Solution 5.1.2:

Part a)

For the case of FM:

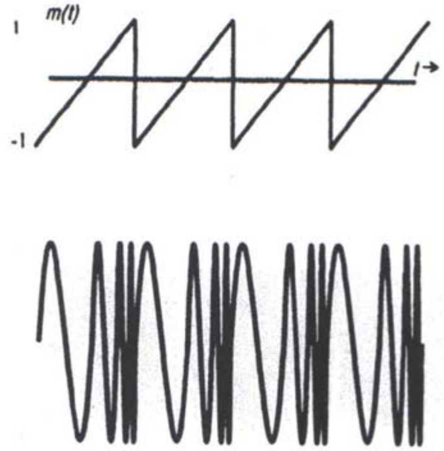
$$\varphi_{FM}(t) = \cos\left[2\pi f_c t + k_f \int_{-\infty}^{t} m(t)dt\right] = \cos\left[2\pi \times 10^6 t + 2000\pi \int_{-\infty}^{t} m(t)dt\right]$$

$$f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^6 + \frac{2000\pi}{2\pi} m(t) = 10^6 + 10^3 m(t)$$

$$(f_i)_{\min} = 10^6 + 10^3 \times (-1) = 999{,}000 = 999KHz$$

$$(f_i)_{\max} = 10^6 + 10^3 \times (+1) = 1001{,}000 = 1001KHz$$

Therefore, the instantaneous frequency f_i of the FM signal changes linearly from 999 to 1001 KHz as shown in the following figure.

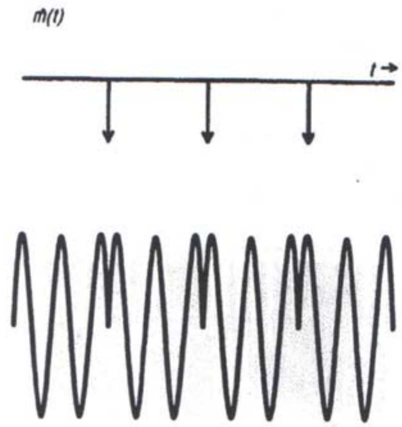


For the case of PM:

$$\varphi_{PM}(t) = \cos[2\pi f_c t + k_p m(t)] = \cos\left[2\pi \times 10^6 t + \frac{\pi}{2} m(t)\right]$$

$$f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t) = 10^6 + \frac{\pi/2}{2\pi} \dot{m}(t) = 10^6 + \frac{1}{4}\frac{d}{dt}\left(\frac{2t}{10^{-3}}\right) = 10^6 + 500 = 1000.5KHz$$

Therefore, the instantaneous frequency is 1000.5 KHz. Obviously, there will be a phase change at discontinuities of $m(t)$. The phase jump is $2k_p = 2 \times \frac{\pi}{2} = \pi$.



Part b)

If we have a jump of Δ in the value of $m(t)$ in some instance, we will have a phase change of Δk_p which should not be larger than 2π because of ambiguity. In our example $\Delta = 2$, therefore we should have $k_p < \pi$.

This PM signal is equivalent to another PM signal with $f_c = 1000.5KHz$ and periodic rectangular message with period of 2×10^{-3} that switches from 1 to -1 and -1 to 1.

Solution 5.2.7:

5.2-7 $m(t) = \sin 2000\pi t$, $B = 1$ kHz, $k_f = 200000\pi$, and $k_p = 10$.

(a) **PM:** $\dot{m}(t) = 2000\pi \cos 2000\pi t$, $\dot{m}_p = 2000\pi$,

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(10)(2,000\pi)}{2\pi} = 10\,\text{kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(10 + 1) = 22$ kHz.

FM: $m_p = 1$, $\Delta f = k_f m_p/(2\pi) = 200000\pi/(2\pi) = 100$ kHz. Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(100 + 1) = 202$ kHz.

(b) $m(t) = 2\sin 2000\pi t$, $B = 1$ kHz. **PM:** $\dot{m}(t) = 4000\pi \cos 2000\pi t$, $\dot{m}_p = 4000\pi$;

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(10)(4,000\pi)}{2\pi} = 20\,\text{kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(20 + 1) = 42$ kHz.

FM: $m_p = 2$, $\Delta f = k_f m_p/(2\pi) = 400000\pi/(2\pi) = 200$ kHz. Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(200 + 1) = 402$ kHz.

(c) $m(t) = \sin 4000\pi t$, $B = 2$ kHz. **PM:** $\dot{m}(t) = 4000\pi \cos 4000\pi t$, $\dot{m}_p = 4000\pi$;

$$\Delta f = \frac{k_p \dot{m}_p}{2\pi} = \frac{(10)(4,000\pi)}{2\pi} = 20\ \text{kHz}$$

$B_{\text{PM}} = 2(\Delta f + B) = 2(20 + 2) = 44$ kHz.

FM: $m_p = 1$, $\Delta f = k_f m_p/(2\pi) = 200,000\pi/2\pi) = 100$ kHz; Thus, $B_{\text{FM}} = 2(\Delta f + B) = 2(100 + 2) = 204$ kHz.

(d) Doubling the amplitude of $m(t)$ roughly doubles the bandwidth of both FM and PM. Doubling the frequency of $m(t)$ (i.e., expanding the spectrum of $M(\omega)$ by a factor of 2) has hardly any effect on the FM bandwidth. However, it roughly doubles the bandwidth of PM, indicating that the PM spectrum is sensitive to the shape of the baseband spectrum. The FM spectrum is relatively insensitive to the nature of the spectrum $M(\omega)$.

Solution 5.3.3:

5.3-3 The design is shown in Fig. S5.3-3. In this case, the NBFM generator generates $f_{c_1} = 150$ kHz, and $\Delta f_1 = 10$ Hz. The final WBFM should have $f_{c_4} = 96.3$ MHz, and $\Delta f_4 = 20.48$ kHz. The total factor of frequency multiplication needed is $M_1 \cdot M_2 = \frac{\Delta f_4}{\Delta f_1} = 2048$. Because only frequency doublers are available, we find that $M_1 \cdot M_2 = 2^{11} = 2048$. Now, $M_1 = 2^{n_1}$, $M_2 = 2^{n_2}$, $n_1 + n_2 = 11$, $f_{c_2} = 2^{n_1} f_{c_1}$, and $f_{c_4} = 2^{n_2} f_{c_3}$. In order to find f_{LO}, there are three possible relationships: $f_{c_3} = f_{c_2} \pm f_{LO}$, $f_{LO} - f_{c_2}$. Each should be tested to determine the one that will require $13 \times 10^6 \le f_{LO} \le 14 \times 10^6$.

First, we test $f_{c_3} = f_{c_2} - f_{LO}$. This case leads to $96.3 \text{ MHz} = f_{c_4} = 2^{n_2} f_{c_3} = 2^{n_2}(f_{c_2} - f_{LO}) = 2^{n_2}(2^{n_1} f_{c_1} - f_{LO}) = 2^{n_1+n_2} f_{c_1} - 2^{n_2} f_{LO} = 2^{11}(150 \times 10^3) - 2^{n_2} f_{LO}$. Thus, we have $f_{LO} = 2^{-n_2}(3.072 \times 10^8 - 9.63 \times 10^7) = 2^{-n_2}(2.109 \times 10^8)$.

In this case, if $n_2 = 4$, then $f_{LO} = 13.1813$ MHz, which is in the desired range. We won't test the other cases since this one works. Thus, the final design is $M_1 = 128$, $M_2 = 16$, and $f_{LO} = 13.1813$ MHz. This gives $f_{c_2} = 2^{n_1} f_{c_1} = 19.2$ MHz, $\Delta f_2 = M1 \cdot \Delta f_1 = 1280$ Hz, $f_{c_3} = f_{c_2} - f_{LO} = 19.2 - 13.1813 = 6.0187$ MHz, $\Delta f_3 = 1280$ Hz. The bandpass filter used will be centered at 6.0187 MHz.

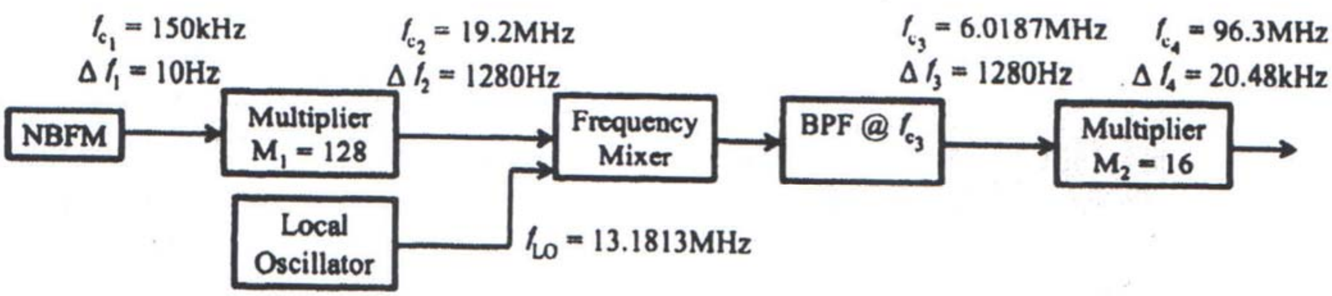


Fig. S5.3-3

Solution 5.4.2:

5.4-2 Given that $f_c = 10$ kHz, $\Delta f = 1$ kHz, and the message is periodic square wave of period T_0, the resulting FM signal simply switches instantaneous frequency from 11 kHz to 9 kHz and back over one period. Thus,

$$\varphi_{FM}(t) = A \cos [20000\pi t \pm 2000\pi t]$$

over any given half period. Now, after the ideal differentiator,

$$\dot{\varphi}_{FM}(t) = -(20000\pi \pm 2,000\pi) A \sin [2,000\pi t \pm 2000\pi t]$$

Next, after the envelope detector, the output will be a periodic square wave proportional to $(20000\pi \pm 2,000\pi)A$, with a dc offset. After dc blocking, the result is a periodic square wave proportional to $m(t)$. This is illustrated in Fig. S5.4-2.

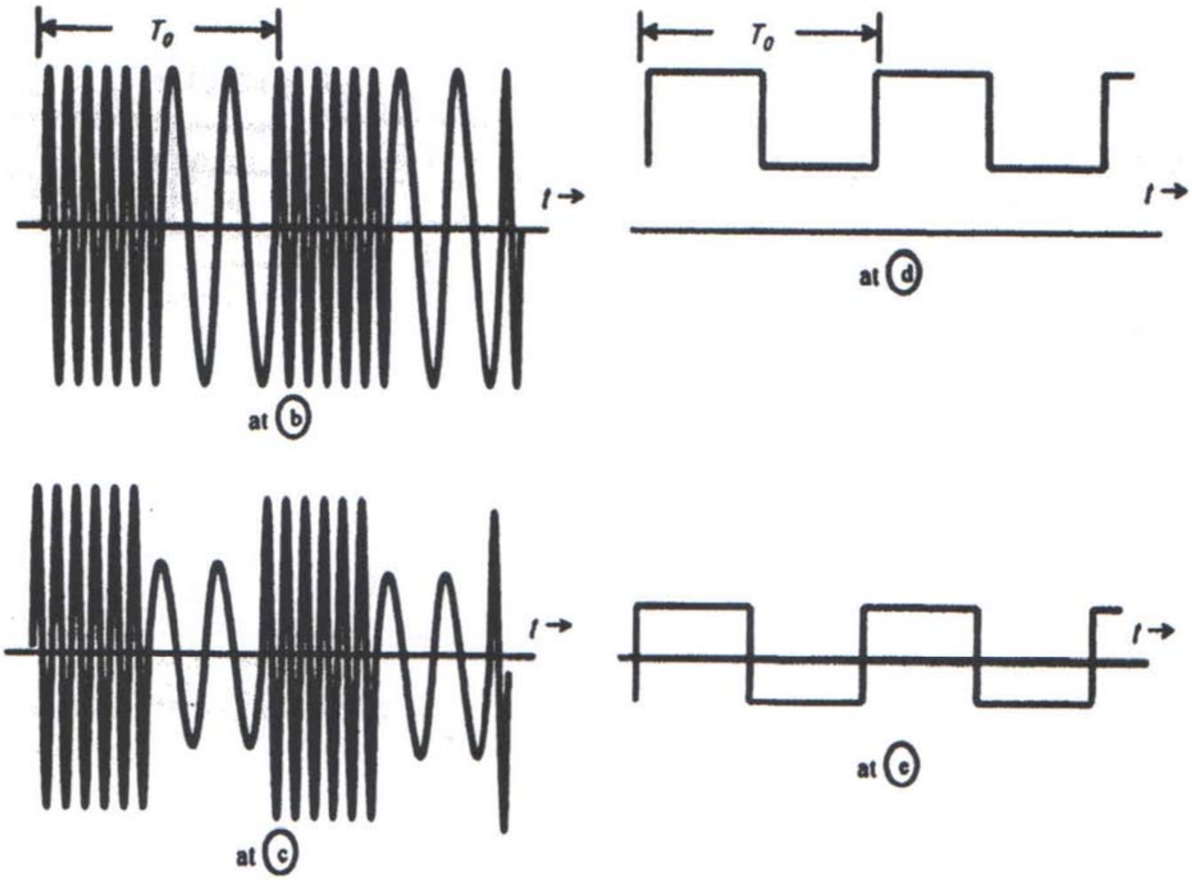


Fig. S5.4-2

Solution 5.6.2:

5.6-2 With $f_{IF} = 10.7$ MHz and a desired range of 88 to 108 MHz, $[f_{LO}]_{min} = 88 + 10.7 = 98.7$ MHz and $[f_{LO}]_{max} = 108 + 10.7 = 118.7$ MHz. Image stations exist at frequencies $2f_{IF} = 21.4$ MHz apart. Since $88 + 21.4 = 109.4$ MHz and $108 - 21.4 = 86.6$ MHz, all of the image stations will be out of the frequency band used for this FM system, so it is not possible to pick up a signal from a second one of these FM stations.