{1}{1-\frac{1}{4}e^{-j\Omega}} - 4\,\frac{1}{1-\frac{3}{4}e^{-j\Omega}} = \frac{-2(1+\frac{1}{4}e^{-j\Omega})}{(1-\frac{1}{4}e^{-j\Omega})(1-\frac{3}{4}e^{-j\Omega})}$$

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{-(1+\frac{1}{4}e^{-j\Omega})(1-\frac{1}{3}e^{-j\Omega})}{(1-\frac{3}{4}e^{-j\Omega})(1-\frac{7}{24}e^{-j\Omega})}$$

ii) partial fraction expansion

$$+\left(\tfrac{1}{12}+\frac{25\times 32}{84\times 24}\right)e^{-j\Omega}$$

$$H(\Omega) = \frac{-1+\frac{1}{12}e^{-j\Omega}+\frac{1}{12}e^{-j2\Omega}}{1-\frac{25}{24}e^{-j\Omega}+\frac{21}{96}e^{-j2\Omega}} = \frac{\frac{32}{84}\left(1-\frac{25}{24}e^{-j\Omega}+\frac{7}{32}e^{-j2\Omega}\right)+\left(-1-\frac{32}{84}\right)}{1-\frac{25}{24}e^{-j\Omega}+\frac{7}{32}e^{-j2\Omega}}$$

7/32

$$= \frac{8}{21} + \frac{-\frac{29}{21}+\frac{121}{252}e^{-j\Omega}}{(1-\frac{3}{4}e^{-j\Omega})(1-\frac{7}{24}e^{-j\Omega})} = \frac{8}{21} + \frac{A}{1-\frac{3}{4}e^{-j\Omega}} + \frac{B}{1-\frac{7}{24}e^{-j\Omega}}$$

$$A = \left.\frac{-\frac{29}{21}+\frac{121}{252}e^{-j\Omega}}{1-\frac{7}{24}e^{-j\Omega}}\right|_{e^{-j\Omega}=\frac{4}{3}} = -1.2121$$

$$B = \left.\frac{-\frac{29}{21}+\frac{121}{252}e^{-j\Omega}}{1-\frac{3}{4}e^{-j\Omega}}\right|_{e^{-j\Omega}=\frac{24}{7}} = -0.1688$$

$$\Rightarrow h[k] = \frac{8}{21}\delta[k] - 1.2121\left(\tfrac{3}{4}\right)^{k}u[k] - 0.1688\left(\tfrac{7}{24}\right)^{k}u[k]$$

#19 Difference equation?

(iii)

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{-1 + \frac{1}{12} e^{-j\Omega} + \frac{1}{12} e^{-j2\Omega}}{1 - \frac{25}{24} e^{j\Omega} + \frac{7}{32} e^{-j2\Omega}}$$

$$Y(\Omega) - \frac{25}{24} e^{-j\Omega} Y(\Omega) + \frac{7}{32} e^{-j2\Omega} Y(\Omega) = -X(\Omega) + \frac{1}{12} e^{-j\Omega} X(\Omega) + \frac{1}{12} e^{-j2\Omega} X(\Omega)$$

$$\Big\downarrow F^{-1}$$

$$y[k] - \frac{25}{24} y[k-1] + \frac{7}{32} y[k-2] = -x[k] + \frac{1}{12} x[k-1] + \frac{1}{12} x[k-2]$$

(iv) Causal ?

$h[k]$ from part (ii) $\Rightarrow$ $h[k] = 0$ for $k < 0$

$\Rightarrow$ the system is causal.

13.1
(iv)

$$x_4[k] = 3^{k+1}\cos\left(\frac{\pi}{3}k - \frac{\pi}{4}\right) u[-k+5] \xrightarrow{Z} ?$$

$$x_4[k] = 3^{k+1} \cdot \frac{e^{j\pi/3\, k - j\pi/4} + e^{-j\pi/3\, k + j\pi/4}}{2}\, u[-k+5]$$

$$= \frac{3e^{-j\pi/4}}{2} \cdot \left(3e^{j\pi/3}\right)^k u[-k+5] + \frac{3e^{j\pi/4}}{2}\left(3e^{-j\pi/3}\right)^k u[-k+5]$$

$$-a^k u[-k-1] \xleftrightarrow{Z} \frac{1}{1-az^{-1}} \qquad |z|<|a|$$

time shift $\Rightarrow$ $$-a^{k-6} u[-(k-6)-1] \xleftrightarrow{Z} \frac{z^{-6}}{1-az^{-1}} \qquad |z|<|a|$$

$$= -a^{k-6} u[-k+5]$$

$$\Rightarrow \quad -a^k u[-k+5] \xleftrightarrow{Z} \frac{(z/a)^{-6}}{1-az^{-1}} \qquad |z|<|a|$$

$$\Rightarrow \quad X_4(z) = \frac{3e^{-j\pi/4}}{2} \cdot \frac{-\left(z/3e^{j\pi/3}\right)^{-6}}{1-3e^{j\pi/3}z^{-1}} + \frac{3e^{j\pi/4}}{2} \cdot \frac{-\left(z/3e^{-j\pi/3}\right)^{-6}}{1-3e^{-j\pi/3}z^{-1}}$$

ROC: $|z| < |3e^{j\pi/3}| = 3$

$$\Rightarrow \quad X_4(z) = \frac{3^7}{2\sqrt{2}}(1-j)\, \frac{-z^{-6}\, e^{j6\pi/3}}{1-3e^{j\pi/3}z^{-1}} + \frac{3^7}{2\sqrt{2}}(1+j)\, \frac{-z^{-6}\, e^{-j6\pi/3}}{1-3e^{-j\pi/3}z^{-1}}$$

$$= -\frac{3^7}{2\sqrt{2}}\left[(1-j)\,\frac{z^{-6}}{1-3e^{j\pi/3}z^{-1}} + (1+j)\,\frac{z^{-6}}{1-3e^{j\pi/3}z^{-1}}\right]$$ ✓

ROC: $|z| < 3$ ✓✓

13.3

(iii) Partial fraction expantion

$$X_3(Z) = \frac{Z^2+2}{(Z-0.3)(Z+0.4)(Z-0.7)} \qquad \text{Causal} \quad |Z| > \overset{\Downarrow}{0.7}$$

order of num. is less than denom.

$$\Rightarrow X_3(Z) = \frac{A}{Z-0.3} + \frac{B}{Z+0.4} + \frac{C}{Z-0.7}$$

$$A = \left.\frac{Z^2+2}{(Z+0.4)(Z-0.7)}\right|_{Z=0.3} = \frac{0.09+2}{0.7 \times (-0.4)} = -\frac{2.09}{0.28} = -7.464$$

$$B = \left.\frac{Z^2+2}{(Z-0.3)(Z-0.7)}\right|_{Z=-0.4} = \frac{0.16+2}{-0.7 \times (-1.1)} = 2.8052$$

$$C = \left.\frac{Z^2+2}{(Z-0.3)(Z+0.4)}\right|_{Z=0.7} = \frac{2.49}{0.4 \times 1.1} = 5.6591$$

$$X_3(Z) = \frac{-7.464\,Z^{-1}}{1-0.3Z^{-1}} + \frac{2.8052\,Z^{-1}}{1+0.4Z^{-1}} + \frac{5.6591\,Z^{-1}}{1-0.7Z^{-1}}$$

$$\Rightarrow x_3[k] = -7.464(0.3)^{k-1}u[k-1] + 2.8052(-0.4)^{k-1}u[k-1]$$

$$+ 5.6591(0.7)^{k-1}u[k-1]$$

13.3

(iv)

$$X_4(z) = \frac{z^2+2}{(z-0.3)(z+0.4)^2} \qquad \text{Causal}$$

$$X_4(z) = \frac{A}{z-0.3} + \frac{B}{(z+0.4)^2} + \frac{C}{(z+0.4)}$$

$$A = \left.\frac{z^2+2}{z+0.4}\right|_{z=+0.3} = \frac{2.09}{0.7} = 2.9857$$

$$B = \left.\frac{z^2+2}{z-0.3}\right|_{z=-0.4} = \frac{2.16}{-0.7} = -3.0857$$

$$A(z+0.4)^2 + B(z-0.3) + C(z+0.4)\overset{(z-0.3)}{\wedge} \equiv z^2+2$$

$$.Az^2 + Cz^2 \equiv z^2 \Rightarrow A+C=1 \Rightarrow C = 1-A = -1.9857$$

$$\text{Page } 589 \Rightarrow k a^k u[k] \xleftrightarrow{z} \frac{az^{-1}}{(1-az^{-1})^2} \qquad \text{Roc: } |z|>|a|$$

$$\Rightarrow k a^{k-1} u[k] \xleftrightarrow{z} \frac{z^{-1}}{(1-az^{-1})^2}$$

$$X_4(z) = \frac{Az^{-1}}{1-0.3z^{-1}} + \frac{Bz^{-2}}{(1+0.4z^{-1})^2} + \frac{Cz^{-1}}{1+0.4z^{-1}}$$

$$\xrightarrow{z^{-1}} x_4[k] = 2.9857\,(0.3)^{k-1}u[k-1] - 3.0857\,(k-1)(-0.4)^{k-2}u[k-1] +$$

$$+ (-1.9857)(-0.4)^k u[k]$$

ROC: $|z| > 0.4$ causal

13.5

(b) $$x[k] = r\alpha^k \sin(\Omega_0 k + \theta)\, u[k] \xleftrightarrow[?]{z} \frac{A + Bz^{-1}}{1 + 2\gamma z^{-1} + \alpha^2 z^{-2}}$$

$$X(z) = \frac{1}{1 - z^{-1} + z^{-2}} \equiv \frac{A + Bz^{-1}}{1 + 2\gamma z^{-1} + \alpha^2 z^{-2}}$$

$$\Rightarrow A = 1,\ B = 0,\ \gamma = -1/2,\ \alpha = 1$$

$$\Rightarrow \Omega_0 = \cos^{-1}(-\gamma/\alpha) = \cos^{-1}(1/2) = \pi/3 \text{ rad}$$

$$\theta = \tan^{-1}\left(\frac{A\sqrt{\alpha^2 - \gamma^2}}{B - A\gamma}\right) = \tan^{-1}\left(\frac{1\sqrt{1 - 1/4}}{1/2}\right) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

$$r = \sqrt{\frac{A^2\alpha^2 + B^2 - 2AB\gamma}{\alpha^2 - \gamma^2}} = \sqrt{\frac{1 + 0 - 2\times 0}{1 - 1/4}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow x[k] = \frac{2}{\sqrt{3}} \sin(\pi/3\, k + \pi/3)\, u[k] \quad ✓$$

13.4 (iv)

$$X_4(z) = \frac{z^2+2}{(z-0.3)(z+0.4)^2} = \frac{z^2+2}{(z-0.3)(z^2+0.8z+0.16)}$$

$$= \frac{z^2+2}{z^3+0.5z^2-0.08z-0.048} \quad |z| > 0.4 \text{ due to causality}$$

$$\begin{array}{r|l} z^3+0.5z^2-0.08z-0.048 & z^{-1}-0.5z^{-2}+2.33z^{-3}-1.157z^{-4}+\cdots \\ \hline & z^2+0z+2 \\ \ominus & z^2+0.5z-0.08-0.048z^{-1} \\ \hline & -0.5z+2.08+0.048z^{-1} \\ \ominus & -0.5z-0.25z^0+0.040z^{-1}+0.024z^{-2} \\ \hline & 2.33+0.008z^{-1}-0.024z^{-2} \\ \ominus & 2.33+1.165z^{-1}-0.1864z^{-2}-0.1118z^{-3} \\ \hline & -1.157z^{-1}+0.1624z^{-2}+0.1118z^{-3} \end{array}$$

$$\Rightarrow X_4(z) = z^{-1}-0.5z^{-2}+2.33z^{-3}+1.157z^{-4}+\cdots$$

$$\Rightarrow x_4[k] = \delta[k]-0.5\,\delta[k-2]+2.33\,\delta[k-3]-1.157\,\delta[k-4]+\cdots$$

13.7

$$x_5[k] = \begin{cases} 1 & k=0,1 \\ 2 & k=2,5 \\ 0 & \text{o.w.} \end{cases} \xrightarrow[\text{Deffinition}]{z} X_5(z) = 1 + z^{-1} + 2z^{-2} + 2z^{-5}$$

ROC: entire z-plane $z \neq 0$

$$g[k] = x_5[k-10] \longrightarrow G(z) = z^{-10} X_5(z)$$

$$= z^{-10} + z^{-9} + 2z^{-12} + 2z^{-15} \quad \forall z \neq 0$$

13.10

(i) $x[k] = \left(\frac{5}{6}\right)^k u[k-6]$

$$a^k u[k] \longrightarrow \frac{1}{1-az^{-1}} \quad |z| > |a|$$

$$a^{k-6} u[k-6] \longleftrightarrow \frac{z^{-6}}{1-az^{-1}} \quad \Rightarrow \quad a^k u[k-6] \longleftrightarrow \frac{a^6 z^{-6}}{1-az^{-1}} \quad |z| > |a|$$

$$\Rightarrow X(z) = \frac{(5/6)^6 z^{-6}}{1 - 5/6\, z^{-1}} \qquad |z| > 5/6$$

(ii) $x[k] = k\,(2/9)^k u[k]$

$$(2/9)^k u[k] \longleftrightarrow \frac{1}{1 - 2/9\, z^{-1}} \quad |z| > 2/9$$

$$\Downarrow d/dz$$

$$k\,(2/9)^k u[k] \longrightarrow -z \frac{d}{dz}\left(\frac{1}{1-2/9\, z^{-1}}\right) = -z \frac{-2/9\, z^{-2}}{(1-\frac{2}{9}z^{-1})^2} \quad |z| > 2/9$$

$$\Rightarrow X(z) = \frac{2/9\, z^{-1}}{(1-2/9\, z^{-1})^2} \qquad \text{ROC: } |z| > 2/9$$

13.10
(iii)

$$x[k] = kU[k]$$

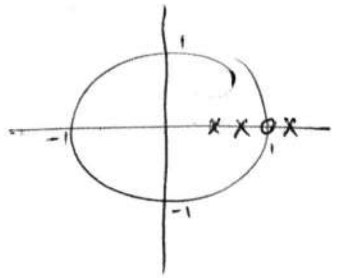
Time accumulation $\sum_{m=0}^{k} x_1[m] \longrightarrow \frac{z}{z-1} X_1(z)$

$$x[k] = kU[k] = \sum_{m=0}^{k} U[m] - U[k] \rightarrow X(z) = \frac{z}{z-1} \cdot Z\{U[k]\} - Z\{U[k]\}$$

$$\Rightarrow X(z) = \frac{z}{z-1} * \frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1}} = \frac{1}{(1-z^{-1})^2} - \frac{1}{(1-z^{-1})} = \frac{z^{-1}}{(1-z^{-1})^2}$$

13.11

$$H(z) = \frac{1-z^{-1}}{(1-0.5z^{-1})(1-0.75z^{-1})(1-1.25z^{-1})}$$



$$H(z) = \frac{A}{1-0.5z^{-1}} + \frac{B}{1-0.75z^{-1}} + \frac{C}{1-1.25z^{-1}}$$

$$A = H(z)\cdot(1-0.5z^{-1})\Big|_{z=0.5} = \frac{1-0.5^{-1}}{(1-0.75\times2)(1-1.25\times2)} = \frac{-1}{-0.5\times(-1.5)} = -4/3$$

$$B = \frac{1-4/3}{(1-3/3)(1-1.25\cdot 4/3)} = \frac{-1/3}{1/3(-2/3)} = 3/2$$

$$C = 1 - A - B = 1 + 4/3 - 3/2 = 7/3 - 3/2 = 5/6$$

Stable $\Rightarrow$ ROC Contains the unit circle

$$\Rightarrow \boxed{0.75 < |z| < 1.25}$$

$$\Rightarrow h[k] = -4/3\,(0.5)^k U[k] + 3/2\,(0.7)^k U[k] - 5/6\,(1.25)^k U[-k-1]$$

the system is not causal : $h[k] \neq 0 \quad k<0$ eg. $h[-1] = -5/6\,(1.25)^{-1}$

13.13

$$x[k] = \left(1/3\right)^{k} U[k] - \left(\frac{1}{4}\right)^{k-1} U[k] = \left(\frac{1}{3}\right)^{k} U[k] - 4\left(\frac{1}{4}\right)^{k} U[k]$$

$$y[k] = \left(1/4\right)^{k} U[k]$$

(i)

$$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}} - \frac{4}{1-\frac{1}{4}z^{-1}} = \frac{-3+\left(4/3 - 1/4\right)z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$Y(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-1/3\,z^{-1}}{-3+\frac{13}{12}z^{-1}} = \frac{-1/3 + 1/9\,z^{-1}}{1-\frac{13}{36}z^{-1}} = \frac{-1/3}{1-\frac{13}{36}z^{-1}} + \frac{1/9\,z^{-1}}{1-\frac{13}{36}z^{-1}}$$

(ii)

ROC: $|z| > \frac{13}{36}$

$$H(z) \xrightarrow{Z^{-1}} h[k] = -1/3 \cdot \left(\frac{13}{36}\right)^{k} U[k] + 1/9 \left(\frac{13}{36}\right)^{k-1} U[k-1]$$

(iii)

$$-3Y(z) + \frac{13}{12}z^{-1}Y(z) = X(z) - 1/3\,z^{-1}X(z)$$

$$\Rightarrow \quad -3y[k] + \frac{13}{12}y[k-1] = x[k] - 1/3\,x[k-1]$$

13.15 $\quad y[k] + y[k-1] + \frac{1}{4} y[k-2] = x[k] - x[k-2]$

(i) $\Rightarrow \quad Y(z) + z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) = X(z) - z^{-2} X(z)$

$$\Rightarrow \quad H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-2}}{1 + z^{-1} + \frac{1}{4} z^{-2}} = \frac{1 - z^{-2}}{\left(1 + \frac{1}{2} z^{-1}\right)^2} \qquad \begin{array}{c} \text{Causal} \\ |z| > \frac{1}{2} \end{array}$$

(ii) $\quad k a^{k-1} u[k] \longleftarrow \dfrac{z^{-1}}{(1 + a z^{-1})^2} \qquad |z| > |a|$

$$\Longrightarrow \quad h[k] = (k+1)\left(-\tfrac{1}{2}\right)^{k} u[k+1] - (k-1)\left(-\tfrac{1}{2}\right)^{k-2} u[k-1]$$

(iii) $\quad x[k] = \left(\frac{1}{2}\right)^k u[k] \rightarrow X(z) = \dfrac{1}{1 - \frac{1}{2} z^{-1}}$

$$\Rightarrow Y(z) = X(z) H(z) = \frac{1 - z^{-2}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 + \frac{1}{2} z^{-1}\right)^2}$$

$$= \frac{A}{1 - \frac{1}{2} z^{-1}} + \frac{B}{\left(1 + \frac{1}{2} z^{-1}\right)^2} + \frac{C}{1 + \frac{1}{2} z^{-1}} \rightarrow \left\{ \begin{array}{l} A = \left.\dfrac{1 - z^{-2}}{\left(1 + \frac{1}{2} z^{-1}\right)^2}\right|_{z = \frac{1}{2}} = \dfrac{-3}{4} \\[2ex] B = \left.\dfrac{1 - z^{-2}}{1 - \frac{1}{2} z^{-1}}\right|_{z = -\frac{1}{2}} = \dfrac{-3}{2} \\[2ex] A + B + C = 1 \Rightarrow C = 1 + \frac{3}{4} + \frac{3}{2} = \frac{13}{4} \end{array} \right.$$

$$\Rightarrow y[k] = -\frac{3}{4} \left(\tfrac{1}{2}\right)^k u[k] - \frac{3}{2} \underbrace{(k+1)}_{=0 \text{ at } k=-1} \left(-\tfrac{1}{2}\right)^k u[k+1] + \frac{13}{4} \left(-\tfrac{1}{2}\right)^k u[k]$$

$$y[k] = \left[-\frac{3}{4} \left(\tfrac{1}{2}\right)^k - \frac{3}{2} (k+1) \left(-\tfrac{1}{2}\right)^k + \frac{13}{4} \left(-\tfrac{1}{2}\right)^k \right] u[k]$$

$\frac{13.15}{(iv)}$

$$h[k] = (k+1)\left(-\tfrac{1}{2}\right)^k u[k+1] + (k-1)\left(-\tfrac{1}{2}\right)^{k-2} u[k-1]$$

$$= (k+1)\left(-\tfrac{1}{2}\right)^k u[k] - \underbrace{(k-1)\left(-\tfrac{1}{2}\right)^{k-2} u[k-1]}$$

$$= \quad + (k+1)\left(-\tfrac{1}{2}\right)^k u[k] - (k-1)\left(-\tfrac{1}{2}\right)^{k-2} u[k] + (-1)\left(-\tfrac{1}{2}\right)^{-2}\delta[k]$$

$$= (-3k+5)\left(-\tfrac{1}{2}\right)^k u[k] - 4\,\delta[k]$$

$$y[k] = h[k] * x[k] = \underbrace{(-3k+5)\left(-\tfrac{1}{2}\right)^k u[k] * \left(\tfrac{1}{2}\right)^k u[k]}_{y_1[k]} - 4\,x[k]$$

$$y_1[k] = \sum_{m=-\infty}^{\infty} (-3m+5)\left(-\tfrac{1}{2}\right)^m u[m]\left(\tfrac{1}{2}\right)^{k-m} u[k-m]$$

$$= \left(\tfrac{1}{2}\right)^k u[k] \sum_{m=0}^{k} (-3m+5)(-1)^m = \left(\tfrac{1}{2}\right)^k u[k]\left[-3\underbrace{\sum_{m=0}^{k} m(-1)^m}_{} + 5\underbrace{\sum_{m=0}^{k}(-1)^m}_{}\right]$$

$$= \begin{cases} \frac{k}{2} & k: \text{even} \\ -\frac{(k+1)}{2} & k: \text{odd} \end{cases} \qquad = \begin{cases} 1 & k: \text{even} \\ 0 & k: \text{odd} \end{cases}$$

$$= \left(\tfrac{1}{2}\right)^k u[k] \times \begin{cases} -\frac{3k}{2} + 5 & k: \text{even} \\ +\frac{3(k+1)}{2} & k: \text{odd} \end{cases}$$

$$\Rightarrow y[k] = \begin{cases} \left(\tfrac{1}{2}\right)^k \left[\left(\frac{-3k+10}{2}\right) - 4\right] u[k] & k: \text{even} \\ \left(\tfrac{1}{2}\right)^k \left[\frac{3(k+1)}{2} - 4\right] u[k] & k: \text{odd} \end{cases}$$

13.16
(iv) $x[k] = u[k]$ $\quad h[k] = 4^{-|k|}$

$$h[k] = 4^{-k} u[k] + 4^{k} u[-k-1]$$

$$\Rightarrow H(z) = \frac{1}{1-4^{-1}z^{-1}} - \frac{1}{1-4z^{-1}} \quad \tfrac{1}{4}<|z|<4$$

$$X(z) = \frac{1}{1-z^{-1}} \quad |z|>1$$

$$Y(z) = X(z)H(z) = \frac{1}{(1-z^{-1})(1-\frac{1}{4}z^{-1})} - \frac{1}{(1-z^{-1})(1-4z^{-1})} \quad 1<|z|<4$$

$$= \frac{+4/3}{1-z^{-1}} + \frac{-1/3}{1-\frac{1}{4}z^{-1}} - \left(\frac{-1/3}{1-z^{-1}} + \frac{4/3}{1-4z^{-1}} \right)$$

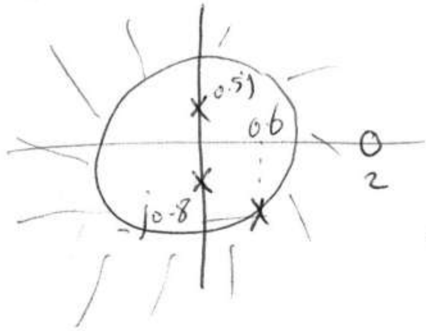
$$= \frac{5/3}{1-z^{-1}} - \frac{1/3}{1-\frac{1}{4}z^{-1}} - \frac{4/3}{1-4z^{-1}} \quad 1<|z|<4$$

$$\Rightarrow y[k] = 5/3\, u[k] - 1/3\, (1/4)^{k} u[k] + 4/3\, (4)^{k} u[-k-1]$$

13.19

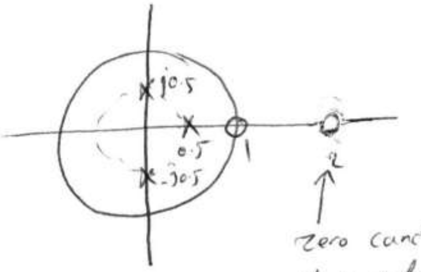
Causal

(i) $$H(z) = \frac{z-2}{(z-0.6+j0.8)(z+j0.5)(z-j0.5)}$$



ROC: $|z|>1$ ⇒ Not stable

(ii) $$H(z) = \frac{(z-2)(z-1)}{(z-2)(z-0.5)(z+j0.5)(z-j0.5)}$$

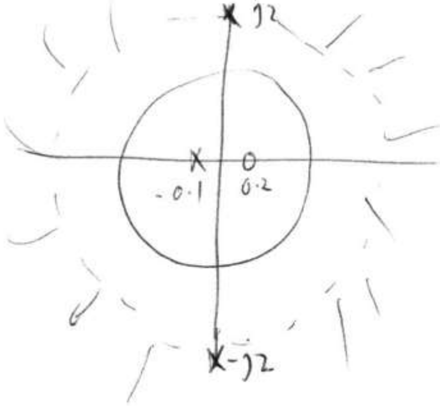


zero cancels the pole ⇒ ther is no zero or pole here

ROC: $|z|>0.5$

⇒ contain $|z|=1$ ⇒ stable

(iii) $$H(z) = \frac{z-0.2}{(z+0.1)(z^2+4)} = \frac{z-0.2}{(z+0.1)(z+j2)(z-j2)}$$

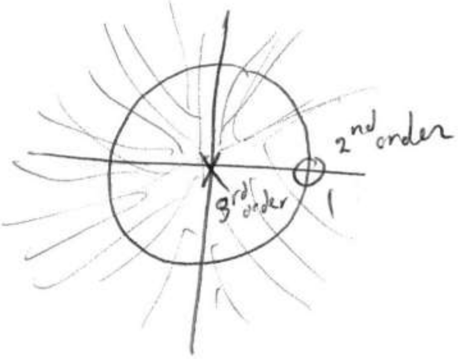


ROC: $|z|>2$ ⇒ Not stable

beause $|z|=1$ is not in ROC

13.19

(iv) $H(z) = z^{-1} - 2z^{-2} + z^{-3} = \frac{z^2 - 2z + 1}{z^3} = \frac{(z-1)^2}{z^3}$



ROC: $\forall z \neq 0$

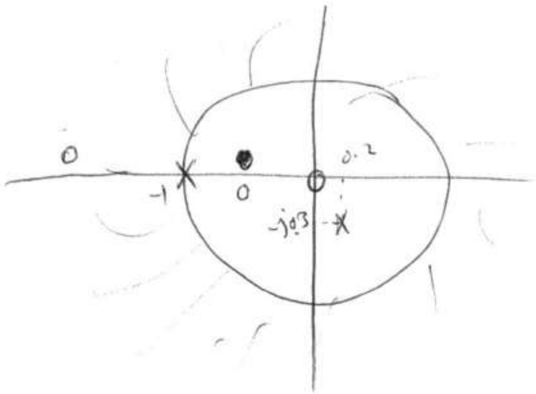
$\Rightarrow$ stable

(v) $$H(z) = \frac{(z^2 + 2.5z + 0.9 \cancel{+} j0.15)z}{z^3 + (1.8 + j0.3)z^2 + (0.6 + j0.6)z - 0.2 + j0.3}$$

sum of coefficient of denominator with even power is equal to the sum of them with odd power $\Rightarrow$ there is a pole at $z = -1$

$$\Rightarrow H(z) = \frac{z(z + 0.4309 + j0.0916)(z^{\bullet} + 2.0691 - j0.0916)}{(z+1)(z^2 + (0.8 \cancel{+} j0.3)z - 0.2 + j0.3)}$$

$$= \frac{z(z + 0.4309 + j0.0916)(z^{\bullet} + 2.0691 - j0.0916)}{(z+1)^2 (z - 0.2 + j0.3)}$$



ROC: $|z| > 1$ $\rightarrow$ system is unstable

14.1

i) $H(z) = 0.7 + 0.2z^{-1} + 0.8z^{-2}$

$\Rightarrow \begin{cases} h[0] = 0.7 \\ h[1] = 0.2 \\ h[2] = 0.8 \end{cases}$ FIR ✓ $h[k]=0$ @ $k<0 \Rightarrow$ causal ✓

$H(\Omega) = 0.7 + 0.2e^{-j\Omega} + 0.8e^{-j2\Omega}$

$= [0.7 + 0.2\cos\Omega + 0.8\cos(2\Omega)] + j[-0.2\sin\Omega - 0.8\sin(2\Omega)]$

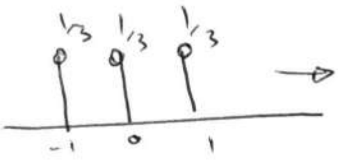
$\measuredangle H(\Omega) = \tan^{-1}\dfrac{-0.2\sin\Omega - 0.8\sin(2\Omega)}{0.7 + 0.2\cos\Omega + 0.8\cos(2\Omega)}$ → its phase is <u>not</u> linear

$\neq -\alpha\Omega + \beta \quad \nexists \alpha, \beta$

ii) $H(z) = \underset{\substack{\uparrow \\ h[-1]}}{\frac{1}{3}z} + \underset{\substack{\uparrow \\ h[0]}}{\frac{1}{3}} + \underset{\substack{\uparrow \\ h[1]}}{\frac{1}{3}z^{-1}}$

FIR ✓

$h[-1] = 1/3 \Rightarrow$ not causal



→ symetric about zero ⇒ linear phase ✓

Proposition 14.1
Page 629

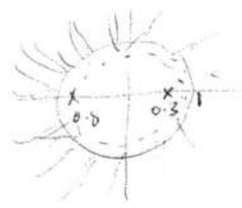
iii) $H(z) = \dfrac{0.7 + 0.2z^{-1} + 0.8z^{-2}}{1 + 0.5z^{-1} - 0.24z^{-2}} \underset{\uparrow}{=} A + \dfrac{B + Cz^{-1}}{1 + 0.5z^{-1} - 0.24z^{+2}}$

order of top and bottom are the same
$\exists A, B, C$

long division
$\overset{\downarrow}{=} a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots$

⇒ IIR ✓ and causal ✓

$1 + 0.5z^{-1} - 0.24z^{-2} = 0 \rightarrow \begin{matrix} z_1 = -0.8 \\ z_2 = 0.3 \end{matrix}$



$\measuredangle H(\Omega) \neq \alpha + \beta\Omega$ → <u>not</u> linear phase

<u>14.1</u>

Continue...

(iv) $$H(z) = \frac{1-0.1z^{-1}-0.06z^{-2}}{1+0.2z^{-1}}$$

$$H(z) = \frac{\cancel{(1+0.2z^{-1})}(1-0.3z^{-2})}{\cancel{1+0.2z^{-1}}} = 1-0.3z^{-2}$$

$\Rightarrow$ FIR ✓ and causal ✓

$$\measuredangle H(\Omega) = \measuredangle\left(1-0.3e^{-j\Omega}\right) = \measuredangle\left(1-0.3\cos\Omega + j\sin\Omega\right) =$$

$$= \tan^{-1}\left(\frac{\sin\Omega}{1-0.3\cos\Omega}\right) \neq \beta - \alpha\Omega$$

<u>not</u> linear phase

14.3 $h[k] = \begin{cases} 1/3 & -1 \leq k \leq 1 \\ 0 & \text{o.w} \end{cases}$

FIR
symmetric signal
⇒ linear phase

(i)
$H(z) = \frac{1}{3}z + \frac{1}{3} + \frac{1}{3}z^{-1} \Rightarrow H(\Omega) = \frac{1}{3}e^{j\Omega} + \frac{1}{3} + \frac{1}{3}e^{-j\Omega}$

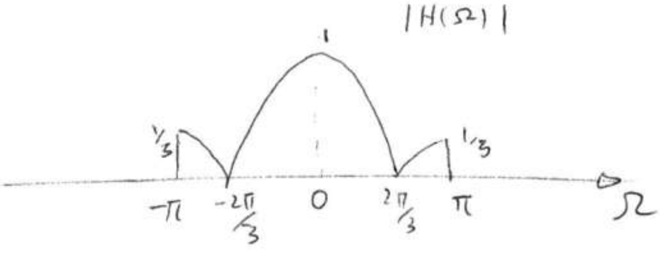
ROC: $\forall z \neq 0$

(ii)

$$H(\Omega) = \left[\frac{2}{3}\cos\Omega + \frac{1}{3}\right] + j\overset{\to 0}{\left[\frac{1}{3}\sin\Omega - \frac{1}{3}\sin\Omega\right]} = \frac{2}{3}\cos\Omega + \frac{1}{3}$$

$$\Rightarrow \measuredangle H(\Omega) = 0 \qquad |H(\Omega)| = \left|\frac{2}{3}\cos\Omega + \frac{1}{3}\right| = \frac{|2\cos\Omega + 1|}{3}$$

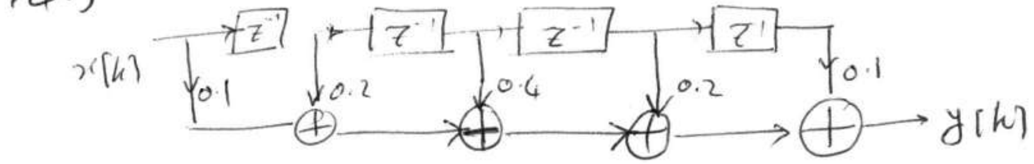
$2\cos\Omega + 1 = 0 \rightarrow \cos\Omega = -\frac{1}{2}$



(iii) It is a low-pass filter with side lobes about −π and π

(iv) It is linear phase fiter because $\measuredangle H(\Omega) = -\underset{\substack{\uparrow \\ 0}}{\alpha}\Omega + \underset{\substack{\uparrow \\ 0}}{\beta} = 0$

14.5



$$Y(Z) = 0.1\,X(Z) + 0.2Z^{-1}X(Z) + 0.4Z^{-2}X(Z) + 0.2Z^{-3}X(Z) + 0.1Z^{-4}X(Z)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = \underline{0.1} + \underline{\underline{0.2}}\,Z^{-1} + 0.4Z^{-2} + \underline{\underline{0.2}}\,Z^{-3} + \underline{0.1\,Z}^{-4} \quad : \text{FIR}$$

Symmetry : $N = 5$ (type I) $\Rightarrow \angle H(\Omega) = -\frac{N-1}{2}\Omega + 0 = -2\Omega$

$\underline{\text{or}}$ directly

$$H(\Omega) = 0.1 + 0.2\,e^{-j\Omega} + 0.4\,e^{-j2\Omega} + 0.2\,e^{-j3\Omega} + 0.1\,e^{-j4\Omega}$$

$$= e^{-j2\Omega}\left[\underline{\underline{0.1e}}^{j2\Omega} + \underline{0.2\,e}^{j\Omega} + 0.4 + \underline{0.2\,e}^{-j\Omega} + \underline{\underline{0.1\,e}}^{-j2\Omega}\right]$$

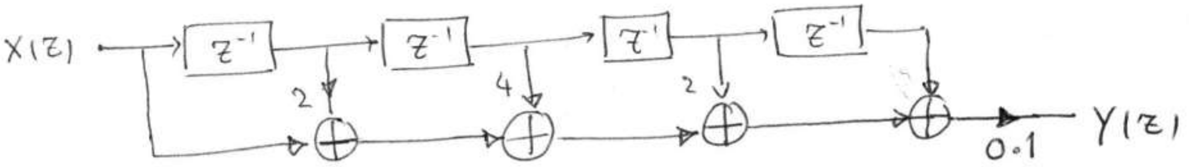
$$= e^{-j2\Omega}\left[0.2\cos(2\Omega) + 0.4\cos(\Omega) + 0.4\right]$$

$$\Rightarrow \angle H(\Omega) = -2\Omega$$

$$|H(\Omega)| = |0.2\cos(2\Omega) + 0.4\cos(\Omega) + 0.4|$$

14.6 from 14.5

$$10\,Y(z) = X(z) + 2z^{-1}X(z) + 4z^{-2}X(z) + 2z^{-3}X(z) + z^{-4}X(z)$$

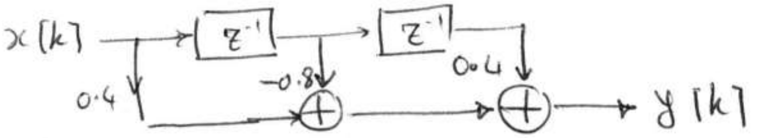


number of multipliers = 4 = 5-1

14.8

$H(z) = 0.4 - 0.8z^{-1} + 0.4z^{-2}$ FIR ; N=3

(i)



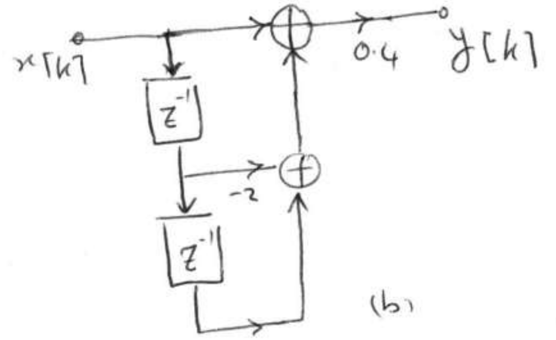
(a)

: Direct form

number of multipliers is 3

$H(z) = 0.4\left(1 - 2z^{-1} + z^{-2}\right)$

(ii)

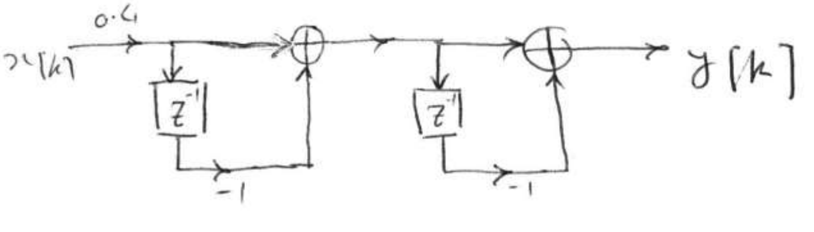


(b)

: Cascade of 2nd order systems

umber of multipliers is 2

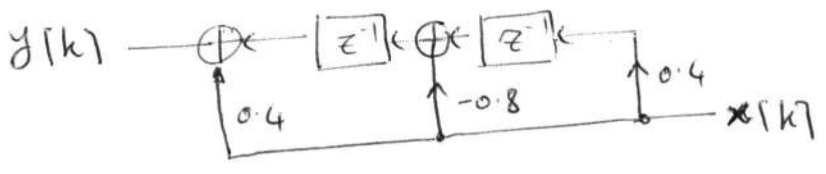
$H(z) = 0.4(1-z^{-1})(1-z^{-1})$



(c)

: Cascade of 1st order systems

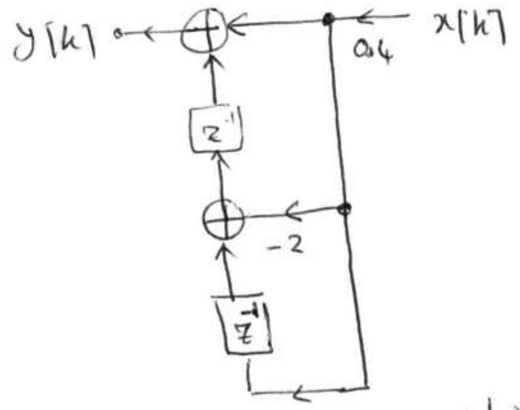
14.9



trnsposed of
14.8 (i)

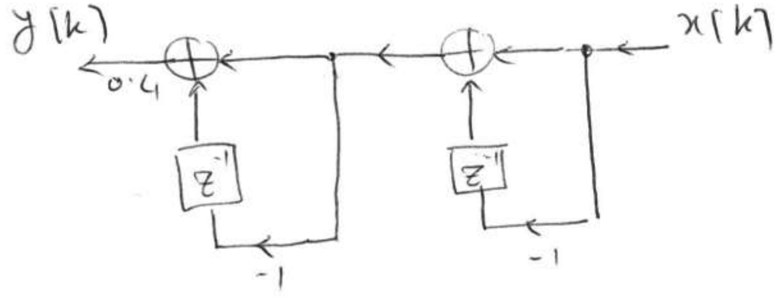
(a)

and



transposed of
14.8 (ii)

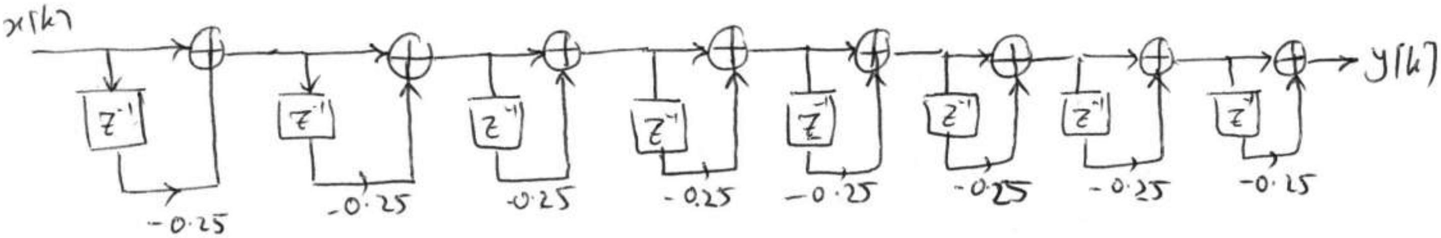
(b)



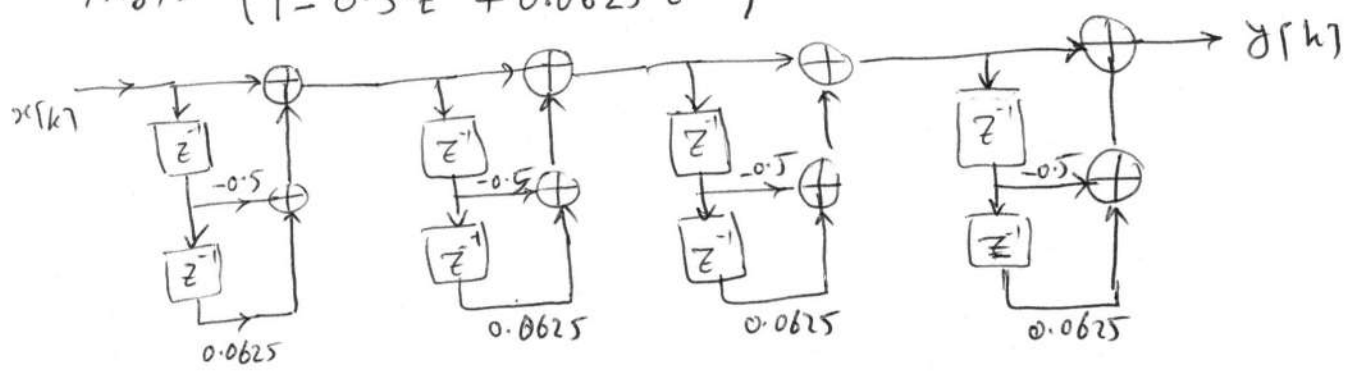
(c)

14.11 $H(z) = (1 - 0.25 z^{-1})^8$

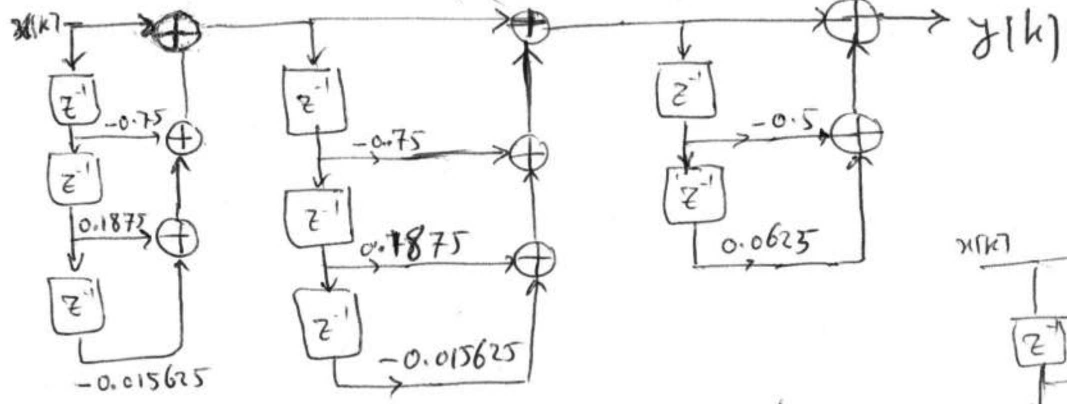
i)



(ii) $H(z) = (1 - 0.5 z^{-1} + 0.0625 z^{-2})^4$

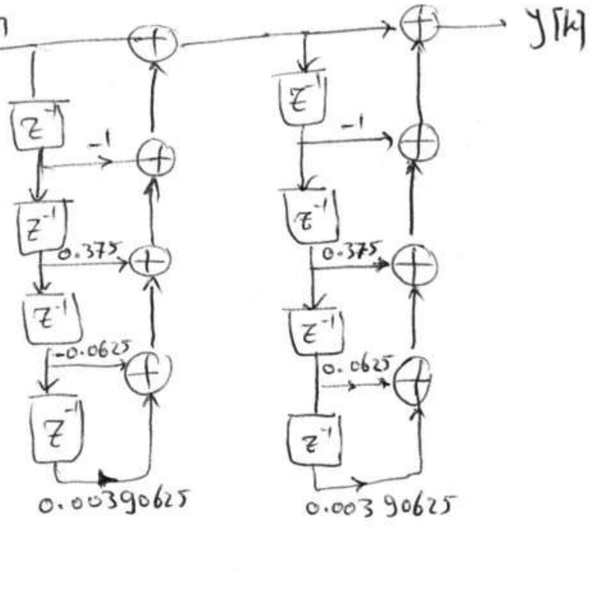


(iii) $H(z) = \underbrace{(1 - 0.75 z^{-1} + 0.1875 z^{-2} - 0.015625 z^{-3})^2}_{((1-0.25z^{-1})^3)^2} \underbrace{(1 - 0.5z^{-1} + 0.0625z^{-2})}_{(1-0.25z^{-1})^2}$



(iv) $H(z) = (1 - 0.25z^{-1})^4 (1 - 0.25z^{-1})^4$

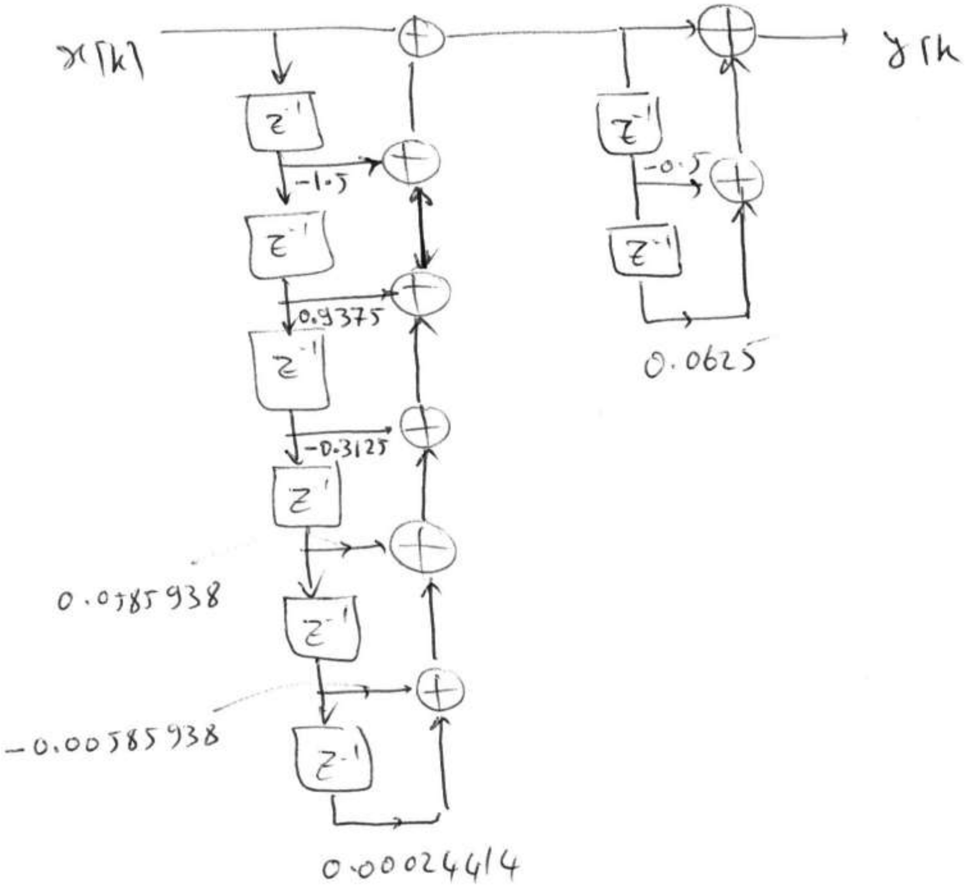
$= [1 - z^{-1} + 0.375 z^{-2} - 0.0625 z^{-3} + 0.00390625 z^{-4}]^2$



14.11 (v)

$$H(z) = (1-0.25z^{-1})^{6}\ (1-0.25z^{-1})^{2}$$

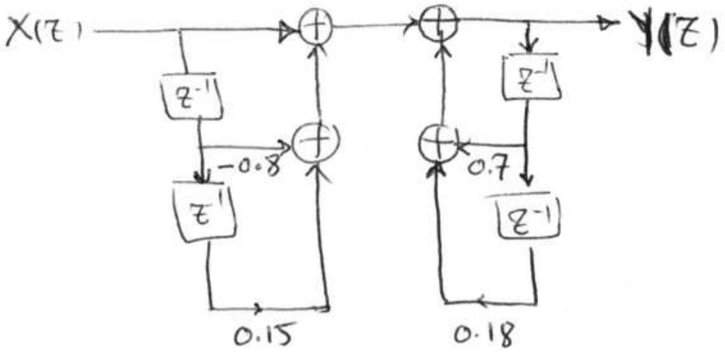
$$= (1-1.5z^{-1}+0.9375z^{-2}-0.3125z^{-3}+0.0585938z^{-4}-0.00585938z^{-5}+0.00024414z^{-6})\ (1-0.5z^{-1}+0.0625z^{-2})$$



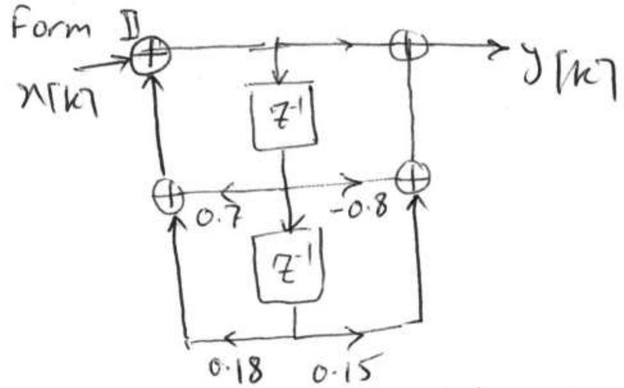
The number of adders, multipliers and delays for all of the realizations are the same, but for implementing higher order systems more precise coefficients are required.

14.14 $H(z) = \dfrac{1-0.8z^{-1}+0.15z^{-2}}{1-0.7z^{-1}-0.18z^{-2}} = \dfrac{Y(z)}{X(z)}$

i) Direct Form I $Y(z) = (1-0.8z^{-1}+0.15z^{-2})X(z)+(0.7z^{-1}+0.18z^{-2})Y(z)$



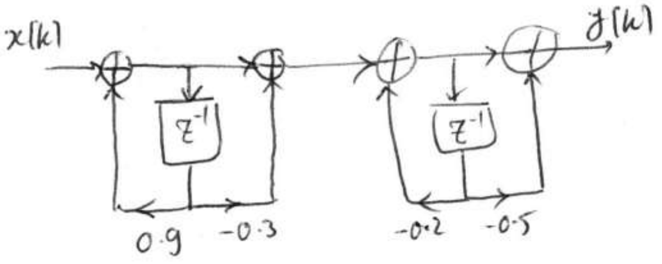
ii) Direct Form II



(iii) Cascade Form

$H(z) = \dfrac{(1-0.3z^{-1})(1-0.5z^{-1})}{(1-0.9z^{-1})(1+0.2z^{-1})}$

$= \dfrac{1-0.3z^{-1}}{1-0.9z^{-1}} \cdot \dfrac{1-0.5z^{-1}}{1+0.2z^{-1}}$



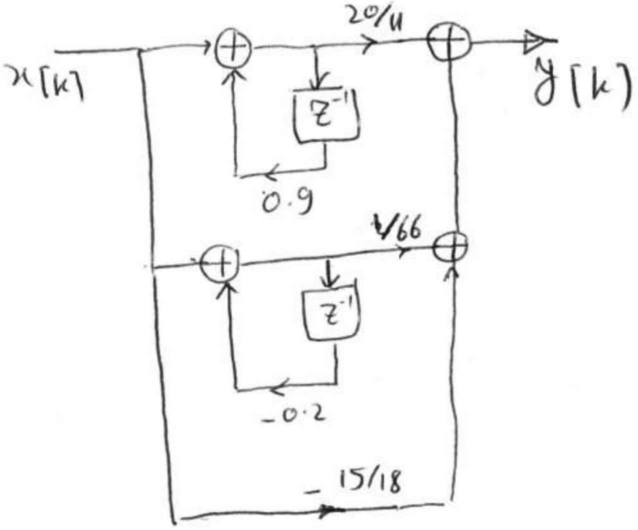
Cascade of two direct from II realizations

(iv) Parallel form

long division

$H(z) = \dfrac{\frac{33}{18} - \frac{49}{30}z^{-1}}{1-0.7z^{-1}-0.18z^{-2}} - \dfrac{15}{18}$

$\Rightarrow H(z) = \dfrac{20/11}{1-0.9z^{-1}} + \dfrac{1/66}{1+0.2z^{-1}} - \dfrac{15}{18}$ Partial fraction



14.16

(i) allpass filter $|H(\Omega)|=1 : \forall\Omega$

$$H_1(z)=\frac{\alpha_1+z^{-1}}{1+\alpha_1 z^{-1}} \Rightarrow H_1(\Omega)=\frac{\alpha_1+e^{-j\Omega}}{1+\alpha_1 e^{-j\Omega}} \Rightarrow H_1^*(\Omega)=\frac{\alpha_1^*+e^{j\Omega}}{1+\alpha_1^* e^{j\Omega}}$$

$$|H_1(\Omega)|^2=H_1(\Omega)H_1^*(\Omega)=\frac{(\alpha_1+e^{-j\Omega})(\alpha_1^*+e^{j\Omega})}{(1+\alpha_1 e^{-j\Omega})(1+\alpha_1^* e^{j\Omega})}=\frac{|\alpha_1|^2+\alpha_1 e^{j\Omega}+\alpha_1^* e^{-j\Omega}+1}{|\alpha_1|^2+\alpha_1 e^{j\Omega}+\alpha_1^* e^{-j\Omega}+1}=1$$

$$\Rightarrow |H_1(\Omega)|=1$$

$$H_2(z)=\frac{\alpha_2+\alpha_1 z^{-1}+z^{-2}}{1+\alpha_1 z^{-1}+\alpha_2 z^{-2}}=\frac{\alpha_2+\alpha_1 z^{-1}+z^{-2}}{z^{-2}(z^2+\alpha_1 z+\alpha_2)}$$

$$\Rightarrow H(\Omega)=\frac{1}{e^{j2\Omega}}\cdot\frac{(\alpha_2+\alpha_1 e^{-j\Omega}+e^{-j2\Omega}) \leftarrow A(\Omega)}{(e^{+j2\Omega}+\alpha_1 e^{j\Omega}+\alpha_2) \leftarrow B(\Omega)}$$

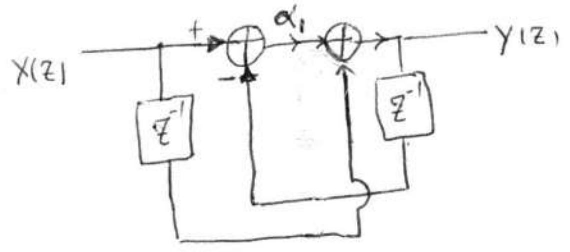
if α_1 and α_2 are <u>real</u> numbers
$A(\Omega)=B^*(\Omega)$

$$\Rightarrow |H(\Omega)|=|e^{j2\Omega}|\cdot\frac{|A(\Omega)|}{|A^*(\Omega)|}=1$$

(ii)

$$H_1(z)=\frac{\alpha_1+z^{-1}}{1+\alpha_1 z^{-1}}=\frac{Y(z)}{X(z)} \Rightarrow Y(z)+\alpha_1 z^{-1}Y(z)=\alpha_1 X(z)+z^{-1}X(z)$$

$$\Rightarrow Y(z)=\alpha_1(X(z)-z^{-1}Y(z))+z^{-1}X(z)$$

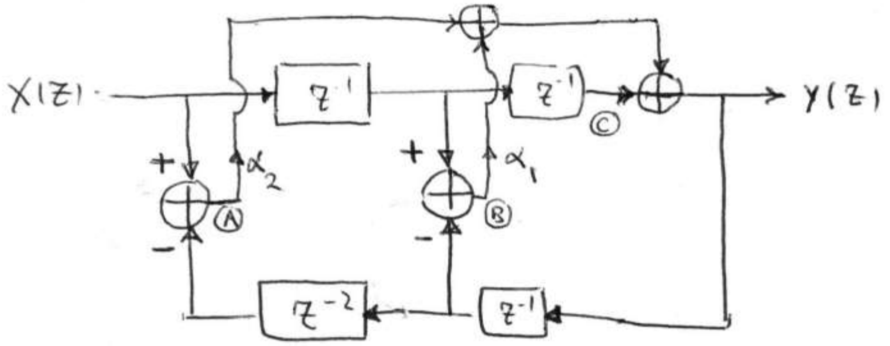


14.16

(iii)

$$H_2(z) = \frac{\alpha_2 + \alpha_1 z^{-1} + z^{-2}}{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}} = \frac{Y(z)}{X(z)}$$

$$\Rightarrow Y(z) = \alpha_2 \underbrace{(X(z) - z^{-2}Y(z))}_{Ⓐ} + \alpha_1 \underbrace{(z^{-1}X(z) - z^{-1}Y(z))}_{Ⓑ} + \underbrace{z^{-2}X(z)}_{Ⓒ}$$

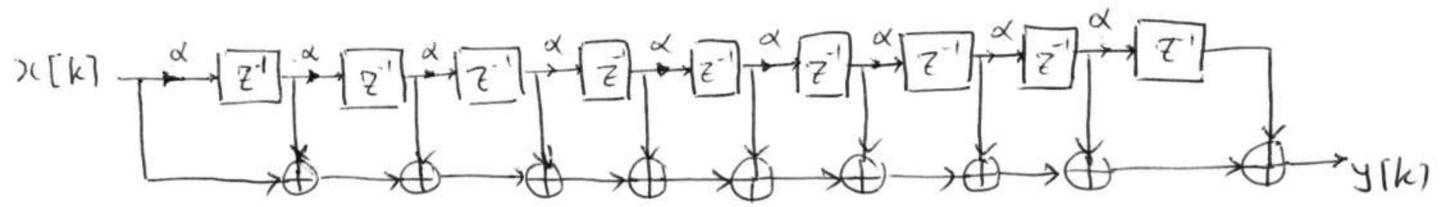


14.17

$$h[k] = \begin{cases} \alpha^k & 0 \leq k \leq 9 \\ 0 & \text{o.w.} \end{cases}$$

(i)

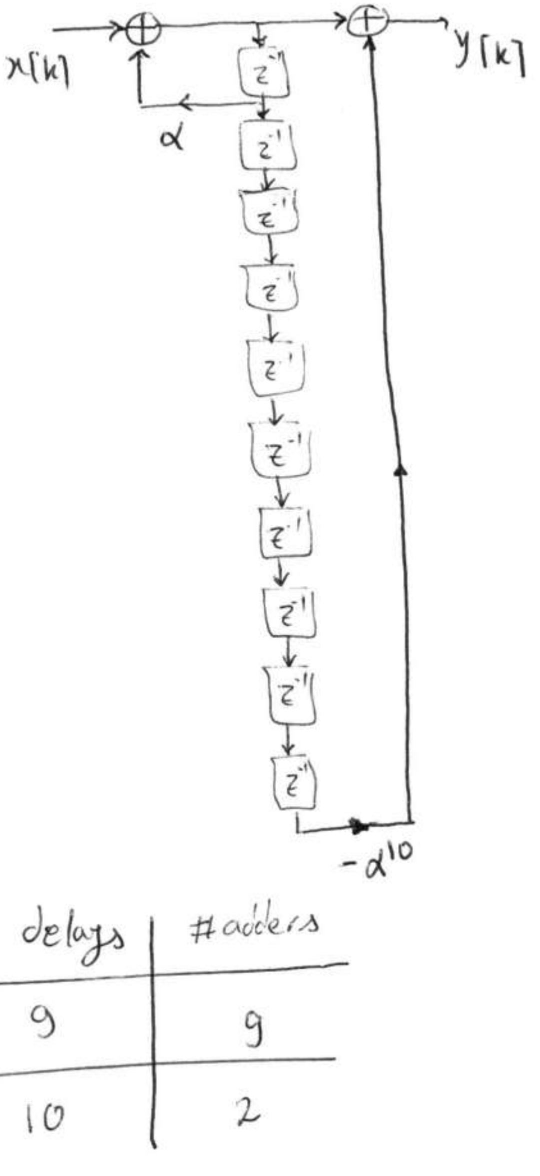
$$H(z) = \alpha^0 + \alpha^1 z^{-1} + \alpha^2 z^{-2} + \cdots + \alpha^9 z^{-9} = \frac{Y(z)}{X(z)}$$



(ii)

$$H(z) = \frac{1 - \alpha^{10} z^{-10}}{1 - \alpha z^{-1}}$$

Direct form II



(iii)

	# multiplier	# delays	# adders
FIR	9	9	9
IIR	2	10	2