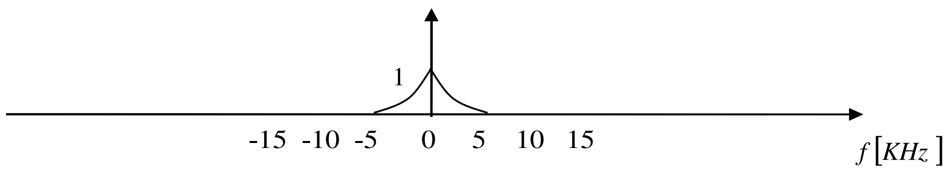
Solution 4.2.7:

Part a)

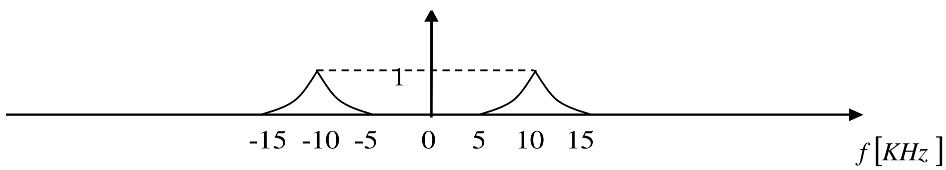
$$a(t) \overset{FT}{\Leftrightarrow} A(f)$$

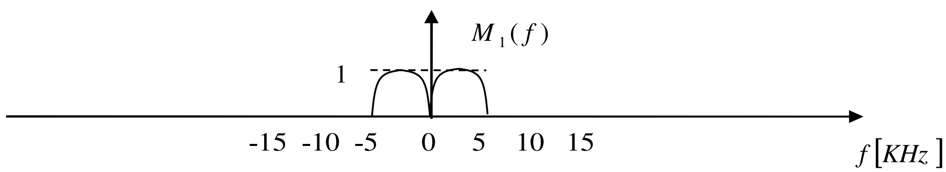
$$a(t) = m_2(t) \times 2\cos(20{,}000\pi t) \overset{FT}{\Leftrightarrow} A(f) = M_2(f - 10{,}000) + M_2(f + 10{,}000)$$

$M_2(f)$

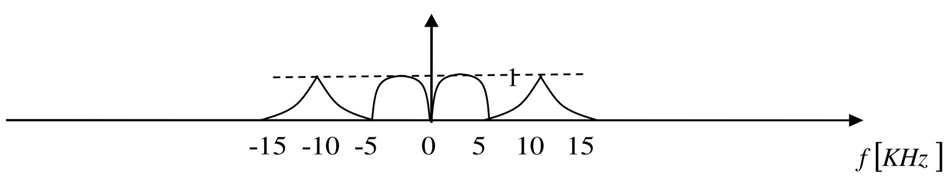


$A(f) = M_2(f - 10{,}000) + M_2(f + 10{,}000)$



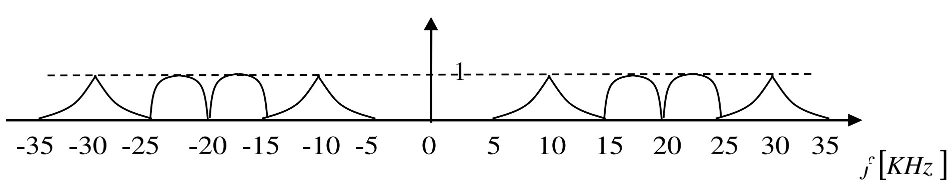


$B(f) = A(f) + M_1(f)$



$$c(t) = b(t) \times 2\cos(40{,}000\pi t) \overset{FT}{\Leftrightarrow} C(f) = B(f - 20{,}000) + B(f + 20{,}000)$$

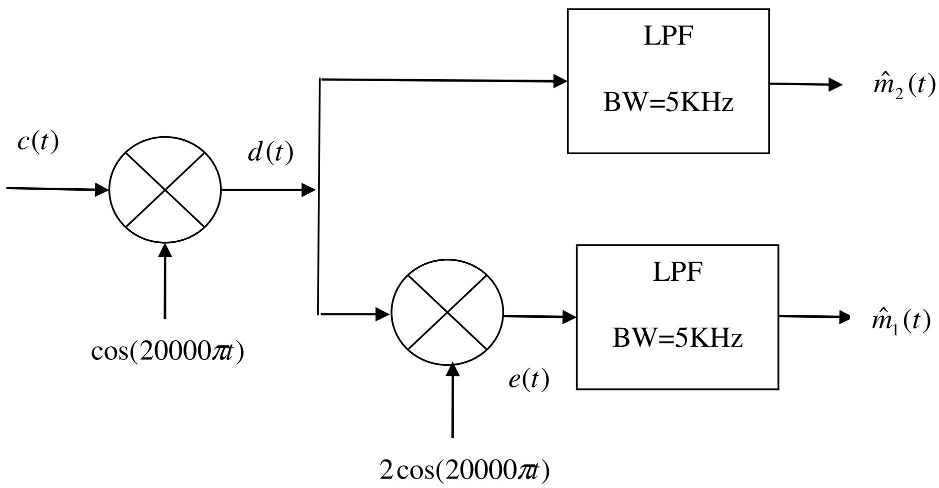
$C(f) = B(f - 20{,}000) + B(f + 20{,}000)$



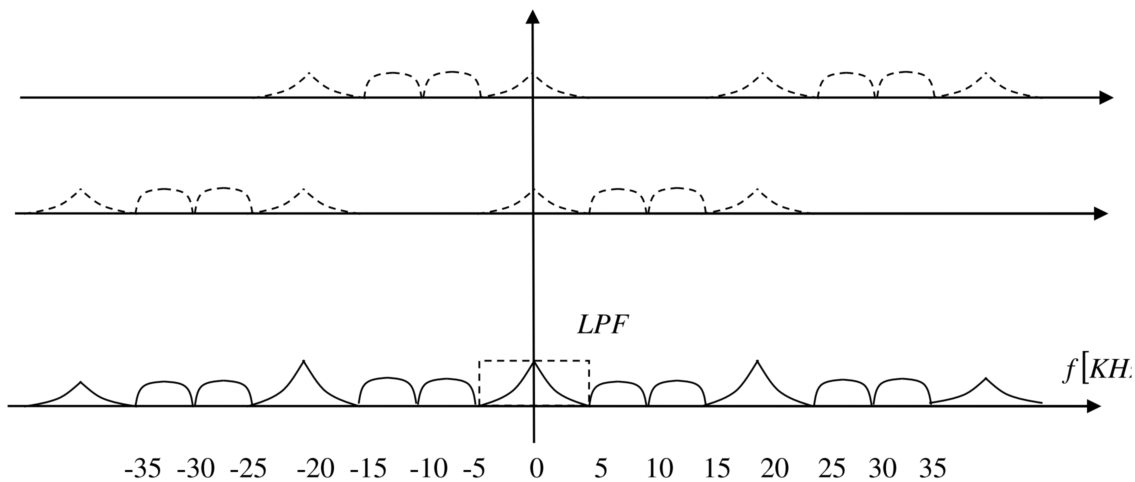
Part b)

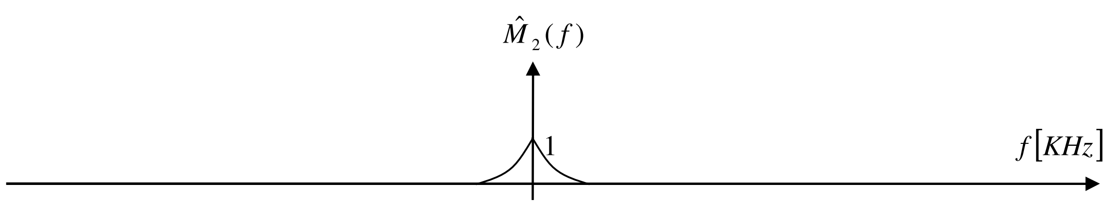
The spectrum of the transmitted signal is $C(f)$ as shown above and obviously is extending from 5 to 35 KHz. Therefore the signal bandwidth is 35-5=30 KHz.

Part c)

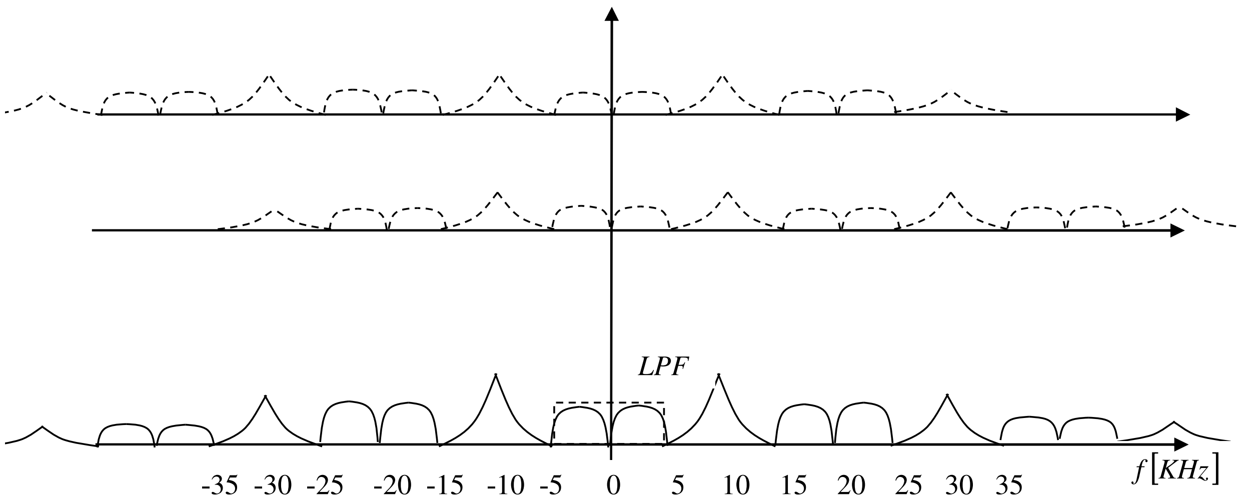


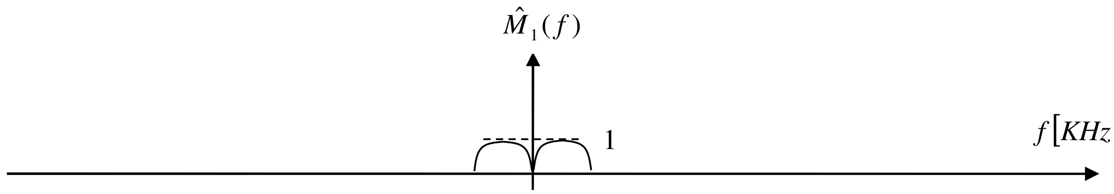
$$D(f) = \frac{1}{2}C(f - 10000) + \frac{1}{2}C(f + 10000)$$





$$E(f) = D(f - 10000) + D(f + 10000)$$





Solution 4.3.1:

Part a)

The message power is:

$$P_m = \frac{1}{T}\int_{-T/2}^{T/2} m^2(t)dt = \frac{2}{0.1}\int_0^{0.05}\left[\frac{2t}{0.05}\right]^2 dt = 20\int_0^{0.05}\frac{4t^2}{0.0025}dt = 32000\left[\frac{t^3}{3}\right]_0^{0.05} = 32000\times\frac{0.05^3}{3} = \frac{4}{3}$$

Part b)

Modulation index: $\mu = \frac{m_p}{A} = \frac{2}{2b}$

$$S_{AM}(t) = 2[b + 0.5m(t)]\cos\omega_c t = 2b\cos(2000\pi t) + m(t)\cos(2000\pi t)$$

Power of carrier is: $P_C = \frac{(2b)^2}{2} = 2b^2$

Power of the modulated signal $m(t)\cos(2000\pi t)$ is $P_S = \frac{1}{2}P_m$ and therefore:

$$P_S = \frac{1}{2}\times\frac{4}{3} = \frac{2}{3}$$

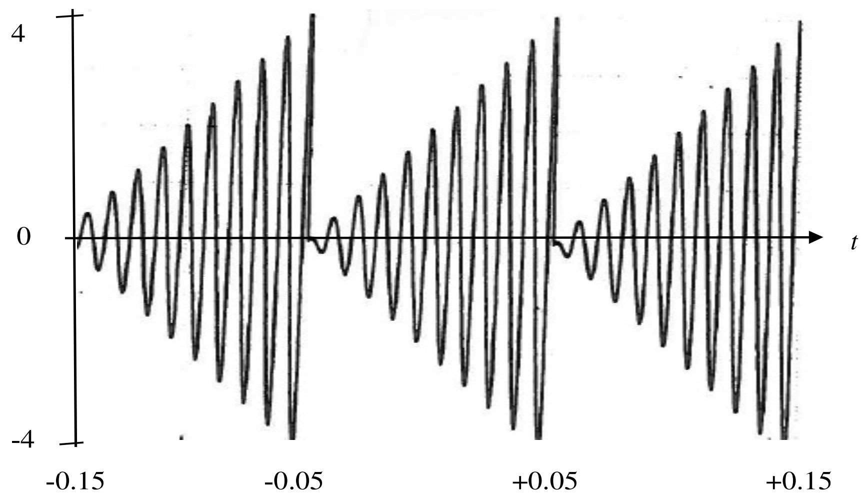
Power efficiency is:

$$\eta = \frac{P_S}{P_C + P_S} = \frac{2/3}{2b^2 + (2/3)} = \frac{1}{3b^2+1}$$

If b=1 then $\mu = \frac{2}{2\times 1} = 1$ and $\eta = \frac{1}{3\times 1^2 + 1} = 0.25 = 25\%$

Part c)

We draw it for the case of carrier frequency of 100Hz since it is difficult to draw it for 1000Hz.



In this case, since $\mu = 1$, we can use envelope detector to recover $m(t)$.

Part d)

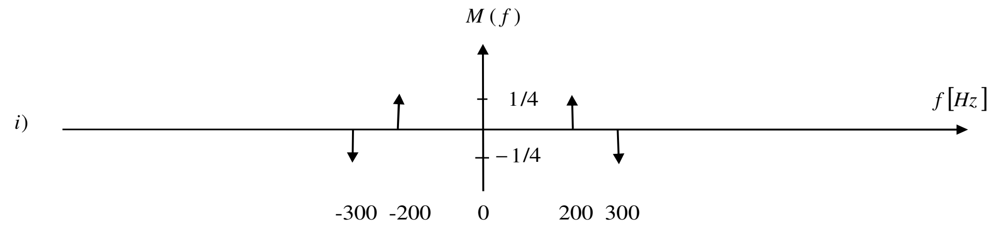
If b=0.5 then $\mu = \frac{2}{2\times 0.5} = 2$ and $\eta = \frac{1}{3\times 0.5^2 + 1} = \frac{4}{7} = 57\%$

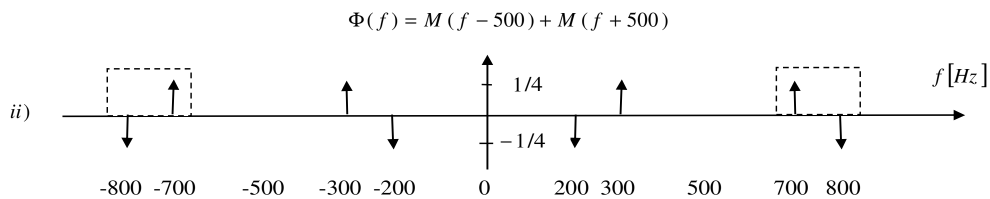
In this case, since $\mu = 2 \geq 1$, therefore we cannot use envelope detector to recover $m(t)$.

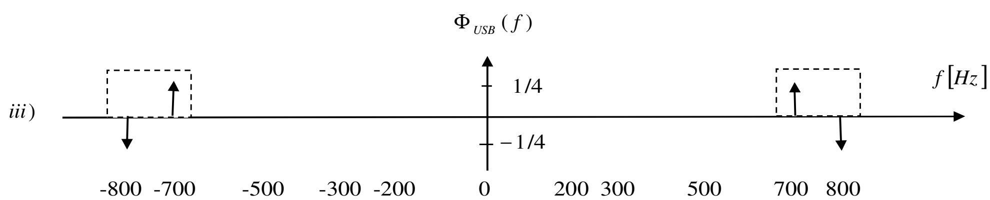
Solution 4.4.2b:

$$m(t) = \sin 100\pi t + \sin 500\pi t = \frac{1}{2}\left[\cos(100\pi t - 500\pi t) - \cos(100\pi t + 500\pi t)\right]$$

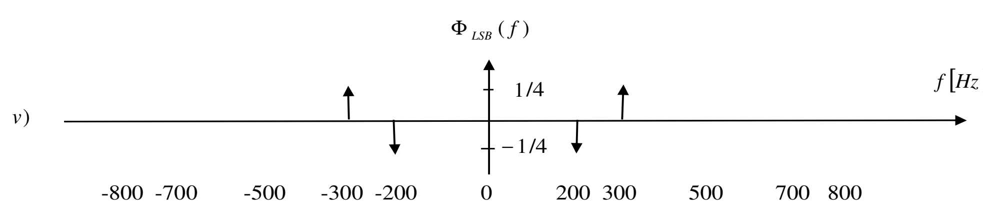
$$m(t) = \frac{1}{2}\cos(400\pi t) - \frac{1}{2}\cos(600\pi t) \overset{FT}{\Leftrightarrow} M(f) = \frac{1}{4}\left[\delta(f-200) + \delta(f+200)\right] - \frac{1}{4}\left[\delta(f-300) + \delta(f+300)\right]$$







iv) From Part iii: $\varphi_{USB}(t) = \frac{1}{2}\cos(1400\pi t) - \frac{1}{2}\cos(1600\pi t)$



And therefore: $\varphi_{LSB}(t) = \frac{1}{2}\cos(600\pi t) - \frac{1}{2}\cos(400\pi t)$

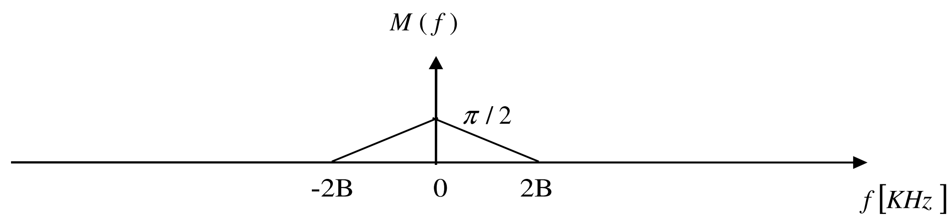
Solution 4.4.4:

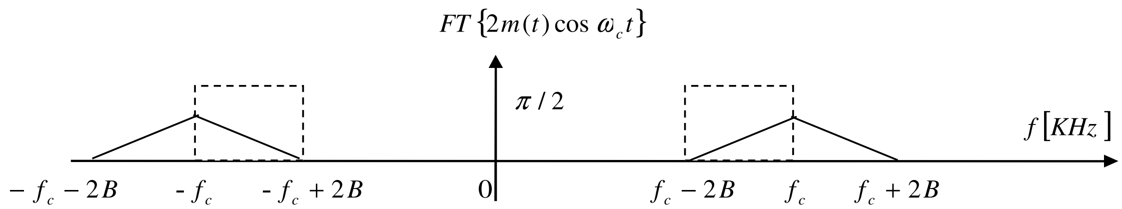
Part a)

$$B\sin c^2(\pi Bt) \overset{FT}{\Leftrightarrow} \Delta\left(\frac{f}{2B}\right)$$

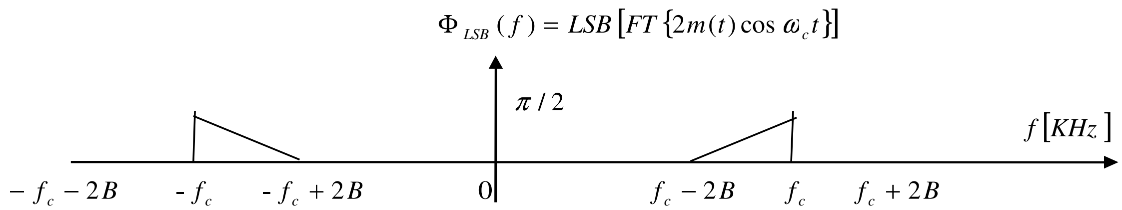
$$2B\sin c^2(2\pi Bt) \overset{FT}{\Leftrightarrow} \Delta\left(\frac{f}{2\times 2B}\right)$$

$$m(t) = \pi B\sin c^2(2\pi Bt) \overset{FT}{\Leftrightarrow} \frac{\pi}{2}\Delta\left(\frac{f}{4B}\right)$$





Part b)



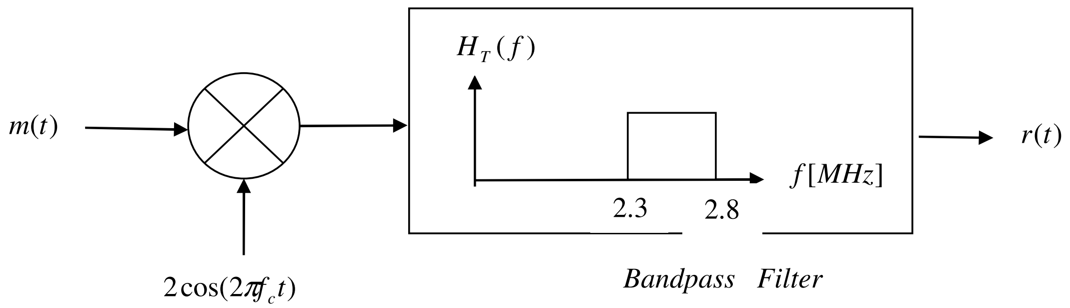
Part c)

$$\varphi_{LSB}(t) = \int_{-\infty}^{\infty}\Phi_{LSB}(f)e^{j2\pi ft}df = \int_{-f_c}^{-f_c+2B}\left[-\frac{\pi}{4B}\left(f+f_c-2B\right)\right]e^{j2\pi ft}df + \int_{f_c-2B}^{f_c}\left[\frac{\pi}{4B}\left(f-f_c+2B\right)\right]e^{j2\pi ft}df$$

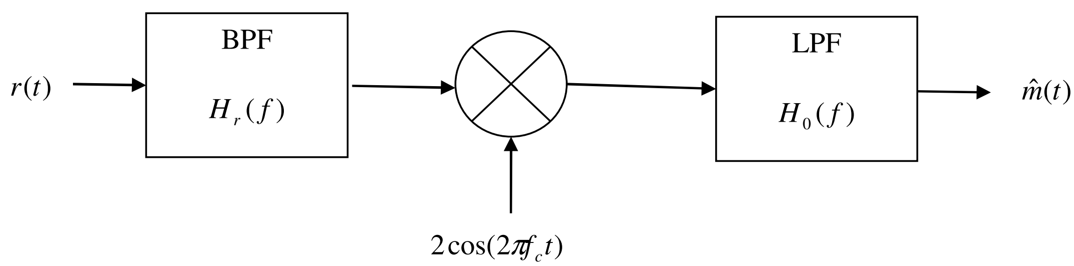
Solution 4.5.3:

Part a)

The massage signal $m(t)$ has bandwidth of 0.4 MHz and the transmit bandpass filter has bandwidth of 0.5 MHz extending from 2.3 MHz to 2.8 MHz. Therefore, we can use VSB amplitude modulation using upper side band and have a carrier frequency of 2.4 MHz. Therefore, the upper side band will extend from 2.4 to 2.8 MHz and we can use the extra bandwidth from 2.3 to 2.4 to implement VSB.

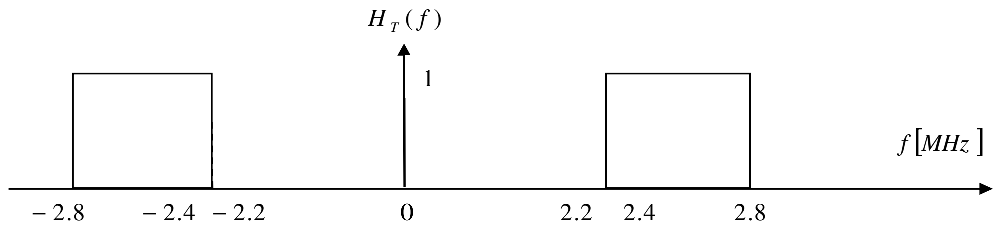


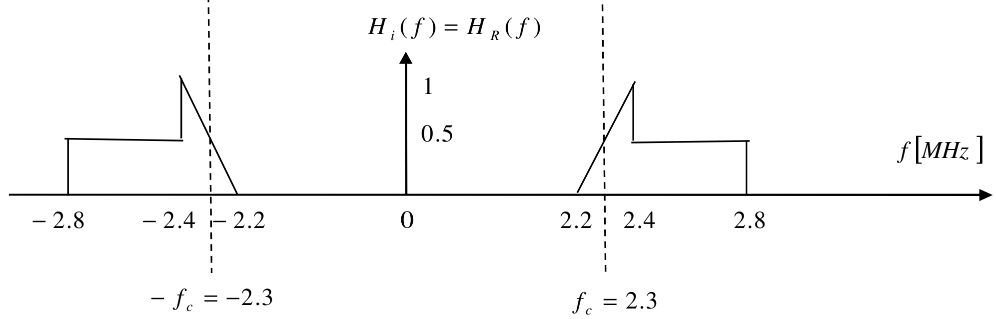
Following figure shows the receiver bock diagram where the carrier frequency is $f_c = 2.4MHz$.

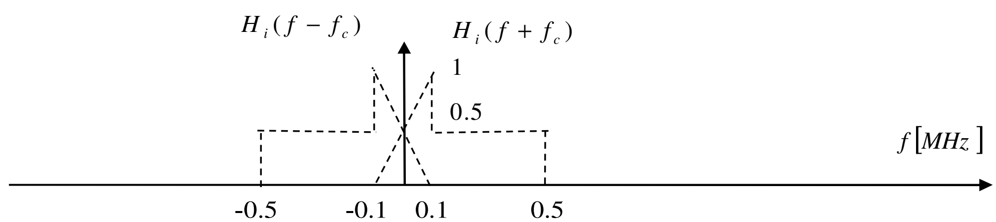


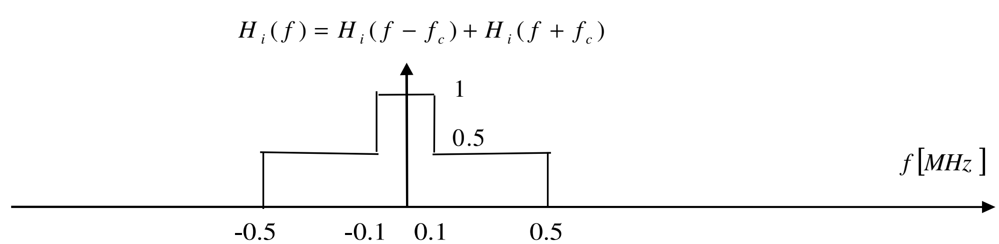
Part b)

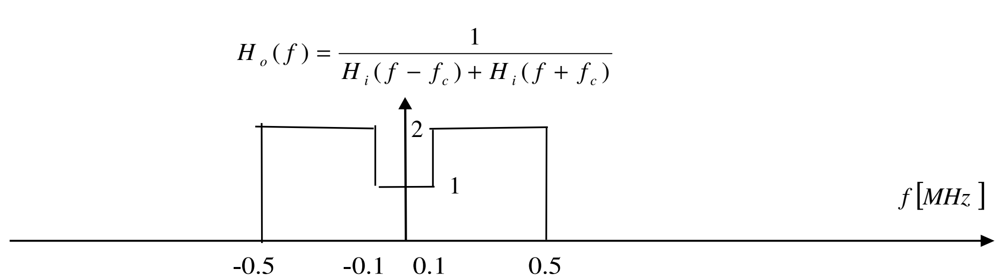
Following figure illustrates one possible solution for $H_R(f)$, $H_i(f - f_C) + H_i(f + f_C)$ and $H_o(f)$.


$H_T(f)$
1
$f[MHz]$
-2.8 -2.4 -2.2 0 2.2 2.4 2.8


$H_i(f) = H_R(f)$
1
0.5
$f[MHz]$
-2.8 -2.4 -2.2 0 2.2 2.4 2.8
$-f_c = -2.3$
$f_c = 2.3$


$H_i(f - f_c)$
$H_i(f + f_c)$
1
0.5
$f[MHz]$
-0.5 -0.1 0.1 0.5


$H_i(f) = H_i(f - f_c) + H_i(f + f_c)$
1
0.5
$f[MHz]$
-0.5 -0.1 0.1 0.5


$H_o(f) = \frac{1}{H_i(f - f_c) + H_i(f + f_c)}$
2
1
$f[MHz]$
-0.5 -0.1 0.1 0.5