Concordia UNIVERSITY

FACULTY OF ENGINEERING AND COMPUTER SCIENCE

COURSE		NUMBER	SECTION
Discrete Time Signals and Systems		ELEC 342	U
EXAMINATION	DATE	TIME & PLACE Room:	# OF PAGES
FINAL	December 15, 2016	2:00am -5:00 pm	5
PROFESSOR		LAB INSTRUCTOR	
Dr. M.R. Soleymani			
MATERIALS ALLOWED ☒ YES (PLEASE SPECIFY) (below)			
CALCULATORS ALLOWED O NO ☒ YES			
SPECIAL INSTRUCTIONS: Answer all questions.			

Name: ______________________________ I.D.: ______________________
Surname, given names

This is a closed book exam.

Only an official calculator with the ENCS stamp is allowed.

Students may bring into the exam three 8.5x11 inch pages on which anything may be written. Both sides may be written on.

If you have any difficulty you may try to make REASONABLE assumptions. State the assumptions and how those assumptions limit your answers. Show all your work in detail and justify your answers.

Marks are given for how an answer is arrived at, not just the answer itself.

Concordia University
ELEC 342 Discrete Time Signals and Systems
Final Exam – Summer 2016

Students are allowed three 8.5*11 inch formula sheets. Anything can be written on these sheets and both sides may be written on. The ENCS calculator, pens, pencils, erasers and straightedges are also allowed.

If you have difficulty you may try making REASONABLE assumptions. State the assumption and how the assumption limits your answer.

Show all your work and justify all your answers. Marks are given for how an answer is arrived at not just the answer itself.

1. A Linear Time Invariant system is described by the following difference equation:

$$y[k] - 2y[k-1] = x[k]$$

Assume that $y[-1] = 0$.

a) Find the output of the system for input $x[k] = \delta[k]$. (3 Marks).
b) Is the system causal? Explain (1 Mark).
c) Is the system stable? Explain (1 Mark).

2. Find the output of the system $y[k] = x[k] - x[k-2]$ for the input $x[k] = u[k]$. (2 Marks)

3. Find $x[k]$ for

$$X(\Omega) = \frac{2e^{-j2\Omega}}{(1 - 4e^{-j\Omega})(1 - 2e^{-j\Omega})}$$

(5 Marks).
Note: The sequence may not be causal.

4. The input output relationship of an LTID system is given as,

$$y[k+2] - \frac{5}{6}y[k+1] + \frac{1}{6}y[k] = x[k+2].$$

a) Find the transfer function of this system (3 Marks)
b) Find the impulse response of the system (5 Marks).

5. When the input

$$x[k] = \left(\frac{1}{2}\right)^k u[k] + \left(\frac{1}{3}\right)^k u[k]$$

is applied to an LTI system the output is

$$y[k] = 4\left(\frac{1}{3}\right)^k \text{u[k]-}3\left(\frac{1}{4}\right)^k u[k].$$

a) Find the transfer function $H(z)$ of the system. (5 Marks).

b) Find the impulse response $h[k]$ (5 Marks).

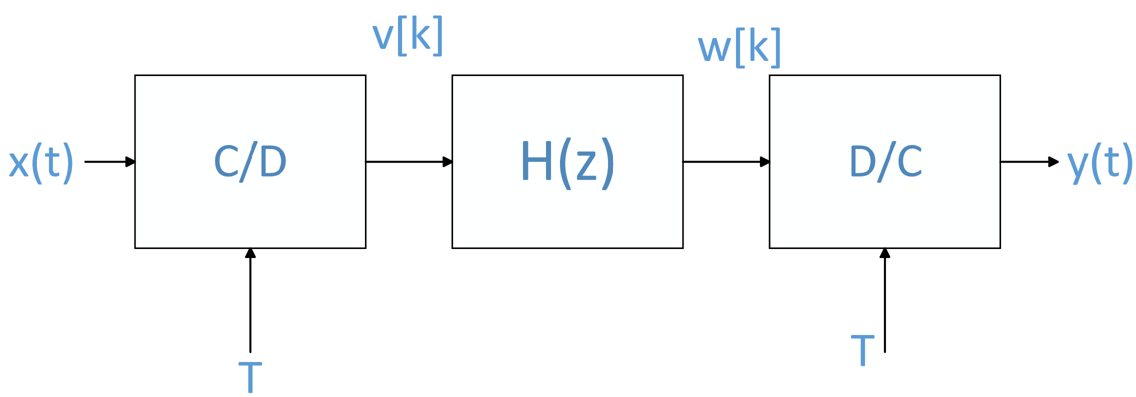
6) A filter is described by the equation $y[k] - \frac{1}{2}y[k-1] = x[k]$.

a) Plot the frequency response of the filter (magnitude and phase plot) (5 Marks).
b) Is the filter FIR or IIR? (2 Marks)
c) Is the filter low pass or high pass? Explain. (2 Marks)

7) Implement the following filter using direct form 2 realization. (6 Marks).

$$H(z0 = \frac{1 - 2z^{-1} + z^{-2}}{1 - 0.1z^{-1} - .07z^{-2} - 0.065z^{-3}}$$

8) Consider the block diagram shown. The discrete time filter H(z) is an ideal bandpass filter with a passband from 0.2π to 0.3π and a pass band gain of 1. If T=0.0005.



If T=0.0005 seconds sketch the spectrum of $v[k]$, $w[k]$ and $y(t)$ if the input is $x(t) = \cos(300\pi t) + \cos(500\pi t) + 2\cos(1000\pi t)$ (5 Marks)

Table 11.2. DTFTs and DTFSs for elementary DT sequences
Note that the DTFS does not exist for aperiodic sequences

Sequence: $x[k]$	DTFS: $D_n = \frac{1}{K_0} \sum_{k=\langle K_0 \rangle} x[k] e^{-jn\Omega_0 k}$	DTFT: $X(\Omega) = \sum_{k=-\infty}^{\infty} x[k] e^{-j\Omega k}$
(1) $x[k] = 1$	$D_n = 1$	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - 2m\pi)$
(2) $x[k] = \delta[k]$	does not exist	$X(\Omega) = 1$
(3) $x[k] = \delta[k - k_0]$	does not exist	$X(\Omega) = e^{-j\Omega k_0}$
(4) $x[k] = \sum_{m=-\infty}^{\infty} \delta(k - mK_0)$	$D_n = \frac{1}{K_0}$ for all n	$X(\Omega) = \frac{2\pi}{K_0} \sum_{m=-\infty}^{\infty} \delta\left(\Omega - \frac{2m\pi}{K_0}\right)$
(5) $x[k] = u[k]$	does not exist	$X(\Omega) = \pi \sum_{m=-\infty}^{\infty} \delta(\Omega - 2m\pi) + \frac{1}{1 - e^{-j\Omega}}$
(6) $x[k] = p^k u[k]$ with $\|p\| < 1$	does not exist	$X(\Omega) = \frac{1}{1 - pe^{-j\Omega}}$
(7) First-order time-rising decaying exponential $x[k] = (k+1)p^k u[k]$, with $\|p\| < 1$.	does not exist	$X(\Omega) = \frac{1}{(1 - pe^{-j\Omega})^2}$
(8) Complex exponential (periodic) $x[k] = e^{jk\Omega_0}$ $K_0 = 2\pi p / \Omega_0$	$D_n = \begin{cases} 1 & n = p \pm rK_0 \\ 0 & \text{elsewhere} \end{cases}$ for $-\infty < r < \infty$	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2m\pi)$
(9) Complex exponential (aperiodic) $x[k] = e^{jk\Omega_0}$, $2\pi/\Omega_0 \neq$ rational	does not exist	$X(\Omega) = 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2m\pi)$
(10) Cosine (periodic) $x[k] = \cos(\Omega_0 k)$ $K_0 = 2\pi p / \Omega_0$	$D_n = \begin{cases} \frac{1}{2} & n = \pm p \pm rK_0 \\ 0 & \text{elsewhere} \end{cases}$ for $-\infty < r < \infty$	$X(\Omega) = \pi \sum_{m=-\infty}^{\infty} \delta(\Omega + \Omega_0 - 2m\pi) + \pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2m\pi)$
(11) Cosine (aperiodic) $x[k] = \cos(\Omega_0 k)$, $2\pi/\Omega_0 \neq$ rational	does not exist	$X(\Omega) = \pi \sum_{m=-\infty}^{\infty} \delta(\Omega + \Omega_0 - 2m\pi) + \pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2m\pi)$
(12) Sine (periodic) $x[k] = \sin(\Omega_0 k)$ $K_0 = 2\pi p / \Omega_0$	$D_n = \begin{cases} \frac{1}{2}j & n = \pm p \pm rK_0 \\ 0 & \text{elsewhere} \end{cases}$ for $-\infty < r < \infty$	$X(\Omega) = j\pi \sum_{m=-\infty}^{\infty} \delta(\Omega + \Omega_0 - 2m\pi) - j\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2m\pi)$
(13) Sine (aperiodic) $x[k] = \sin(\Omega_0 k)$, $2\pi/\Omega_0 \neq$ rational	does not exist	$X(\Omega) = j\pi \sum_{m=-\infty}^{\infty} \delta(\Omega + \Omega_0 - 2m\pi) - j\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2m\pi)$

(cont.)

Table 13.1. Unilateral z-transform pairs for several causal DT sequences

DT sequence $x[k] = \frac{1}{2\pi j}\oint_C X(z)z^{k-1}dz$	z-transform with ROC $X(z) = \sum_{k=-\infty}^{\infty} x[k]z^{-k}$
(1) Unit impulse $x[k] = \delta[k]$	1, ROC: entire z-plane
(2) Delayed unit impulse $x[k] = \delta[k - k_0]$	z^{-k_0},ROC: entire z-plane, except $z = 0$
(3) Unit step $x[k] = u[k]$	$\frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$, ROC: $\lvert z\rvert > 1$
(4) Exponential $x[k] = \alpha^k u[k]$	$\frac{1}{1 - \alpha z^{-1}} = \frac{z}{z - \alpha}$, ROC: $\lvert z\rvert > \lvert\alpha\rvert$
(5) Delayed exponential $x[k] = \alpha^{k-1}u[k - 1]$	$\frac{z^{-1}}{1 - \alpha z^{-1}} = \frac{1}{z - \alpha}$, ROC: $\lvert z\rvert > \lvert\alpha\rvert$
(6) Ramp $x[k] = ku[k]$	$\frac{z^{-1}}{(1 - z^{-1})^2} = \frac{z}{(z - 1)^2}$, ROC: $\lvert z\rvert > 1$
(7) Time-rising exponential $x[k] = k\alpha^k u[k]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2} = \frac{\alpha z}{(z - \alpha)^2}$, ROC: $\lvert z\rvert > \lvert\alpha\rvert$
(8) Causal cosine $x[k] = \cos(\Omega_0 k)u[k]$	$\frac{1 - z^{-1}\cos\Omega_0}{1 - 2z^{-1}\cos\Omega_0 + z^{-2}} = \frac{z[z - \cos\Omega_0]}{z^2 - 2z\cos\Omega_0 + 1}$, ROC: $\lvert z\rvert > 1$
(9) Causal sine $x[k] = \sin(\Omega_0 k)u[k]$	$\frac{z^{-1}\sin\Omega_0}{1 - 2z^{-1}\cos\Omega_0 + z^{-2}} = \frac{z\sin\Omega_0}{z^2 - 2z\cos\Omega_0 + 1}$, ROC: $\lvert z\rvert > 1$
(10) Exponentially modulated cosine $x[k] = \alpha^k\cos(\Omega_0 k)u[k]$	$\frac{1 - \alpha z^{-1}\cos\Omega_0}{1 - 2\alpha z^{-1}\cos\Omega_0 + \alpha^2 z^{-2}} = \frac{z[z - \alpha\cos\Omega_0]}{z^2 - 2\alpha z\cos\Omega_0 + \alpha^2}$, ROC: $\lvert z\rvert > \lvert\alpha\rvert$
(11) Exponentially modulated sine I $x[k] = \alpha^k\sin(\Omega_0 k)u[k]$	$\frac{\alpha z^{-1}\sin\Omega_0}{1 - 2\alpha z^{-1}\cos\Omega_0 + \alpha^2 z^{-2}} = \frac{\alpha z\sin\Omega_0}{z^2 - 2\alpha z\cos\Omega_0 + \alpha^2}$, ROC: $\lvert z\rvert > \alpha$
(12) Exponentially modulated sine II $x[k] = r\alpha^k\sin(\Omega_0 k + \theta)u[k]$, with $\alpha \in R$.	$\frac{A + Bz^{-1}}{1 + 2\gamma z^{-1} + \alpha^2 z^{-2}} = \frac{z(Az + B)}{z^2 + 2\gamma z + \gamma^2}$, ROC: $\lvert z\rvert > \lvert\alpha\rvert^{(a)}$