Question 1:

The message signal $m(t)$ with power of 20 mWatts is applied to an analog-to-digital convertor with dynamic range of -1 volt to 1 volt.

a) To transmit this signal by PCM, uniform quantization is adopted. If the SQNR is required to be at least 43 dB, determine the minimum number of bits required to code the uniform quantizer. Determine the SQNR obtained with this quantizer.
b) Repeat part a, if a $\mu-$law compander is applied with $\mu=100$ to achieve a uniform quantizer.
c) If the power of the signal $m(t)$ is reduced to 5 mWatts, solve parts a and b and discuss the results.

Solution 1:

Part a)

$$\frac{S_o}{N_o}=3L^2\frac{\overline{m^2(t)}}{m_p^2}\Rightarrow 10^{4.3}=3L^2\frac{20\times10^{-3}}{1^2}\Rightarrow L=576.66$$

L should be power of 2 and larger than 576. Therefore, $2^n=1024$ and we need at least $n=10$ bits for the quantizer.

With 10 bits for the quantizer, the SQNR is,

$$\frac{S_o}{N_o}=3L^2\frac{\overline{m^2(t)}}{m_p^2}=3(1024)^2\frac{20\times10^{-3}}{1^2}=62914.56\Rightarrow \qquad SQNR=\frac{S_o}{N_0}=47.98\quad dB$$

Part b)

Let's check the condition: $\mu^2>>\frac{m_p^2}{\overline{m^2(t)}}\Rightarrow 100^2>>\frac{1^2}{20\times10^{-3}}\Rightarrow 10000>>50$

So the condition is satisfied and therefore,

$$\frac{S_o}{N_o}=\frac{3L^2}{[\ln(1+\mu)]^2}\Rightarrow 10^{4.3}=\frac{3L^2}{[\ln(1+100)]^2}\Rightarrow L=376.37$$

L should be power of 2 and larger than 376. Therefore, $2^n=512$ and we need at least $n=9$ bits for the quantizer.

With 9 bits for the quantizer, the SQNR is,

$$\frac{S_o}{N_o}=\frac{3L^2}{[\ln(1+\mu)]^2}=\frac{3\times512^2}{[\ln(1+100)]^2}=36922.83\Rightarrow \qquad SQNR=\frac{S_o}{N_0}=45.67\quad dB$$

Part c)

Without compander:

$$\frac{S_o}{N_o} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2} \Rightarrow 10^{4.3} = 3L^2 \frac{5\times 10^{-3}}{1^2} \Rightarrow L = 1153.33$$

L should be power of 2 and larger than 1153. Therefore, $2^n = 2048$ and we need at least $n = 11$ bits for the quantizer.

With compander, the condition $\mu^2 >> \frac{m_p^2}{\overline{m^2(t)}}$ is still valid and the number of bits should be at least 9 bits which is the same as part b since with compander, the SQNR is independent of power of $m(t)$.

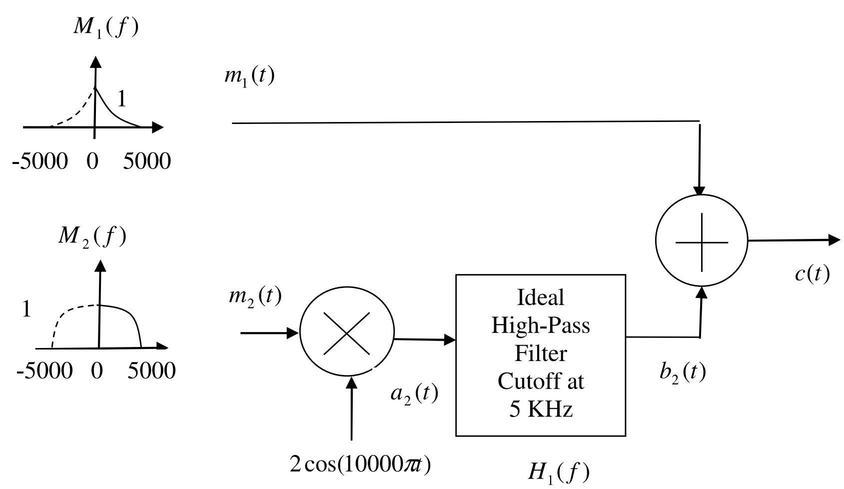
Conclusion: To achieve a SQNR of 43 dB, we have following comparisons:

When compander is not used, the number of bits for the quantizer increases from 10 to 11 by decreasing signal power from 20 to 5 mWatts. However, when compander is used, the number of bits used is 9 and it is constant by changing the signal power.

Question 2:

Two signals $m_1(t)$ and $m_2(t)$ band-limited to 5000 Hz are to be transmitted on a single wireless link. Following steps are considered to prepare these signals for transmission on the wireless link.

a) First following frequency multiplexer is used. Show the frequency spectrum of the output of this system.



b) The output of the system in part "a" is sampled with ideal impulses which have frequency of 150% of the Nyquist rate. Draw the frequency spectrum after the sampler.

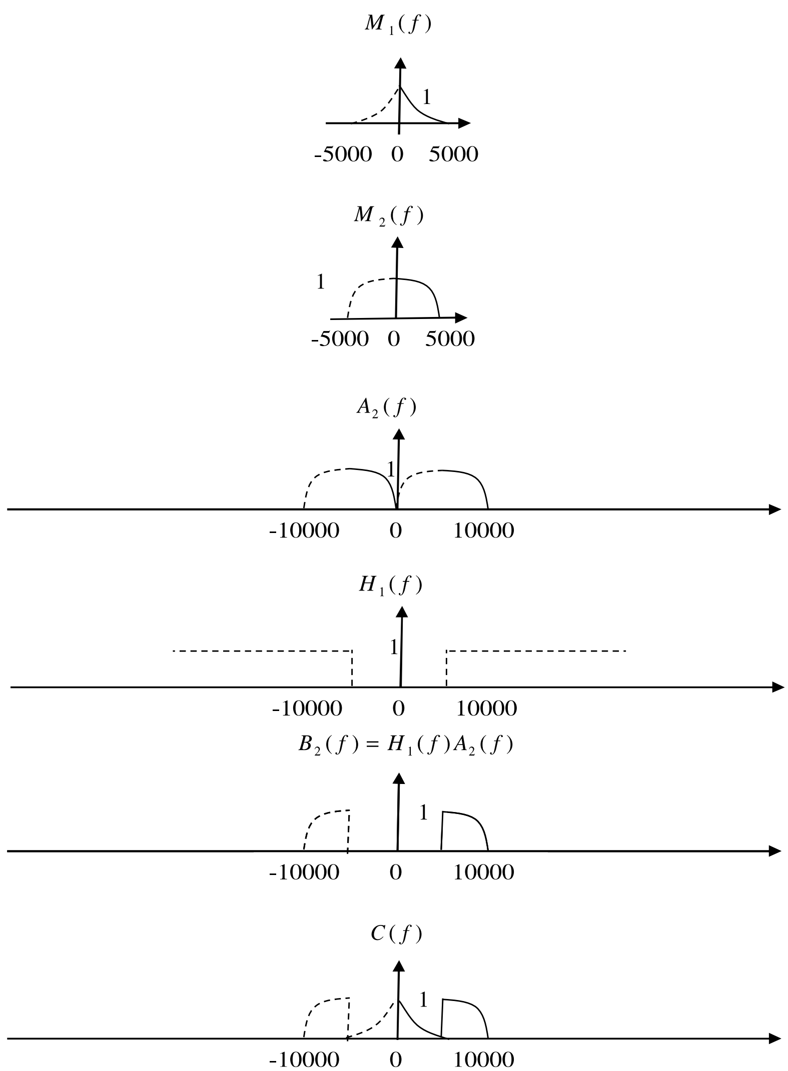
c) The sampler output is applied to a $\mu -$ law compander with $\mu = 100$ and then to a linear quantizer. SQNR of the quantizer has to be at least 45 dB. The output of the quantizer is transferred to a serial bit stream to be transmitted via a digital communication link. Calculate minimum required baseband bandwidth of the wireless link, if the information is sent bit by bit.

Note: Assume that equation 6.36 can be used for SQNR.

d) What is the baseband bandwidth of the wireless link if a Raised cosine pulse shaping with roll-off factor of 20% is used.

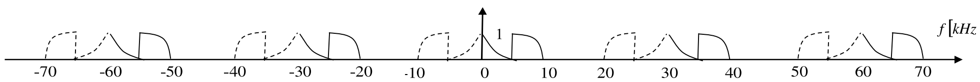
Solution 2:

Part a)



Part b)

Nyquist sampling frequency is double the maximum frequency of $C(f)$ which is $2\times10=20kHz$ and therefore we should sample with 150% of that which is 30 kHz. If we sample $C(f)$ at 30kHz, the spectrum will be as shown below. Note that the spectrum of $C(f)$ will be repeated at $n\times30kHz$, where $n=\ldots,-4,-3,-2,-1,0,1,2,3,4,\ldots$.



Part c)

$$\frac{S_o}{N_o}=\frac{3L^2}{[\ln(1+\mu)]^2}\Rightarrow 10^{4.5}=\frac{3L^2}{[\ln(1+100)]^2}\Rightarrow L=473.82$$

L should be power of 2 and larger than 473. Therefore, $2^n=512$ and we need at least $n=9$ bits for the quantizer.

The sampling frequency is $f_s=30000Hz$ and each sample is $n=9$ bits, therefore the bit rate for bit-by-bit serial transmission is $R_b=f_s\times n=30000\times9=270{,}000\quad bits/\sec$.

Therefore, the minimum bandwidth to transmit this data is $BW_{\min}=\frac{R_b}{2}=\frac{270000}{2}=135{,}000\quad Hz$.

Part d)

The baseband bandwidth is $BW_{bb}=(1+\alpha)\frac{R_b}{2}=(1+0.2)\frac{270000}{2}=162{,}000\quad Hz$.

Question 3:

Two signals $m_1(t)$ and $m_2(t)$ band-limited to 5000 Hz are to be transmitted on a single wireless link. Following steps are considered to prepare these signals for transmission on the wireless link.

a) Each of the signals $m_1(t)$ and $m_2(t)$ are sampled with ideal impulses which have frequency of 150% of the Nyquist rate.
b) Then the output of each sampler output is applied to a $\mu-$law compander with $\mu=100$ and then to a linear quantizer. Calculate the minimum number of bits required to represent each sample (L) of the quantizer, if SQNR of the quantizer has to be at least 45 dB.

   Note: Assume that equation 6.36 can be used for SQNR.

c) The outputs of quantizers are time multiplexed and a bit stream is produced to be transmitted via a digital communication link. Calculate minimum required baseband bandwidth of the wireless link if the information is sent bit by bit.

d) What is the baseband bandwidth of the wireless link if a Raised cosine pulse shaping with roll-off factor of 20% is used.

Solution 3:

Part a)

Nyquist sampling frequency is double the maximum frequency of $M_1(f)$ and $M_2(f)$ which is $2\times 5 = 10kHz$ and therefore we should sample with 150% of that which is 15 kHz.

Part b)

$$\frac{S_o}{N_o} = \frac{3L^2}{[\ln(1+\mu)]^2} \Rightarrow 10^{4.5} = \frac{3L^2}{[\ln(1+100)]^2} \Rightarrow L = 473.82$$

L should be power of 2 and larger than 473. Therefore, $2^n = 512$ and we need at least $n = 9$ bits for the quantizer.

Note that for each signal $n = 9$ bits.

Part c)

The sampling frequency is $f_s = 15000Hz$ and each sample is $n = 9$ bits per signal, therefore the bit rate for bit-by-bit serial transmission is $R_b = 2\times f_s \times n = 2\times 15000 \times 9 = 270{,}000 \quad bits/\sec$.

Therefore, the minimum bandwidth to transmit this data is $BW_{\min} = \frac{R_b}{2} = \frac{270000}{2} = 135{,}000 \quad Hz$.

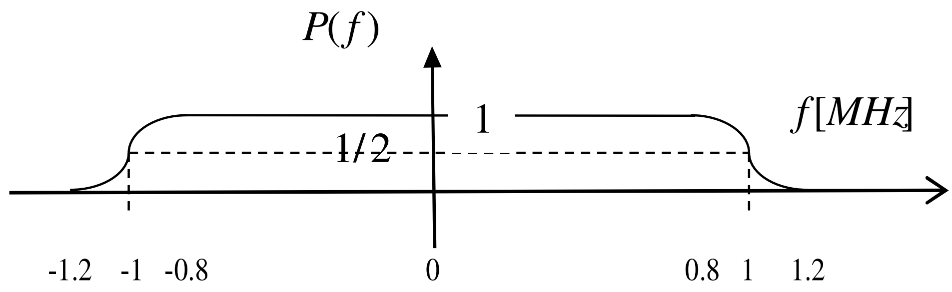
Part d)

The baseband bandwidth is $BW_{bb} = (1+\alpha)\frac{R_b}{2} = (1+0.2)\frac{270000}{2} = 162{,}000 \quad Hz$.

Question 4:

The Fourier transform $P(f)$ of the basic pulse $p(t)$ used in a binary communication system is shown in the figure.

a) From the shape of $P(f)$, explain at what pulse rate this pulse would satisfy Nyquist's first criterion.
b) Explain why using this pulse does not cause inter bit interference (ISI).
c) Explain how much is the excess bandwidth and find the roll-off factor.



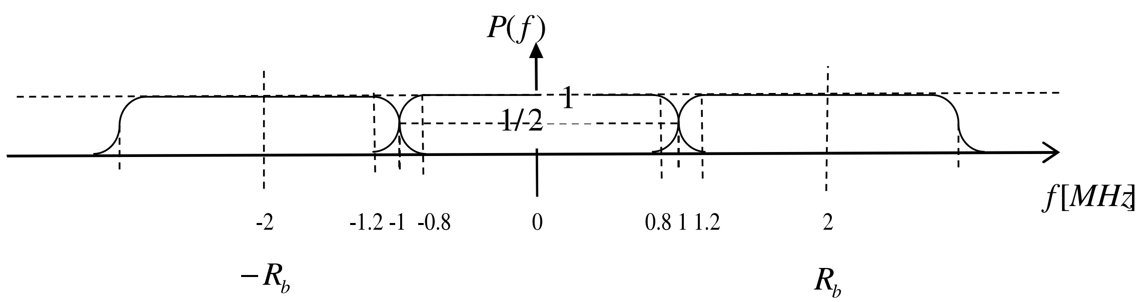
Solution 4:

Part a)

To satisfy first criterion of Nyquist, we should have:

$$\sum_{n=-\infty}^{+\infty} P(f - nR_b) = \text{constant}$$

Obviously, this will be satisfied when $\frac{R_b}{2} = 1 \quad MHz$ as shown below:



Part b)

There is no inter-bit-interference (no ISI) since first Nyquist criteria according to part “a” is satisfied and therefore:

$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm nT_b \quad n = \ldots,-3,-2,-1,+1,+2,+3,\ldots \end{cases}$$

Part c)

The minimum bandwidth is $\frac{R_b}{2} = 1 \quad MHz$ and the base-band bandwidth is $(1+\alpha)\frac{R_b}{2} = 1.2 \quad MHz$

Therefore, the excess bandwidth is $1.2 - 1.0 = 0.2 \quad MHz$ and $\alpha \times 1 = 0.2 \quad \Rightarrow \qquad \alpha = 0.2$