ELEC366

Fundamentals of Telecom. Systems

Summer 2015

Lecture 1

Course outline :

1) Review of Signals & Systems
2) Amplitude Modulation
3) Angle Modulation
4) Noise in AM & FM Systems.
5) Sampling and Quantization
6) Digital Modulation
7) Digital Services and Technologies.

Signals & Systems

<u>Signals</u> are time-varying quantities, i.e., they are functions of time, e.g., speech, video, data.

Signals may be time-<u>discrete</u> or time-continuous, <u>digital</u> or <u>analog</u>.

<u>Energy signals</u>:

if the energy of a signal is finite, i.e., iff

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt < \infty$$

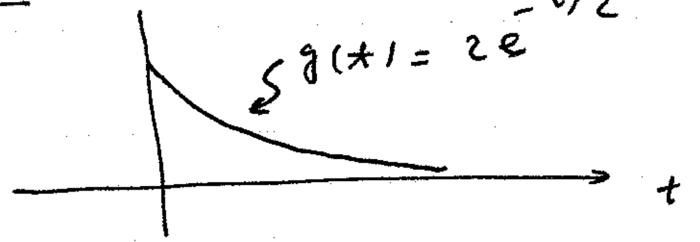
then the signal is called an <u>energy signal</u>.

<u>Power signals</u>: if the Power of a signal:

$$0 < P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty$$

then the signal is called a <u>power signal</u>.

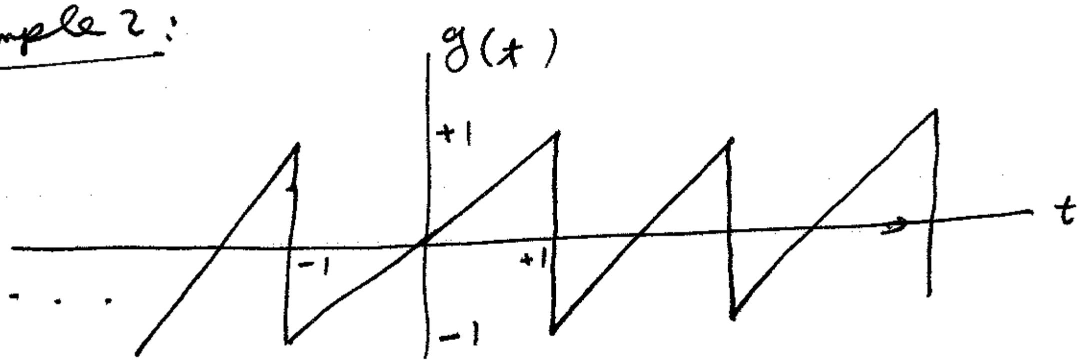
example 1:



$$E_g = \int_0^{\infty} 4e^{-t}\,dt = 4\int_0^{\infty} e^{-t} = 4$$

$$P_g = 0$$

Example 2:



$E_g = \infty \Rightarrow$ not an energy signal

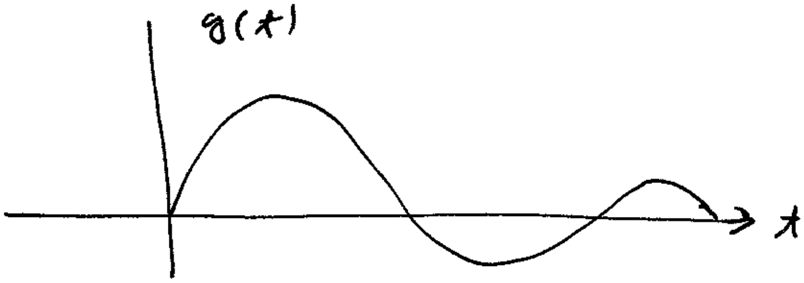
$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g^2(t)\,dt = \frac{1}{2}\int_{-1}^{1} g^2(t)\,dt = \frac{1}{2}\int_{-1}^{1} t^2\,dt$$

$$P_g = \frac{1}{2} \times \frac{t^3}{3}\Big|_{-1}^{1} = \frac{1}{6}(1+1) = \frac{1}{3}$$

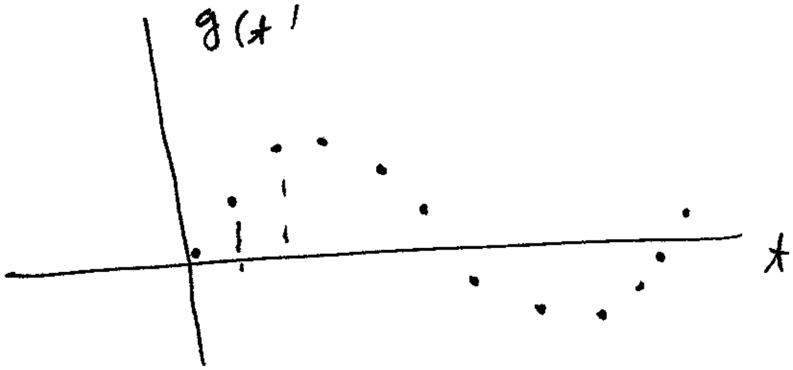
A signal can be Power, energy, non but not both at the same time.

Continuous-time $\longleftrightarrow$ Discrete-time

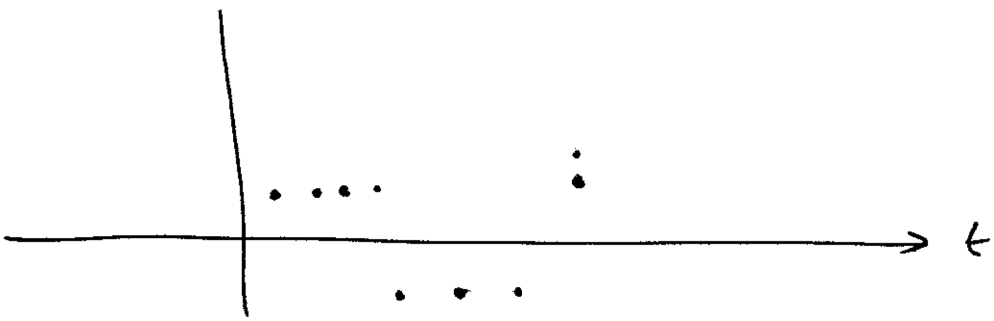
Analog $\longleftrightarrow$ Digital



Continuous time, Analog (continuous value)



discrete-time, Analog.



Digital (Discrete-time, discrete Amplitude)

Periodic $\longleftrightarrow$ aperiodic.

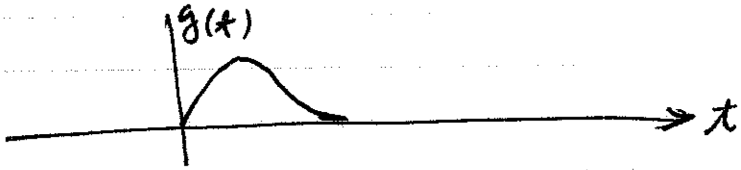
A signal is period with with period T if

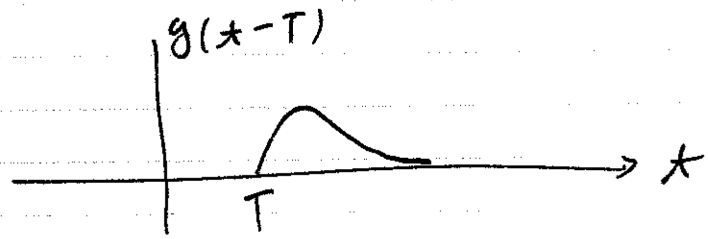
$$g(t) = g(t+T) \quad \text{all } t.$$

Signal operations

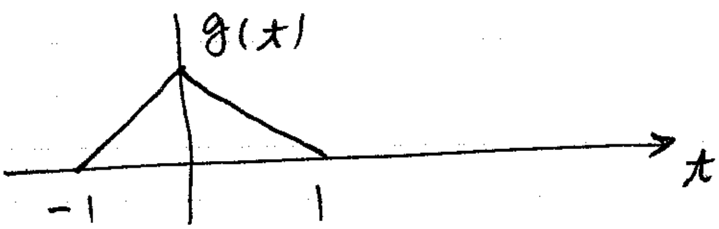
Time - shift

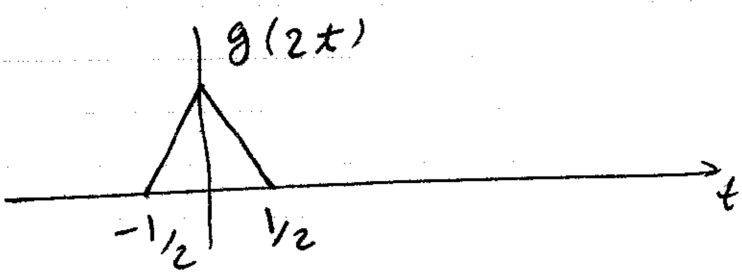
$$f(t) = g(t-T)$$

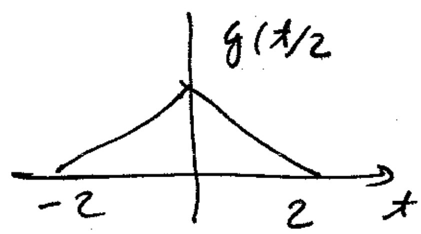




Time-Scaling $\quad f(t) = g(at)$

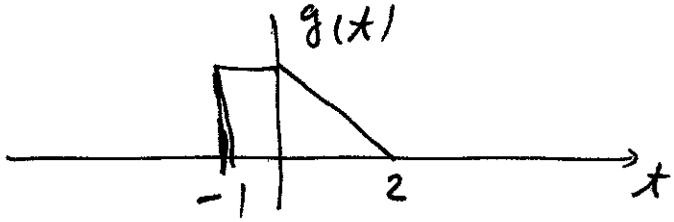


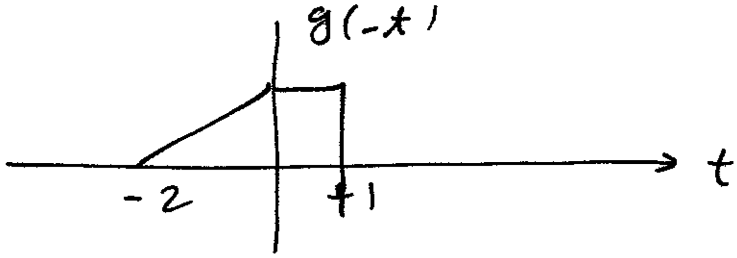




Time - reversal

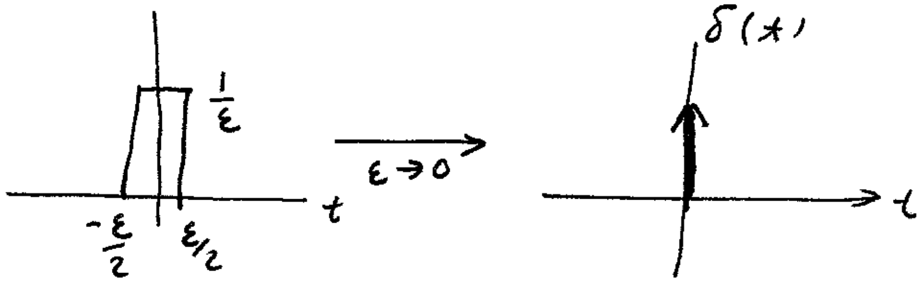
$$f(t) = g(-t)$$





Generalized Functions

Unit impulse function



$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

$$f(t) \cdot \delta(t) = f(0)\,\delta(t)$$

so :

$$\int_{-\infty}^{\infty} f(t)\,\delta(t)\,dt = f(0) \int_{-\infty}^{\infty} \delta(t)\,dt = f(0)$$

Similarly

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-T)\,dt = f(T)$$

this is called the sifting property

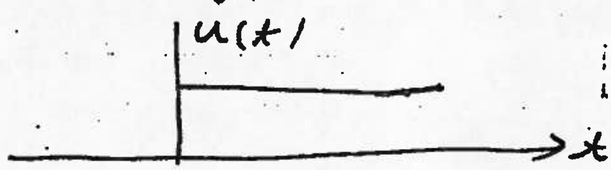
Unit step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} = u(t)$$

or

$$\delta(t) = \frac{d}{dt}\,u(t)$$



Fourier Series

Trigonometric Fourier Series:

$$g(t) = a_0 + a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \cdots + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \cdots$$

or

$$g(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$$

where

$$\omega_0 = \frac{2\pi}{T_0}$$

T_0 is the period of $g(t)$.

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t)\, dt$$

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \cos n\omega_0 t\, dt \qquad n = 1, 2, 3, \cdots$$

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} g(t) \sin n\omega_0 t\, dt \qquad n = 1, 2, 3, \cdots$$

Compact Trigonometric F. Series

$$a_n \cos n\omega_0 t + b_n \sin n\omega_0 t = C_n \cos(n\omega_0 t + \theta_n)$$

where

$$C_n = \sqrt{a_n^2 + b_n^2}$$

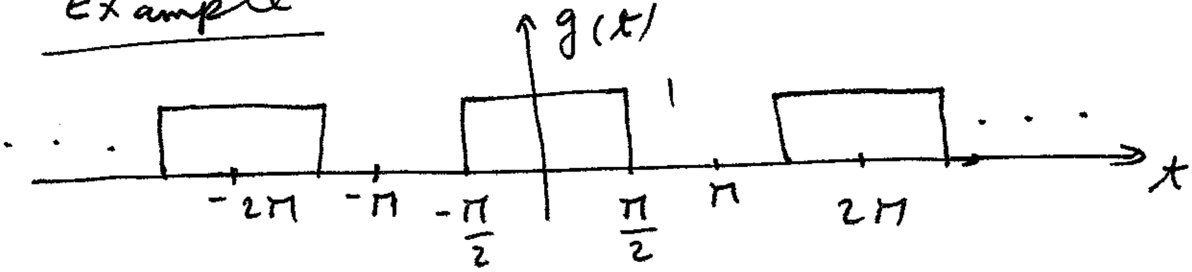
$$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$$

So:

$$g(t) = C_0 + \sum_{n=\phi}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

where $$C_0 = a_0$$

Example



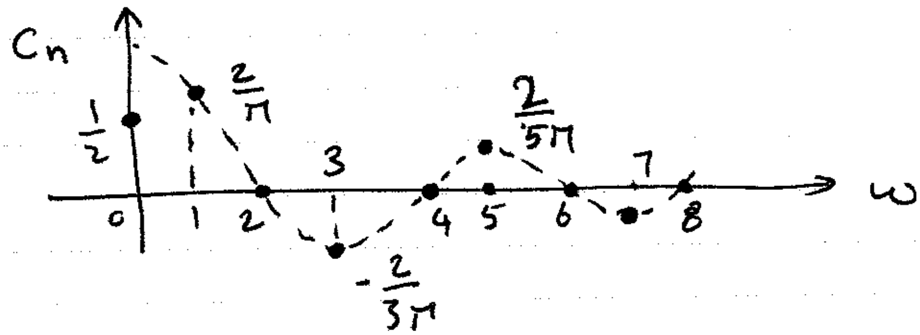
$$T_0 = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{T_0} = 1$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi/2}^{\pi} g(t)\,dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1\,dt = \frac{1}{2}$$

$$a_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} \cos(n\omega_0 t)\,dt = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$b_n = \frac{2}{2\pi}\int_{-\pi/2}^{\pi/2} \sin(n\omega_0 t)\,dt = 0$$

$$g(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \cdots\right)$$



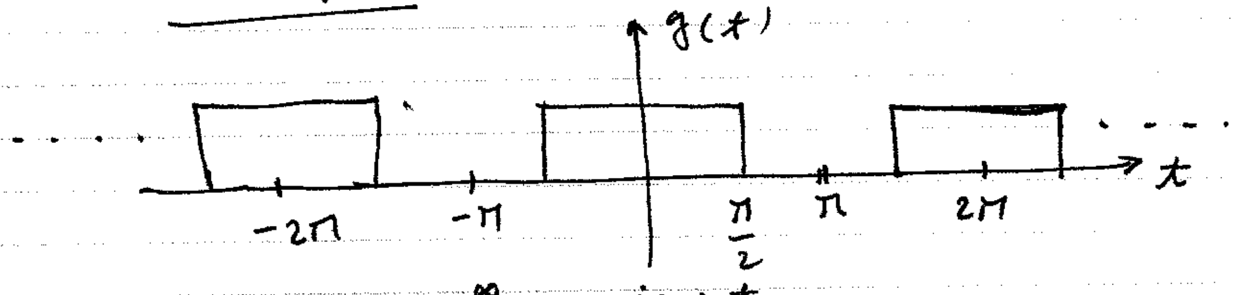
Exponential Fourier Series

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

where

$$D_n = \frac{1}{T_0}\int_{T_0} g(t)\, e^{-jn\omega_0 t}\, dt$$

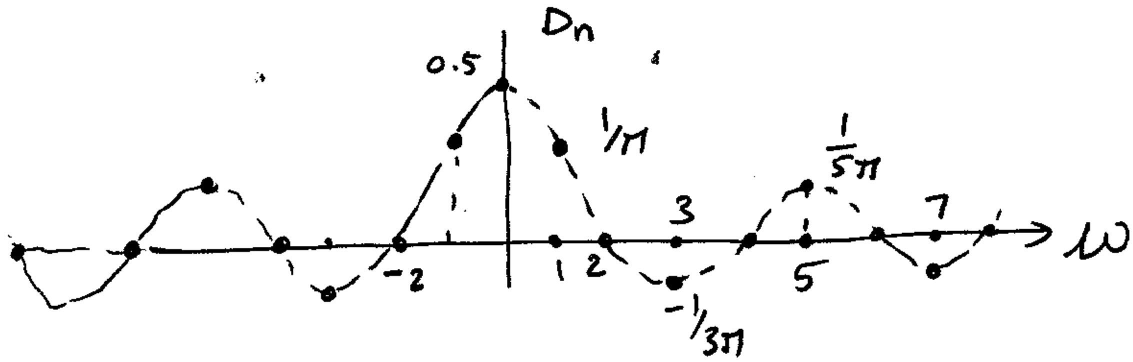
Example:



$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t) e^{-jn\omega_0 t}\, dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{-jn\omega_0 t}\, dt$$

$$D_n = \frac{2}{n\omega_0 T_0} \sin\left(\frac{n\omega_0 T_0}{4}\right) = \frac{1}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$



Parseval's Theorem

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

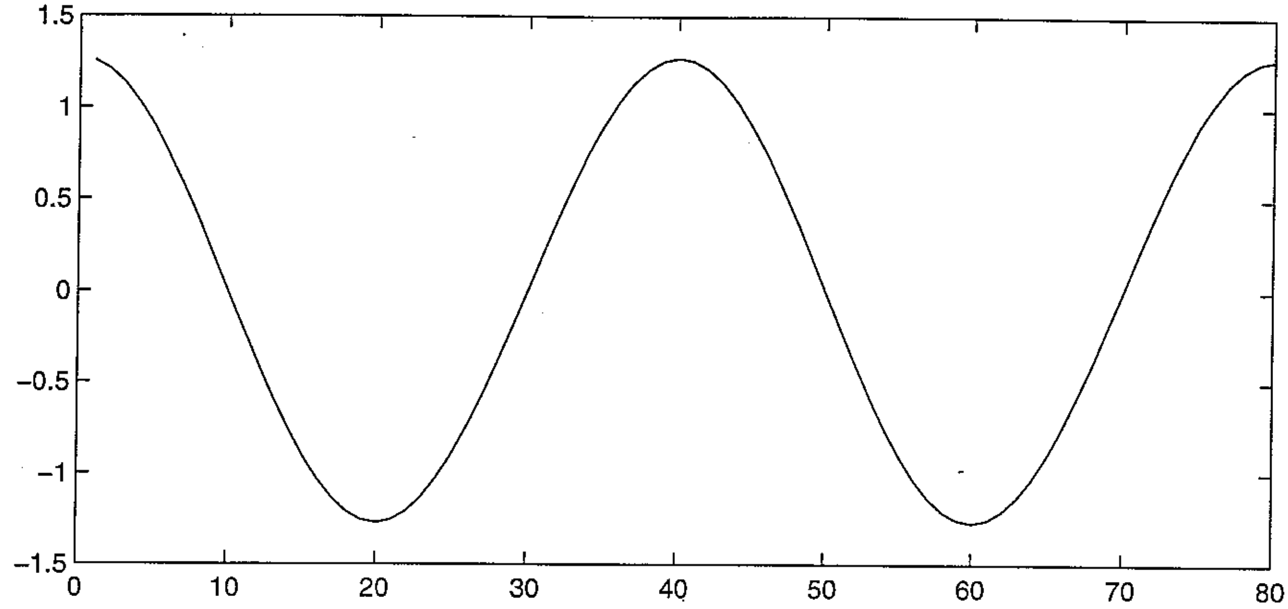
$$P_g = C_0^2 + \sum_{n=1}^{\infty} \frac{C_n^2}{2} = C_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} C_n^2 = a_0^2 + \frac{1}{2}\sum (a_n^2 + b_n^2)$$

Similarly, for

$$g(t) = D_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} D_n e^{jn\omega_0 t}$$

$$P_g = \sum_{n=-\infty}^{\infty} |D_n|^2 = |D_0|^2 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} |D_n|^2$$

```
function sq=sq(length,waves,period)
clf
S=period/2;
temp=1;
c=4/pi;
for k=1:length
x(k)=0;
end
for i=1:waves
j=2*i-1;
for k=1:length
x(k)=x(k)+c*temp*cos(pi*k*j/S)/j;
end
temp=temp*(-1);
hold
plot(x);
%hold
end
```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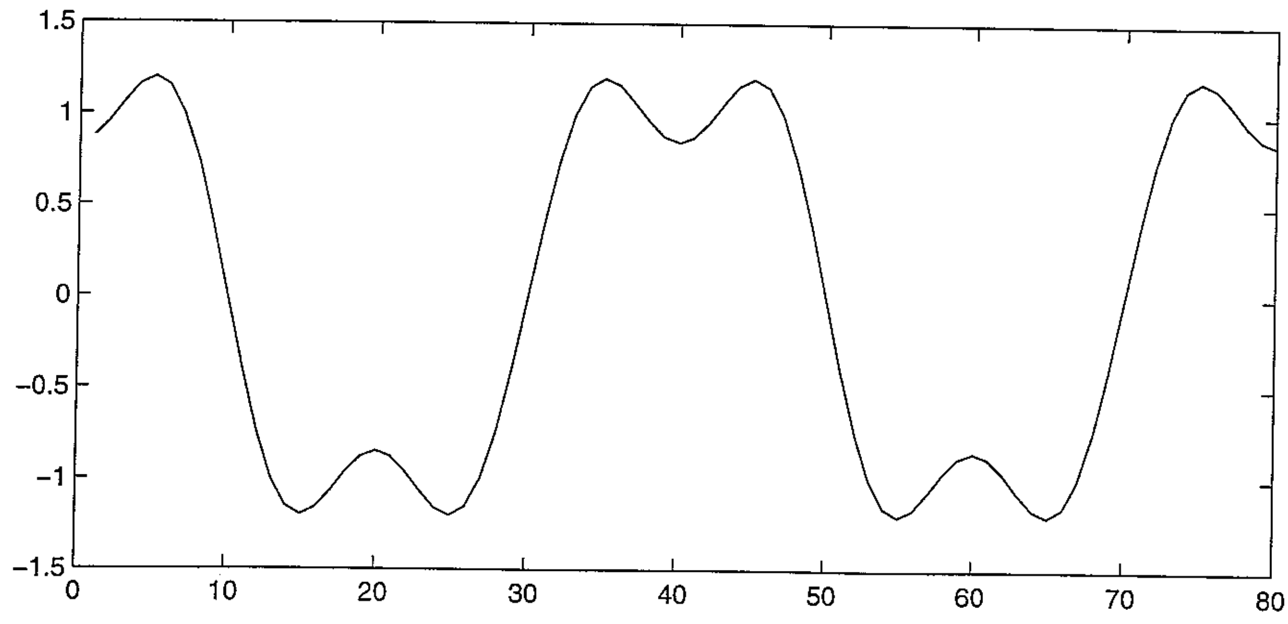
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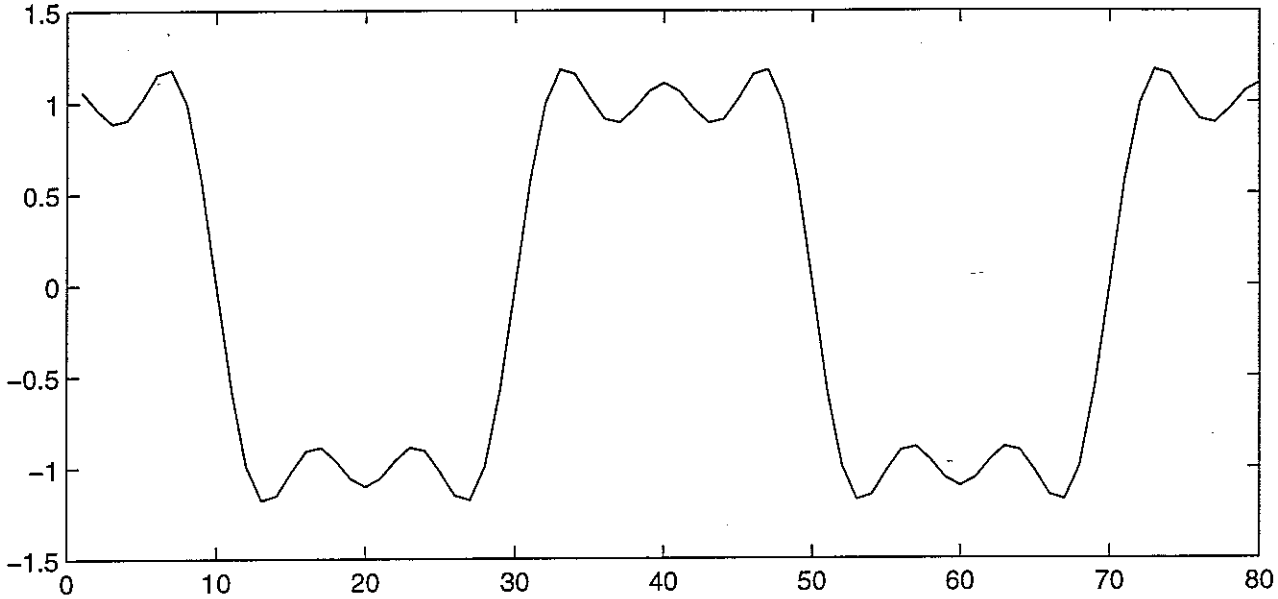
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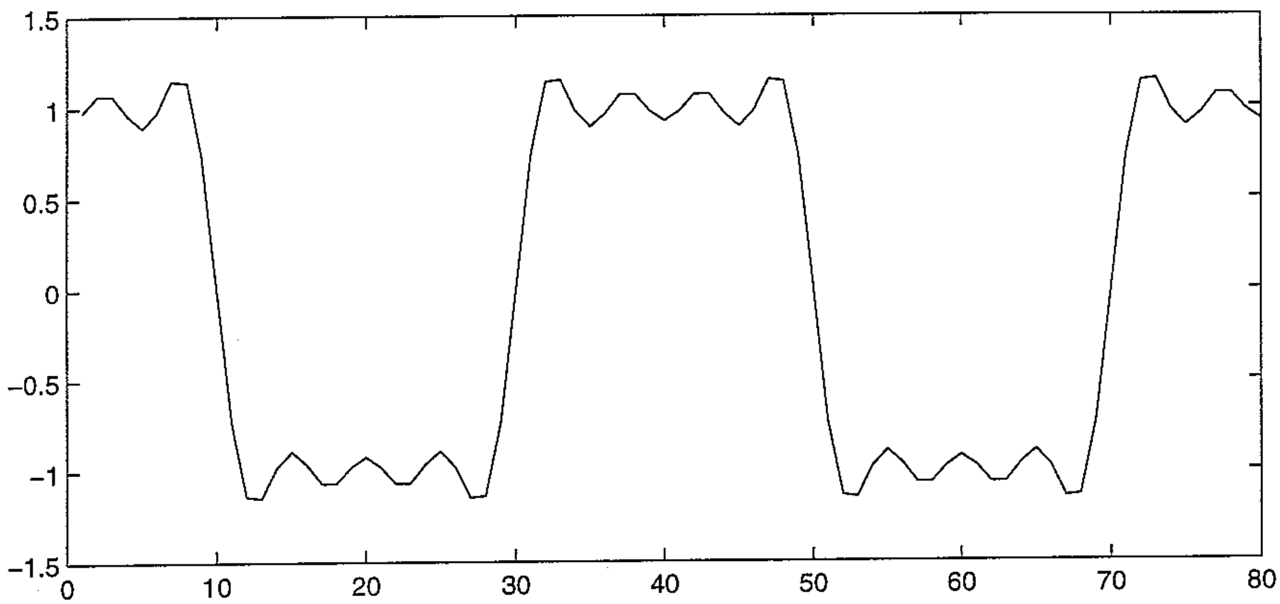
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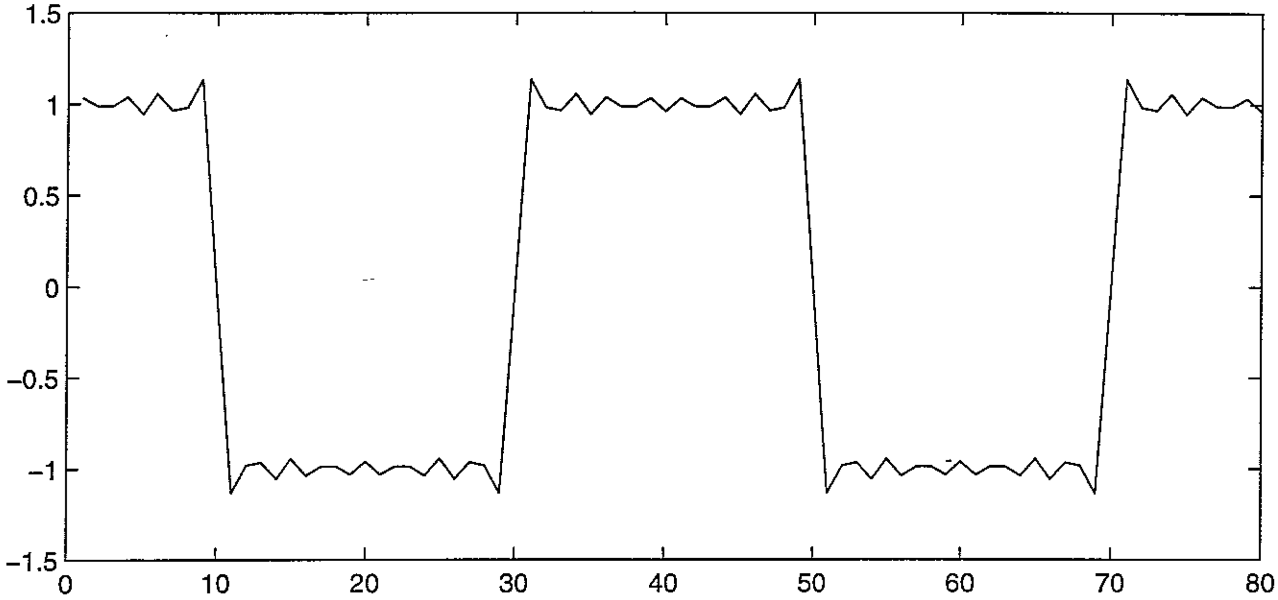
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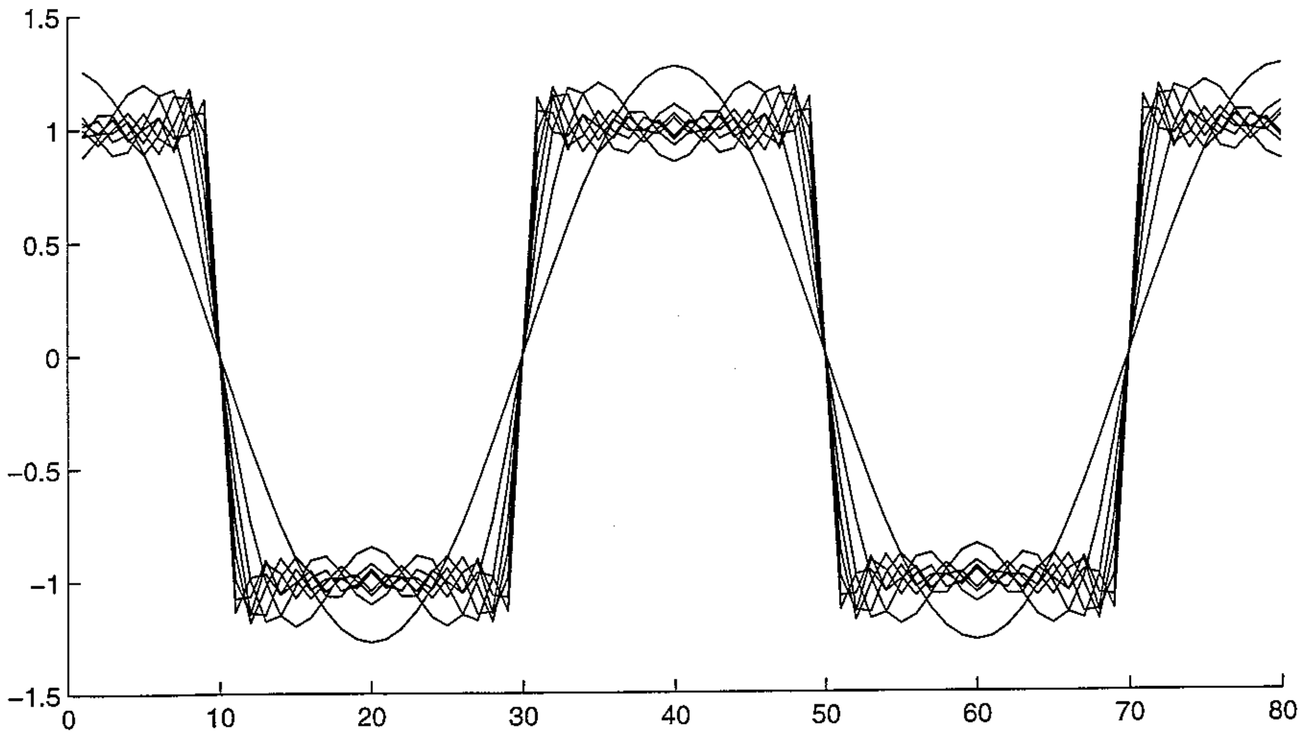
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Fourier Transforms:

Fourier Transforms are used for aperiodic signals. One can derive Fourier Transform relationships by considering an aperiodic function as a periodic function with period ∞ (as $T_0 \to \infty$).

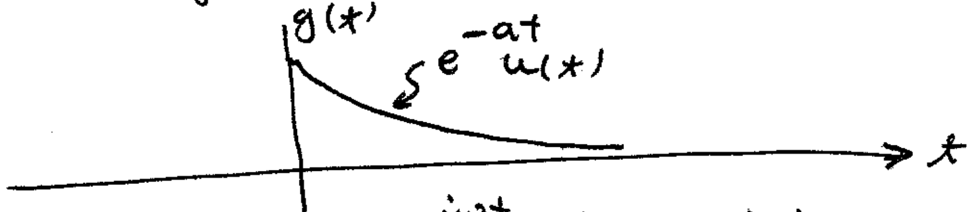
$$g(t) \Longleftrightarrow G(\omega)$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} \, dt$$

$$g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} \, d\omega$$

example:

$$g(t) = e^{-at} u(t)$$



$$G(\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-(a+j\omega)t} \, dt$$

$$G(\omega) = \frac{-1}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty} = \frac{1}{a+j\omega} \quad a > 0$$

$$|G(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}} \qquad \measuredangle G(\omega) = -\operatorname{atan}\left(\frac{\omega}{a}\right)$$

Existence of F.T:

$$\int_{-\infty}^{\infty} |g(t)|\, dt < \infty .$$

This is a sufficient but not necessary condition.

Linearity of F.T.

if $\quad g_1(t) \Leftrightarrow G_1(\omega)$

and $\quad g_2(t) \Leftrightarrow G_2(\omega)$

the

$$a_1 g_1(t) + a_2 g_2(t) \Longleftrightarrow a_1 G_1(\omega) + a_2 G_2(\omega)$$

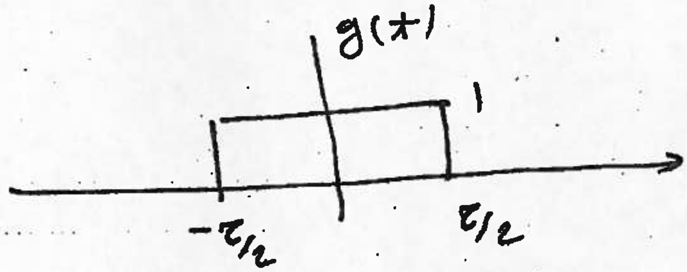
F.T. of some Basic functions:

$g(t) = \delta(t)$

$$\mathcal{F}[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt = 1 \qquad \text{all } \omega$$

X Lecture 2:

gate function:

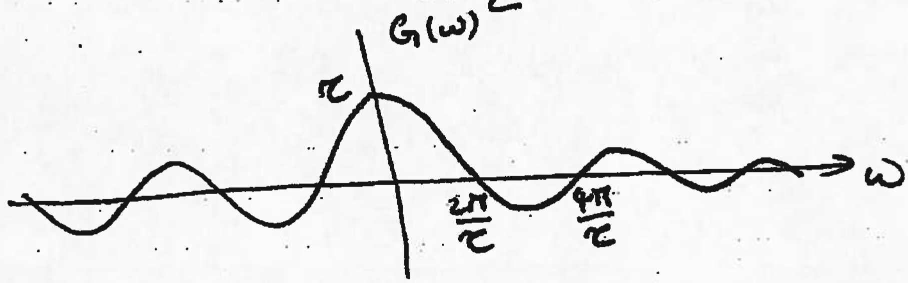


$$g(t) = \begin{cases} 0 & |t| > \frac{\tau}{2} \\ 1 & |\tau| < \tau/2 \end{cases} = \text{rect}\left(\frac{t}{\tau}\right)$$

$$G(\omega) = \int_{-\tau/2}^{\tau/2} 1 \times e^{-j\omega t}\, dt = \frac{-1}{j\omega} e^{-j\omega t}\Big|_{-\tau/2}^{\tau/2}$$

$$G(\omega) = \frac{e^{j\omega\frac{\tau}{2}} - e^{-j\omega\tau/2}}{j\omega} = \frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\omega}$$

$$G(\omega) = \tau\, \frac{\sin\left(\frac{\omega\tau}{2}\right)}{\frac{\omega\tau}{2}} = \tau\, \text{Sinc}\left(\frac{\omega\tau}{2}\right)$$



$\tau \to \infty \Rightarrow g(t) \to 1$ & $G(\omega) \to \overset{2\pi\delta(\omega)}{\cancel{\delta(\omega)}}$

$\tau \to 0 \Rightarrow \frac{g(t)}{\tau} \to \delta(t)$ & $\frac{G(\omega)}{\tau} \to 1$

$$\mathcal{F}^{-1}[\delta(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}$$

$$1 \Longleftrightarrow 2\pi\delta(\omega)$$

$$\mathcal{F}^{-1}[\delta(\omega-\omega_0)] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta(\omega-\omega_0)e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}e^{j\omega_0 t}$$

$$\frac{1}{2\pi}e^{j\omega_0 t} \Longleftrightarrow \delta(\omega-\omega_0)$$

or

$$e^{j\omega_0 t} \Longleftrightarrow 2\pi\delta(\omega-\omega_0)$$

similarly

$$e^{-j\omega_0 t} \Longleftrightarrow 2\pi\delta(\omega+\omega_0)$$

$$\cos\omega_0 t = \frac{1}{2}e^{j\omega_0 t} + \frac{1}{2}e^{-j\omega_0 t}$$

So

$$\mathcal{F}[\cos\omega_0 t] = \pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega_0)$$

Some properties of F.T.

1) $g(t-t_0) \Longleftrightarrow e^{-j\omega t_0} G(\omega)$

2) $g(t) e^{j\omega_0 t} \Longleftrightarrow G(\omega - \omega_0)$

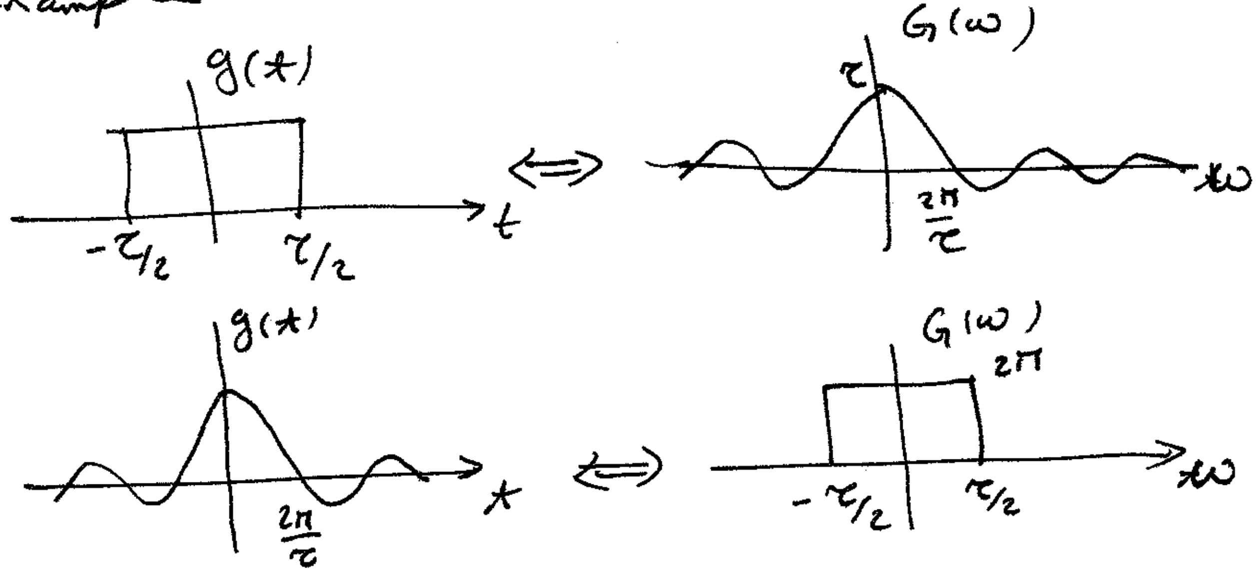
$g(t)\cos\omega_c t = g(t)\left[\frac{1}{2} e^{j\omega_c t} + \frac{1}{2} e^{-j\omega_c t}\right] \Longleftrightarrow \frac{1}{2}\left[G(\omega+\omega_0) + G(\omega-\omega_0)\right]$

3) if $g(t) \Longleftrightarrow G(\omega)$

then

$$G(t) \Longleftrightarrow 2\pi g(-\omega)$$

example



4) Scaling Property

if $g(t) \Longleftrightarrow G(\omega)$

then $g(at) \Longleftrightarrow \frac{1}{|a|} G\left(\frac{\omega}{a}\right)$

5) $$g(t)\cos\omega_0 t = \frac{1}{2}g(t)e^{j\omega_0 t} + \frac{1}{2}g(t)e^{-j\omega_0 t}$$

$$\mathcal{F}[g(t)\cos\omega_0 t] = \pi[G(\omega-\omega_0) + G(\omega+\omega_0)]$$

6) Convolution

$$\mathcal{F}[g(t) * f(t)] = G(\omega)F(\omega)$$

where
$$g(t) * f(t) = \int_{-\infty}^{\infty} g(\tau)f(t-\tau)\,d\tau.$$

7) Derivation

if $\quad g(t) \Longleftrightarrow G(\omega)$

then $\quad \dfrac{dg(t)}{dt} \Longleftrightarrow j\omega G(\omega)$

8) Integration

$$\int_{-\infty}^{t} g(\tau)\,d\tau \Longleftrightarrow \frac{G(\omega)}{j\omega} + \pi G(0)\delta(\omega)$$

9) Parseval's theorem

$$E_g = \int_{-\infty}^{\infty}|g(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|G(\omega)|^2 d\omega$$

Signal Transmission through a L.T.I. System

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

where $h(t)$ is the unit impulse response of the system, i.e.,

$$\mathcal{L}[\delta(t)] = h(t)$$

then

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

and

$$Y(\omega) = X(\omega)\, H(\omega)$$

where $H(\omega)$ is the F.T. of $h(t)$ and is called the <u>transfer function</u> of the system.

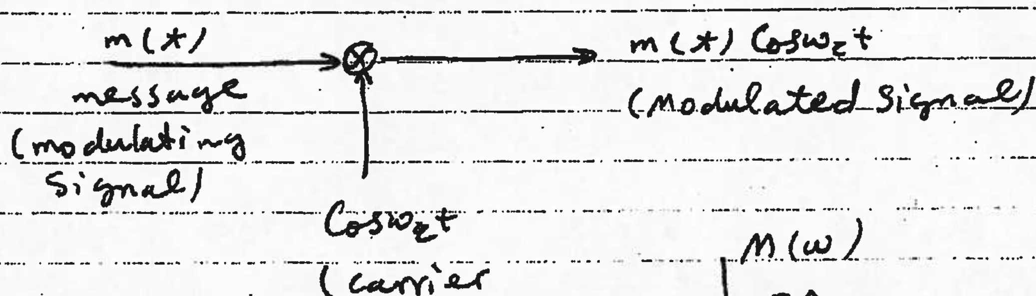
Modulation:

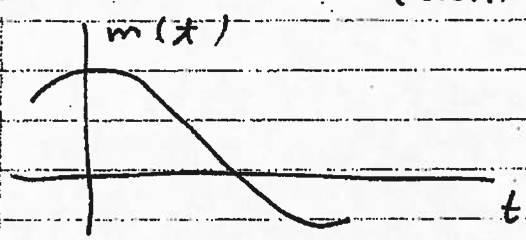
- Why modulation?
  - easy of transmission
  - multiplexing.

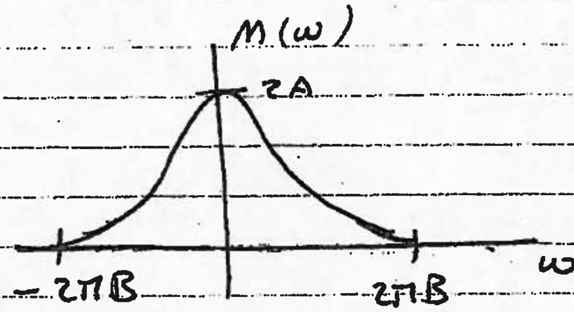
Amplitude Modulation : DSB-SC

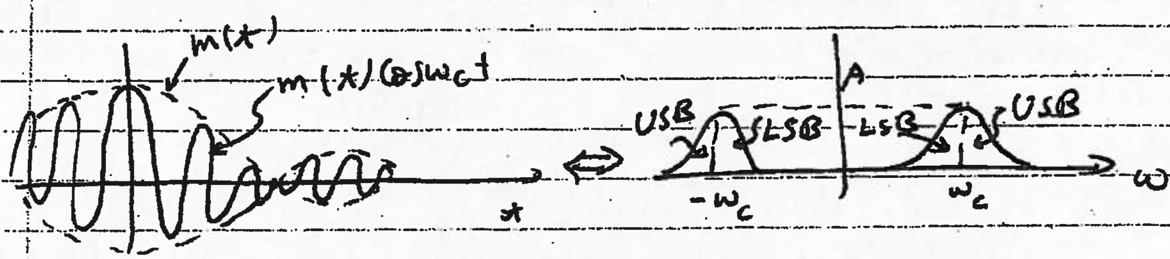
$$m(t) \Longleftrightarrow M(\omega)$$

$$m(t)\cos\omega_c t \Longleftrightarrow \frac{1}{2}\left[M(\omega+\omega_c) + M(\omega-\omega_c)\right]$$

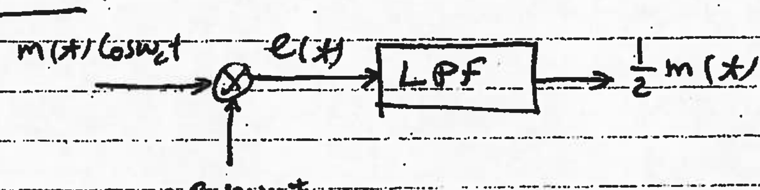


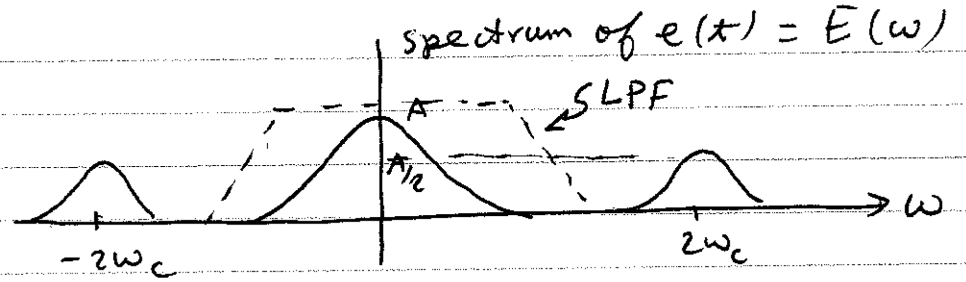






Demodulation:





$$e(t) = m(t)\cos^2 \omega_c t = \frac{1}{2}[m(t) + m(t)\cos 2\omega_c t]$$

$$\xrightarrow{LPF} \frac{1}{2}m(t)$$

$$E(\omega) = \frac{1}{4}[M(\omega) + M(\omega + 2\omega_c) + M(\omega - 2\omega_c) + M(\omega)]$$

$$= \frac{1}{2}M(\omega) + \frac{1}{4}[M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

This is called Synchronous or Coherent detection.

Modulation Techniques:

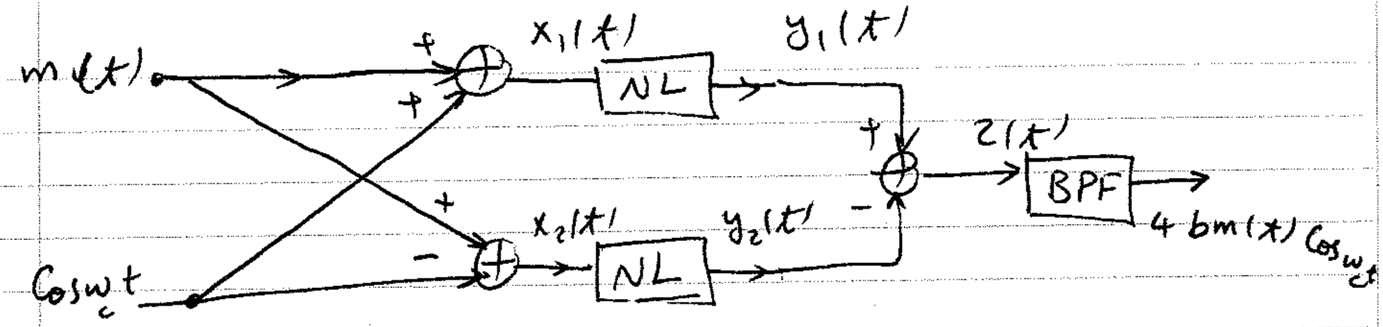
1- Multiplier Modulator

using direct multiplication, e.g., through an amplifier whose gain is controlled by $m(t)$ and amplifying $\cos \omega_c t$.

It is difficult to maintain linearity, therefore, expensive to implement.

2) Non-linearity modulators:

$$y(t) = ax(t) + bx^2(t)$$



$x_1(t) = m(t) + \cos w_c t$

$x_2(t) = m(t) - \cos w_c t$

$y_1(t) = a[m(t) + \cos w_c t] + b[m(t) + \cos w_c t]^2$

$y_2(t) = a[m(t) - \cos w_c t] + b[m(t) - \cos w_c t]^2$

$z(t) = y_1(t) - y_2(t) = 2am(t) + \underbrace{4b\,m(t)\cos w_c t}$

$z(t) \xrightarrow{LPF} 4bm(t)\cos w_c t$

3) Switching Modulator

A periodic wave, e.g., a square wave can be represented as:

$$g(t) = \sum_{n=0}^{\infty} C_n \cos(n w_0 t + \phi_n)$$

so

$$m(t)g(t) = \sum_{n=0}^{\infty} C_n m(t) \cos(n w_c t + \phi_n)$$

Lecture 3:

For example:

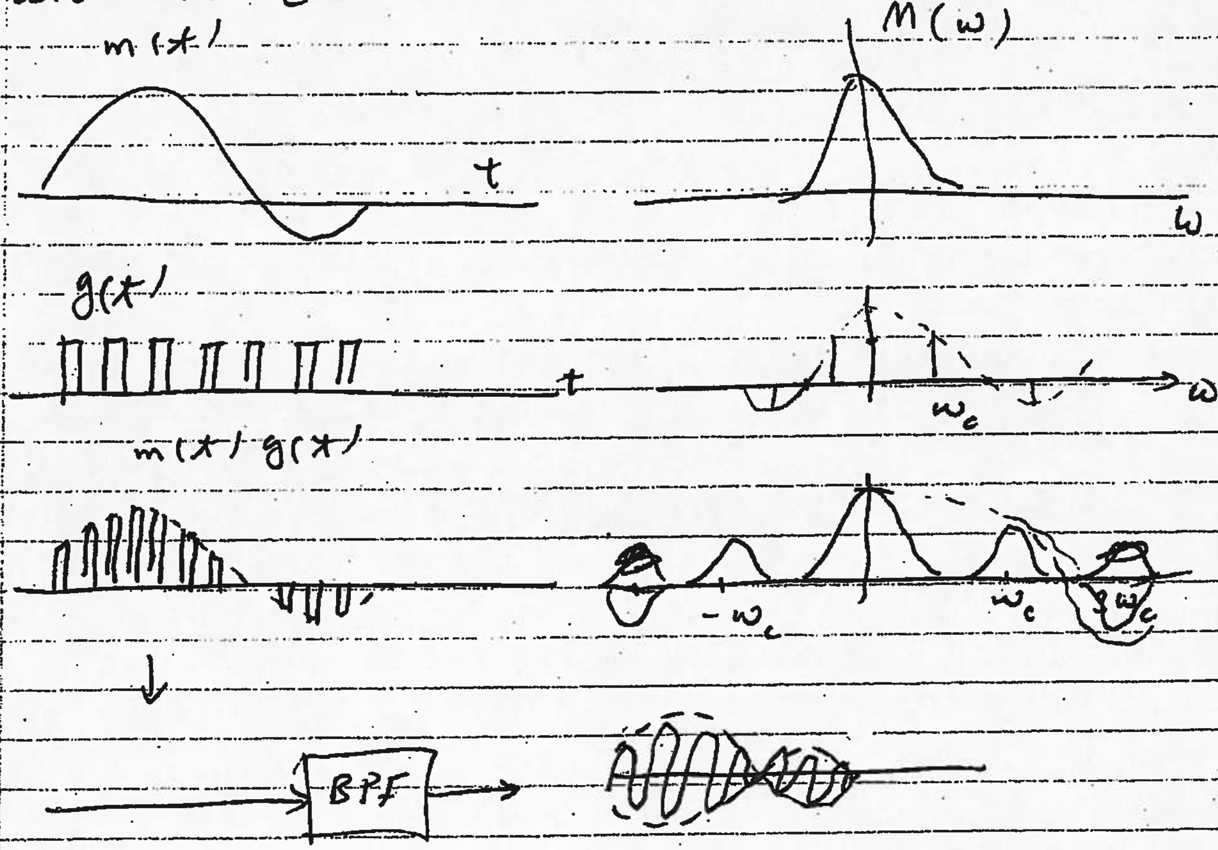
$$g(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \dots\right)$$

is the representation for a square wave

so

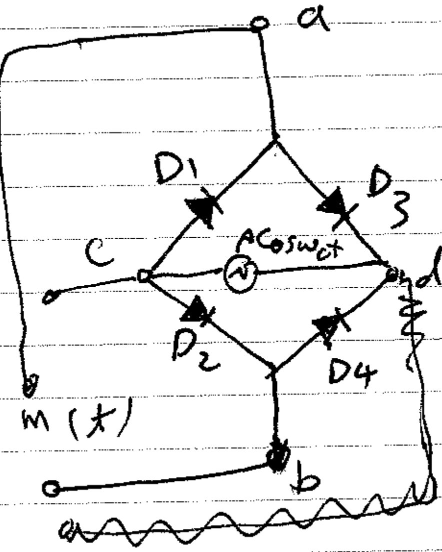
$$m(t)\,g(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos\omega_c t - \frac{2}{3\pi}m(t)\cos 3\omega_c t + \dots$$

we can separate $\frac{2}{\pi}m(t)\cos\omega_c t$ by a BPF centered around ω_c.



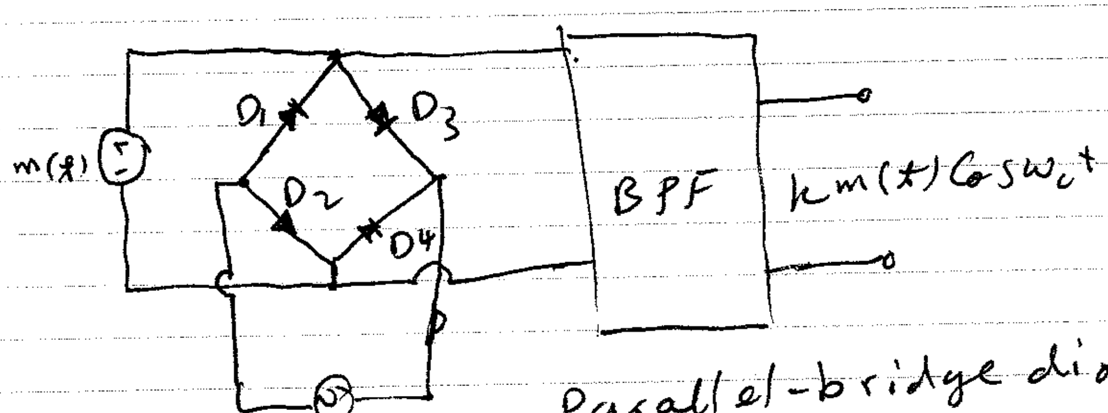
~~Lecture 5, Tue. 12/12/2009~~ 23

Diode-bridge Modulator

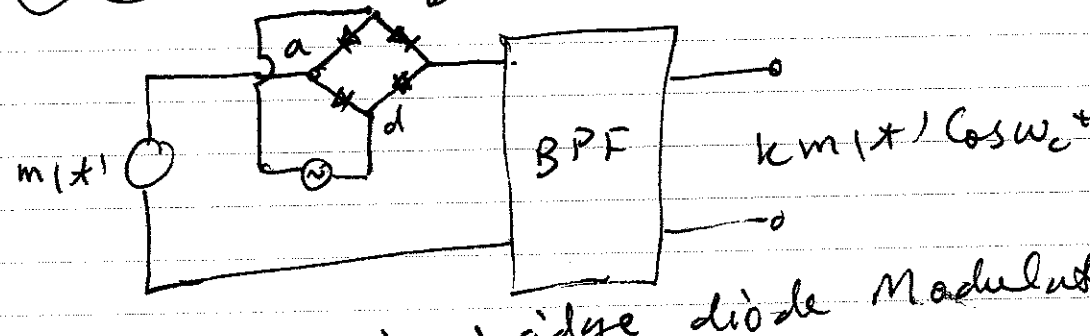


when $V_c > V_d \Rightarrow V_a = V_b$ i.e. short

when $V_c < V_d \Rightarrow V_{ab} = m(t)$, i.e., ab open

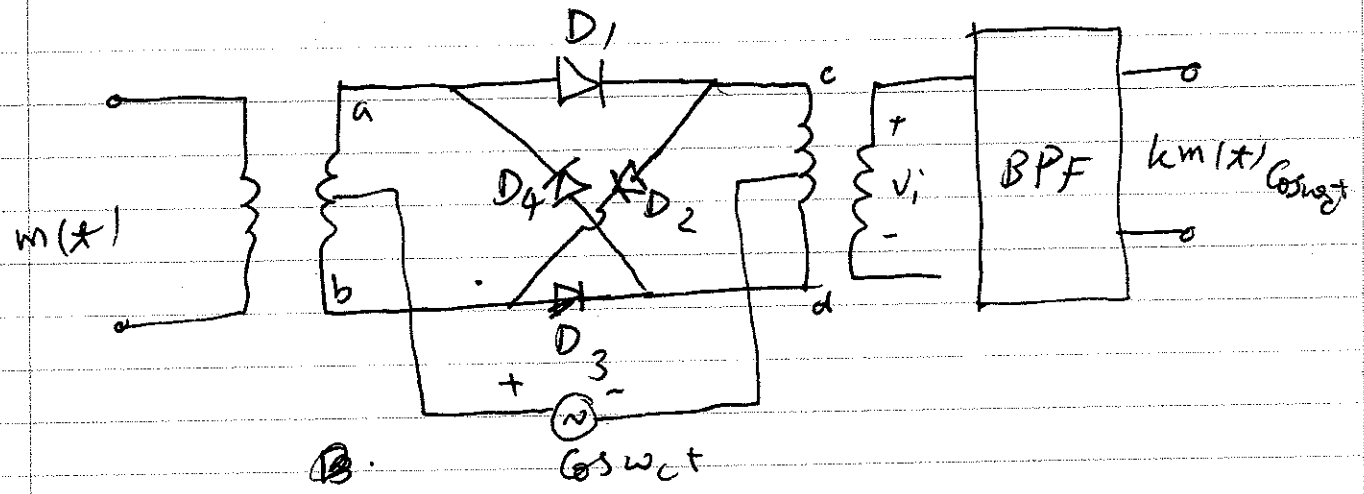


Parallel-bridge diode ~~or~~ modulator

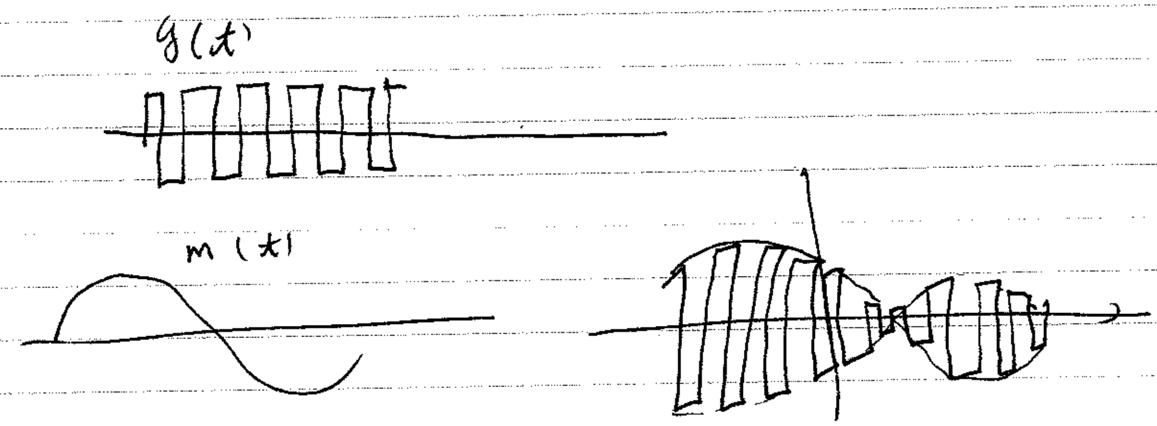


series-bridge diode Modulator

Ring Modulator



Double Balanced Modulator.

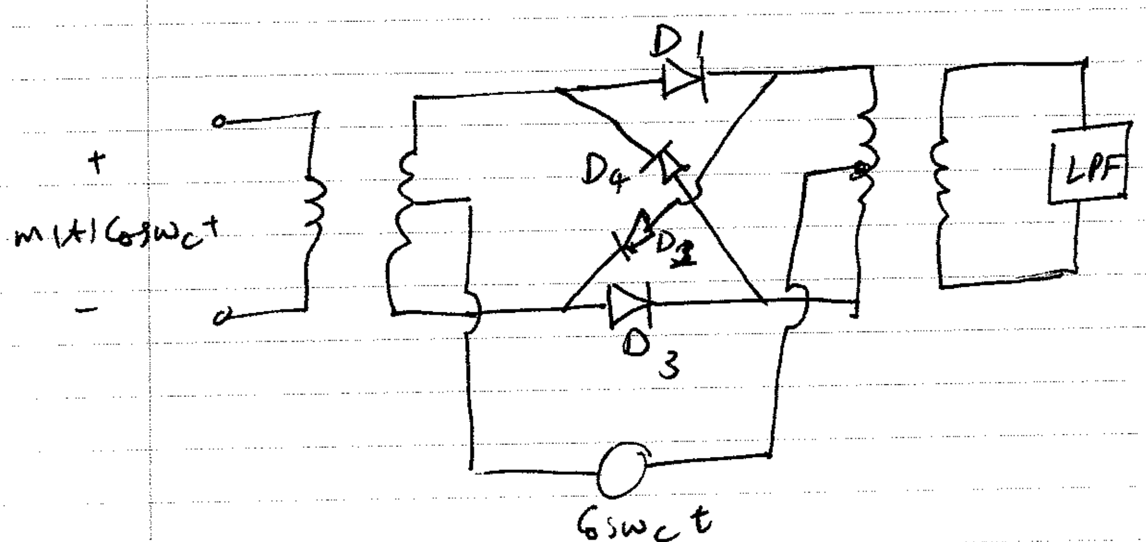


$$g(t) = \frac{4}{\pi}\left(\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t + \cdots\right)$$

$$g(t)\,m(t) = \frac{4}{\pi}\left[m(t)\cos w_c t - \frac{1}{3}m(t)\cos 3w_c t + \cdots\right]$$

Demodulation of DSB-SC signals:

Any of the modulator circuits discussed can be used for demodulation (followed by a LPF). For example, we can use the Ring-Modulator;



The input to the LPF is

$$m(t)\cos w_c t \cdot g(t) = m(t)\cos w_c t\left[\tfrac{1}{2} + \tfrac{4}{\pi}\left(\cos w_c t - \tfrac{1}{3}\cos 3w_c t + \cdots\right)\right]$$

$$= \frac{2}{\pi}m(t) + \frac{2}{\pi}m(t)\cos 2w_c t + \text{other terms}$$

after LPF we get $\frac{2}{\pi}m(t)$

AM (or DSB-LC)

need for carrier acquisition:

To avoid phase recovery (carrier Acquisition) we add unmodulated carrier

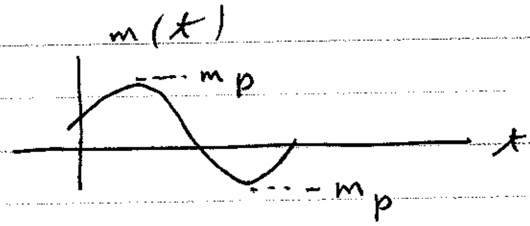
$$\phi_{AM}(t) = A\cos w_c t + m(t)\cos w_c t$$

$$= [A + m(t)]\cos w_c t$$

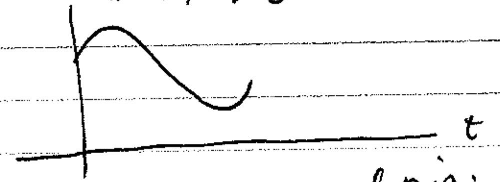
$$\Phi_{AM}(\omega) = \frac{1}{2}\left[M(\omega + \omega_c) + M(\omega - \omega_c) + \pi A\,\delta(\omega + \omega_c) + \pi A\,\delta(\omega - \omega_c)\right]$$

In order to be able to recover $m(t)$ from $\phi_{AM}(t)$ easily, we need $A + m(t) > 0$ all t

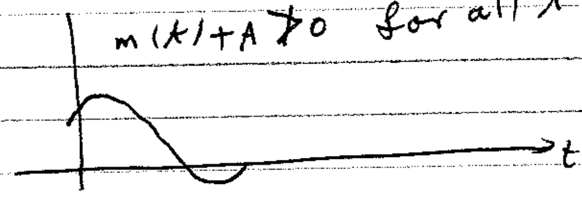
Then we can use envelope detector:



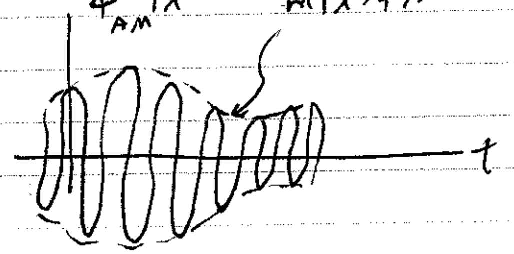
$m(t) + A > 0$



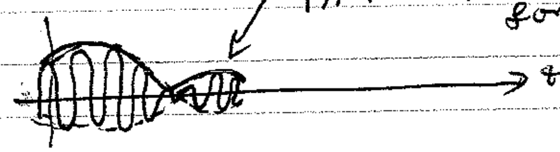
$m(t) + A \ngtr 0$ for all t



Then $\phi_{AM}(t)$ envelop is: $m(t)+A$



envelope $|A + m(t)| \neq A + m(t)$ for all t.



$$A + m(t) \geqslant 0 \quad \text{all } t$$

So

$$A \geqslant m_p$$

where $-m_p$ is the minimum message amplitude;

$$\mu = \frac{m_p}{A} \triangleq \text{modulation index.}$$

So, we need

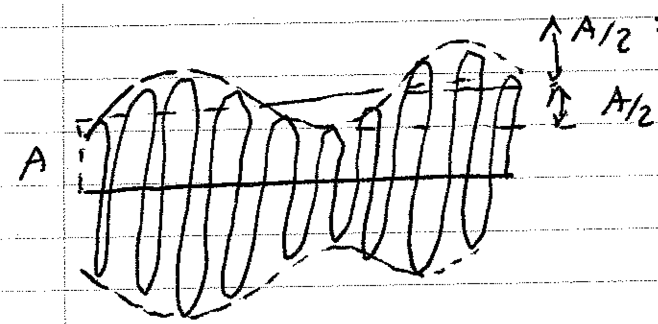
$$0 \leqslant \mu \leqslant 1$$

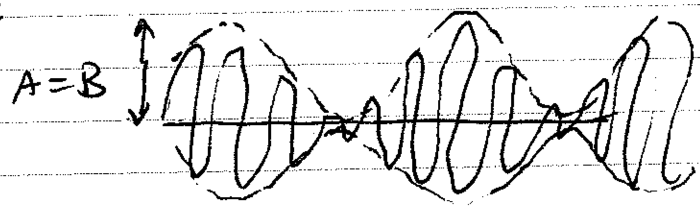
in order to be able to use envelope ~~demodula~~ detector.

when $A < m_p$ then $\mu > 1$ (overmodulation), then we cannot use envelope detector.

example:

Sketch $\varphi_{AM}(t)$ for modulation indexes $\mu = 0.5$ and $\mu = 1$ when $m(t) = B \cos \omega_m t$





$$\mu = 0.5 = \frac{m_p}{A} = \frac{B}{A} \Rightarrow A = 2B$$

$$\mu = 1 = \frac{m_p}{A} = \frac{B}{A} \Rightarrow B = A$$

$$A = B$$

Sideband and Carrier Power

$$\phi_{AM}(t) = \underbrace{A\cos w_c t}_{\text{unmodulated (Pure) Carrier}} + \underbrace{m(t)\cos w_c t}_{\text{modulated information (sidelobes)}}$$

$$P_c = \frac{A^2}{2} \quad \text{and} \quad P_s = \frac{1}{2}\widetilde{m^2(t)}$$

efficiency of modulation:

$$\eta = \frac{\text{useful power}}{\text{Total Power}} = \frac{P_s}{P_c + P_s} = \frac{\widetilde{m^2(t)}}{A^2 + \widetilde{m^2(t)}} \times 100\%$$

For a sinusoidal message (tone)

$$m(t) = \mu A \cos w_c t \Rightarrow \overline{m^2(t)} = \frac{(\mu A)^2}{2}$$

$$\eta = \frac{\mu^2}{2+\mu^2} 100\%$$

the maximum of η is achieved with $\mu = 1$ (since $0 \leq \mu \leq 1$), i.e.,

$$\eta_{max} = 33\%$$

<u>Example</u>: Determine the efficiency of modulation and the per cent of power in the sidelobes when a) $\mu = 0.5$ b) $\mu = 0.3$

a)
$$\eta = \frac{\mu^2}{2+\mu^2} \times 100\% = \frac{(0.5)^2}{2+(0.5)^2} \times 100\% = 11.11\%$$

so 11% of power is in the sidelobes, i.e., is useful power.

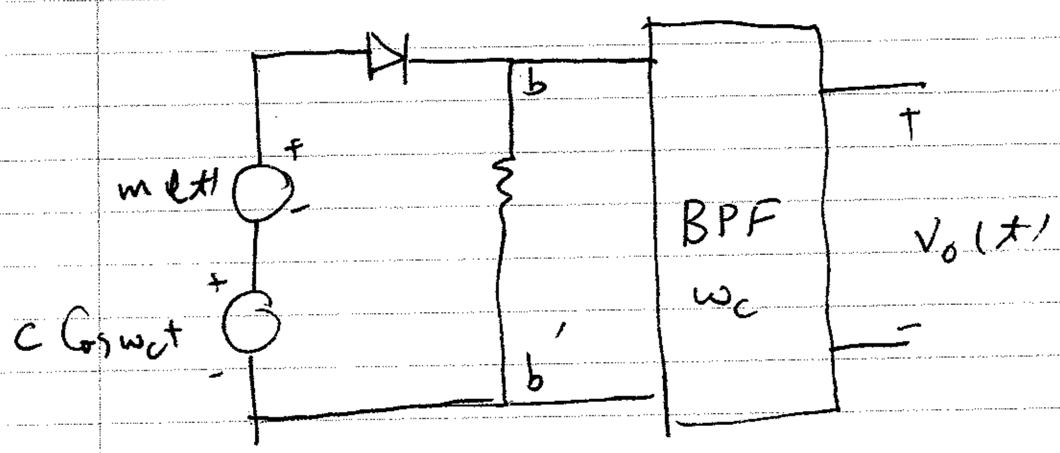
b)
$$\eta = \frac{(0.3)^2}{2+0.3^2} \times 100\% = 4.3\%$$

only 4.3% of the power is in the sidebands.

Generation of AM Signals

The same as DSB-SC but with input as $A+m(t)$ insteade of $m(t)$.
However, since there is no need to suppress the carrier, the circuit does not need to be balanced. So, a single diode can be used.

The following circuit is an example:



$$v_{bb'} = \left[c \cos\omega_c t + m(t)\right] w(t)$$

where $w(t)$ is a square wave, since diode shorts and opens whenever the sinusoidal term is positive/negative, respectively ($c \gg m(t)$)

So:

$$v_{bb'}(t) = \left[c \cos\omega_c t + m(t)\right]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \cdots\right)\right]$$

$$= \underbrace{\frac{c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t}_{\text{desired AM Signal}} + \underbrace{\text{other terms}}_{\text{to be filtered out by BPF}}$$

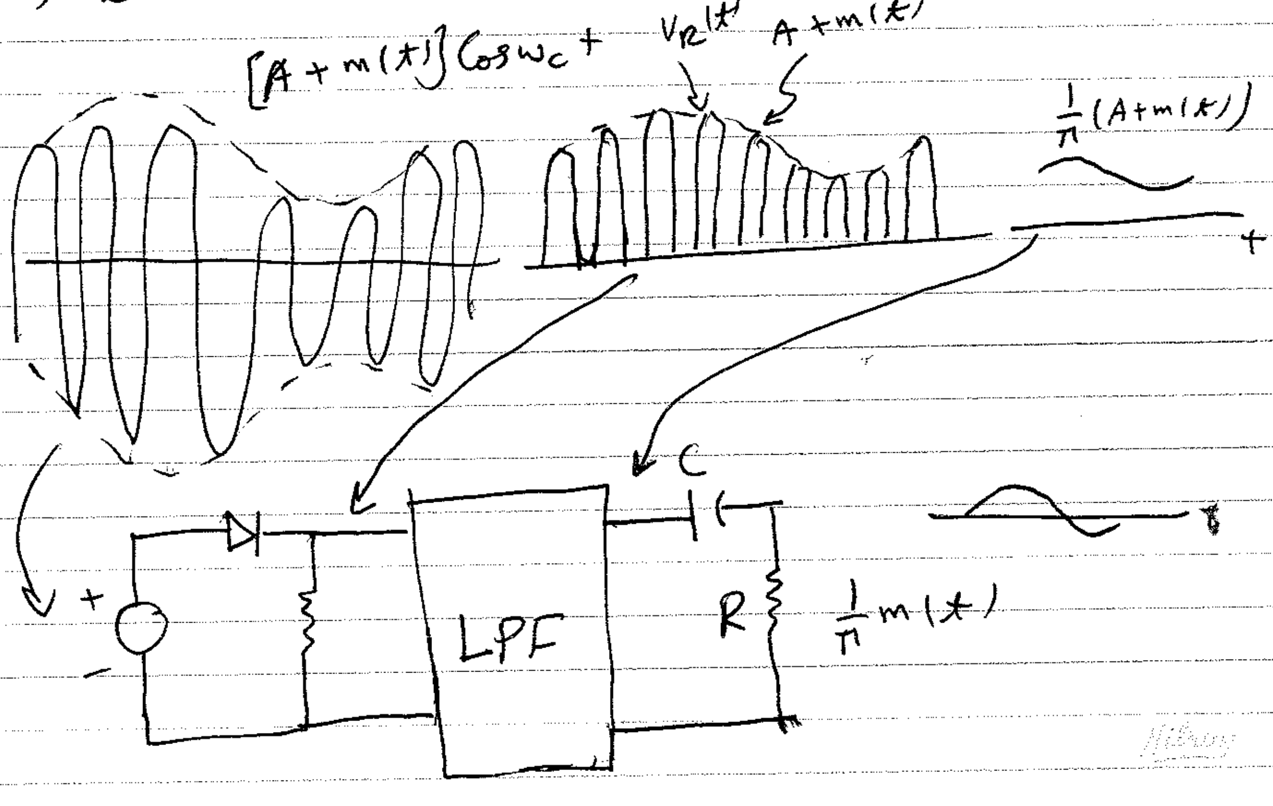
Demodulation of AM Signals:

Coherent demodulation is possible, however, defeats the purpose of having extra carrier added.

So, non-coherent demodulation is preferred. Examples of non-coherent AM demodulator are:

1) Rectifier Detector
2) Envelope Detector.

1) Rectifier Detector



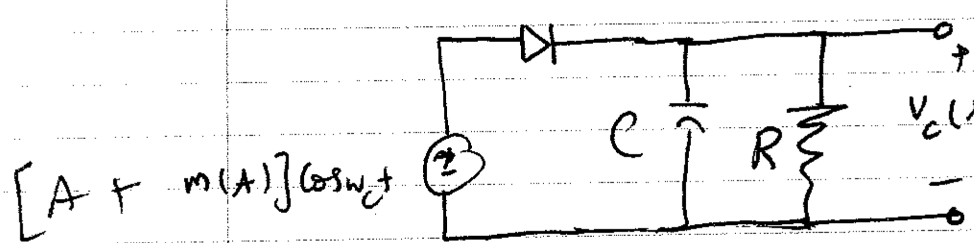
$$V_R = \left\{ [A + m(t)] \cos w_c t \right\} w(t)$$

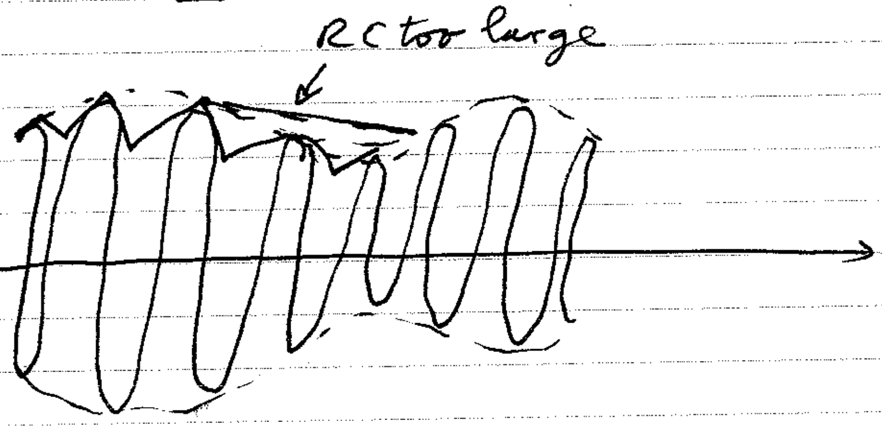
$$= \{A + m(t)\} \cos w_c t \left[\frac{1}{2} + \frac{2}{\pi} \left(\cos w_c t - \frac{1}{3} \cos 3w_c t + \frac{1}{5} \cos 5w_c t \right.\right.$$

$$= \frac{1}{\pi} [A + m(t)] + \text{other terms at higher frequencies.}$$

2) Envelope Detector.

Her again, we have a half-wave rectifier followed by a LPF. However, here the purpose of the LPF is to follow the envelope of the waveform and is designed taking into consideration the modulation index μ.





The operation is as follows:

during positive ~~cycle~~ cycle of $\phi_{AM}(t)$, diode conducts and C charges to the max value of the input signal. As signal falls below this value, C starts discharging, but diode reverses preventing it to discharge fast (C discharges through R and the diode).

The same happens in the next cycle, ...

To see the relationship between the parameters of filter and the modulation index note that,

$$V_c = E\, e^{-t/RC}$$

where E is the peak value at $t=0$.

since $RC \gg \frac{1}{\omega_c}$

then the capacitor voltage can be approximated by a straight line. Using Taylor series, we get:

$$v_c \approx E\left(1 - \frac{t}{RC}\right)$$

The slope of discharge is $-\frac{E}{RC}$

In order for the capacitor to follow the envelope $E(t)$, the magnitude of slop of RC discharge must be greater than the magnitude of slope of the envelope $E(t)$

$$\left|\frac{dV_c}{dt}\right| = \frac{E}{RC} \geq \left|\frac{dE}{dt}\right|$$

But, for a tone : $E = A[1+\mu \cos w_m t]$

$$\frac{dE}{dt} = \frac{d}{dt}\left\{A[1+\mu \cos w_m t]\right\}$$

or $$\frac{dE}{dt} = -\mu A w_m \sin w_m(t)$$

Hence, we need

$$\frac{A[1+\mu \cos w_m t]}{RC} \geq \mu A w_m \sin w_m t \quad \text{for all } t.$$

or

$$RC \leq \frac{1+\mu \cos w_m t}{\mu w_m \sin w_m t}$$

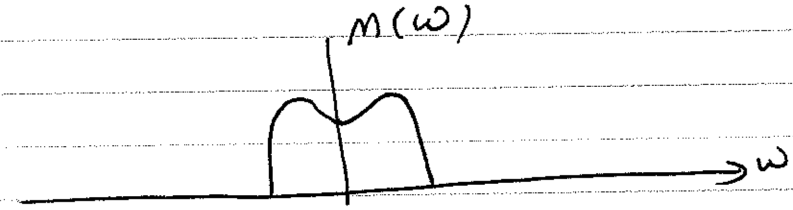
The worst case happens when the right hand side is minimum ~~or~~)

$$\frac{d}{dt}\left[\frac{1+\mu \cos w_m t}{\mu w_m \sin w_m t}\right] = 0 \Rightarrow \cos w_m t = -\mu$$

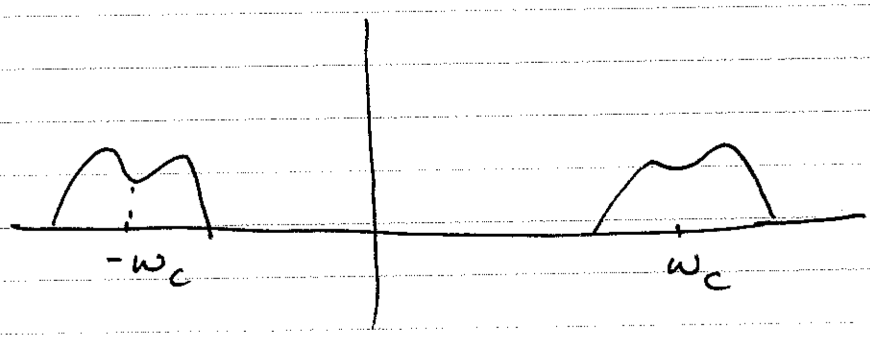
So, we required that:

$$RC \leq \frac{1}{\omega_m}\left(\frac{\sqrt{1-\mu^2}}{\mu}\right)$$

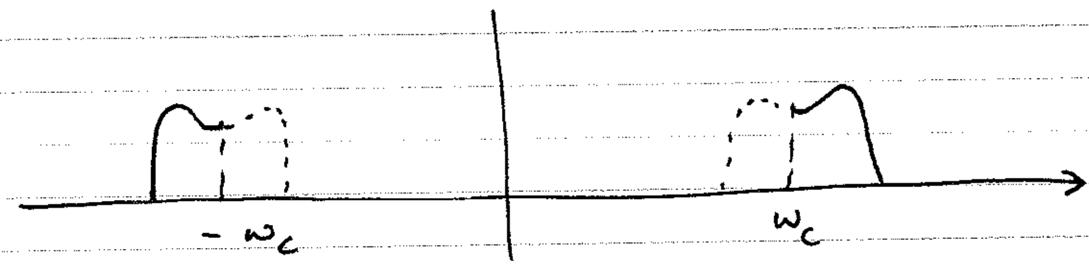
Single Sideband (SSB) Modulation



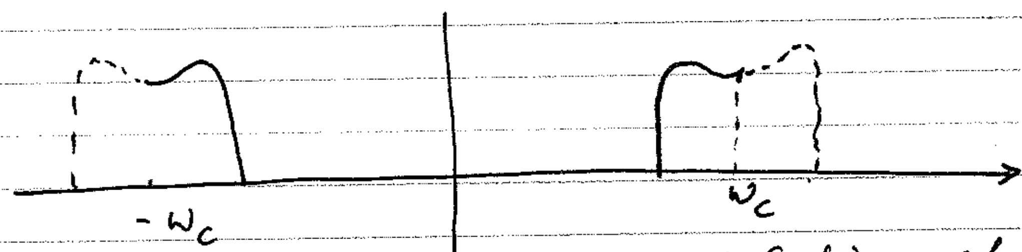
Baseband



DSB

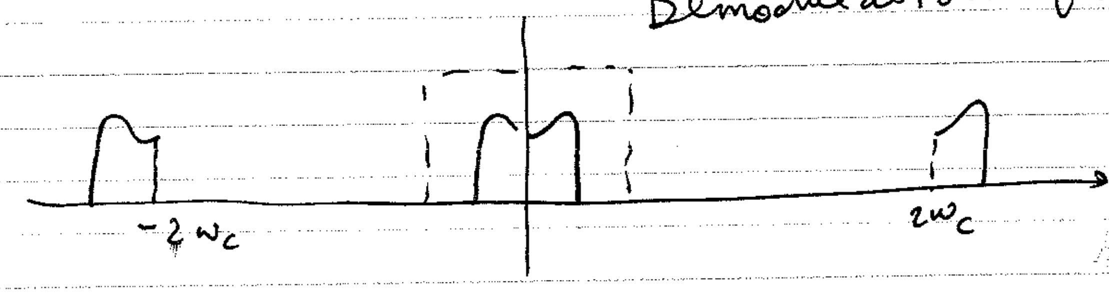


USB



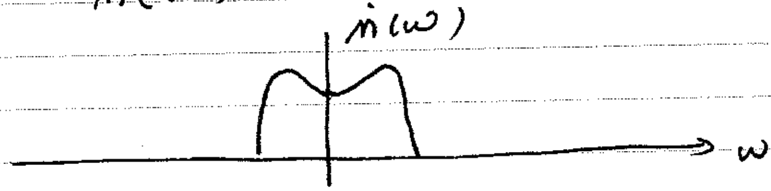
LSB

Demodulation of SSB

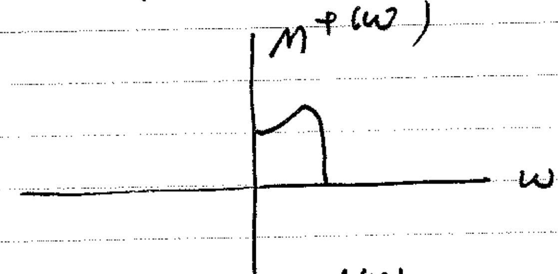


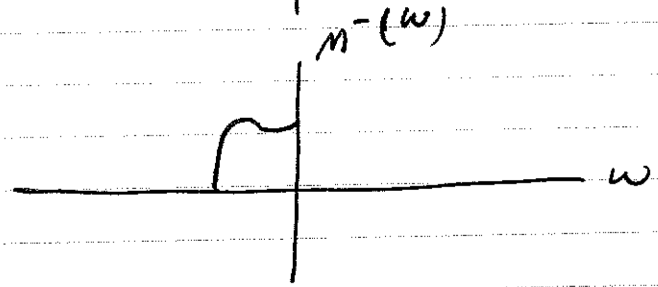
Time-domain representation of SSB

Consider $M(\omega)$ as:



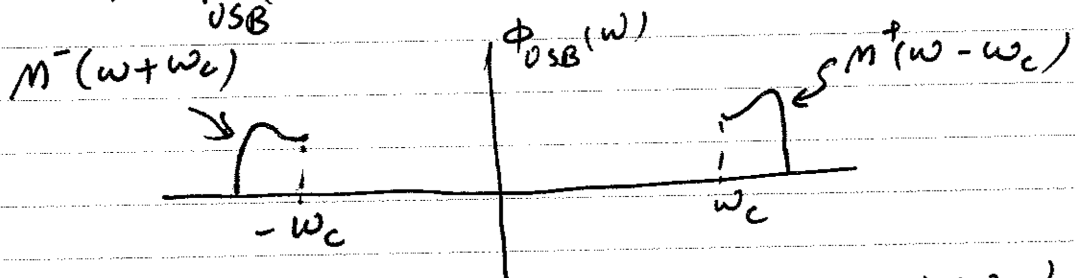
Define $M^+(\omega)$ and $M^-(\omega)$ as the positive an negative frequency portions of $M(\omega)$





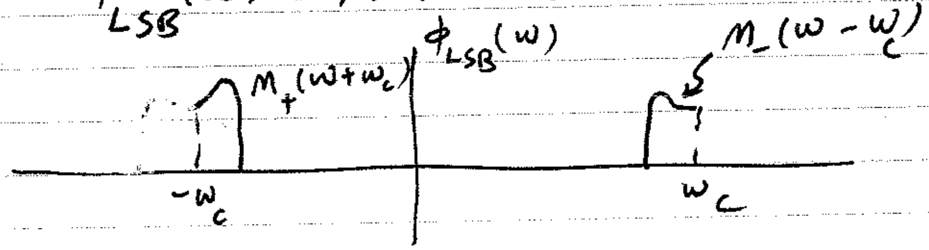
Then USB can be represented as

$$\Phi_{USB}(\omega) = M^+(\omega - \omega_c) + M^-(\omega + \omega_c)$$



and

$$\Phi_{LSB}(\omega) = M^+(\omega + \omega_c) + M_-(\omega - \omega_c)$$



But

$$M_+(\omega) = M(\omega)\,U(\omega)$$

where

$$U(\omega) = \begin{cases} 1 & \omega > 0 \\ 0 & \omega \leq 0 \end{cases}$$

note that

$$U(\omega) = \frac{1}{2}\left[1 + \operatorname{Sgn}(\omega)\right]$$

So:

$$M_+(\omega) = \frac{1}{2} M(\omega) + \frac{1}{2} M(\omega)\operatorname{Sgn}(\omega)$$

$$m_+(t) = \frac{1}{2} m(t) + \frac{1}{2}\mathcal{F}^{-1}\left[M(\omega)\operatorname{Sgn}(\omega)\right]$$

$$F^{-1}\left[M(\omega)\operatorname{Sgn}(\omega)\right] = m(t) * \mathcal{F}^{-1}\left[\operatorname{Sgn}(\omega)\right]$$

$$\mathcal{F}^{-1}\left[\operatorname{Sgn}(\omega)\right] = \frac{j}{\pi t}$$

So:

$$m_+(t) = \frac{1}{2} m(t) + \frac{1}{2}\frac{j}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha}\,d\alpha$$

X Lecture 4:

let

$$m_h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha}\,d\alpha$$

this is called the Hilbert Transform of $m(t)$!

Then

$$m_+(t) = \frac{1}{2}\left[m(t) + j\,m_h(t)\right]$$

Similarly, it can be shown that

$$m_-(t) = \frac{1}{2}\left[m(t) - j\,m_h(t)\right]$$

$$\Phi_{USB}(\omega) = M_+(\omega-\omega_c) + M_-(\omega+\omega_c)$$

implies that

$$\varphi_{USB}(t) = m_+(t)e^{+j\omega_c t} + m_-(t)e^{-j\omega_c t}$$

$$\varphi_{USB}(t) = \frac{1}{2}[m(t)+jm_h(t)]e^{j\omega_c t} + \frac{1}{2}[m(t)-jm_h(t)]e^{-j\omega_c t}$$

$$\boxed{\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t}$$

similarly,

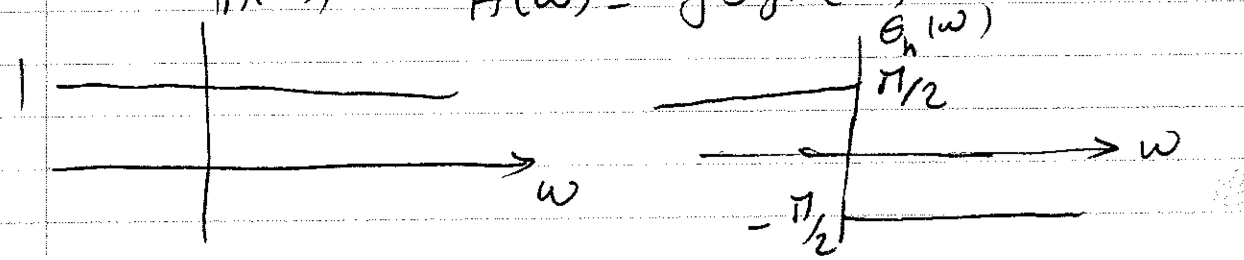
$$\boxed{\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t}$$

$$M_h(\omega) = \mathcal{F}[m_h(t)] = \mathcal{F}\left[m(t) * \frac{1}{\pi t}\right] = -M(\omega)j\,\mathrm{Sgn}(\omega)$$

$$M_h(\omega) = -j\,\mathrm{Sgn}(\omega)M(\omega) = M(\omega)H(\omega)$$

So, finding $m_h(t)$ is equivalent to passing $m(t)$ through a filter with transfer function

$|H(\omega)|$ $\qquad H(\omega) = -j\,\mathrm{Sgn}(\omega)$



since

$$H(\omega) = -j\,\mathrm{sgn}(\omega) = \begin{cases} -j = 1\cdot e^{-j\pi/2} & \omega > 0 \\ j = 1\cdot e^{j\pi/2} & \omega < 0 \end{cases}$$

Example: Tone Modulation

$$m(t) = \cos\omega_m t$$

$$m_h(t) = \cos(\omega_m t - \tfrac{\pi}{2}) = \sin\omega_m t$$

So:

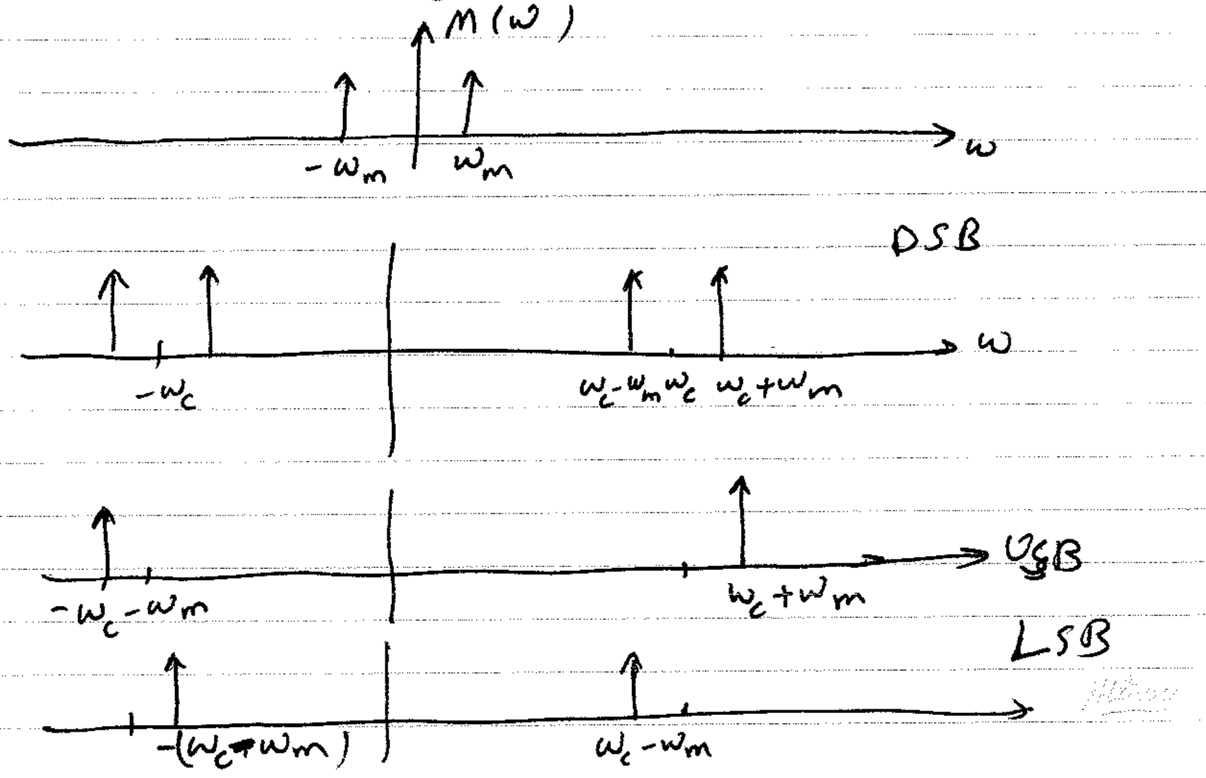
$$\phi_{USB}(t) = \cos\omega_m t\cos\omega_c t - \sin\omega_m t\sin\omega_c t$$

$$= \cos(\omega_c - \omega_m)t$$

and

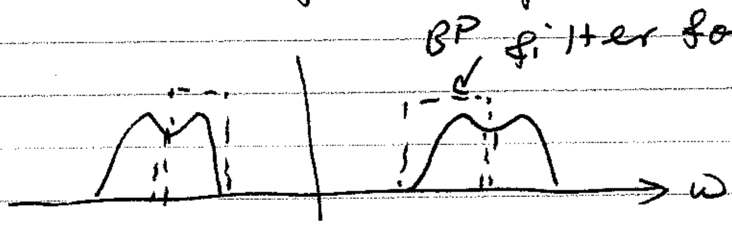
$$\phi_{LSB}(t) = \cos\omega_m t\cos\omega_c t + \sin\omega_m t\sin\omega_c t$$

$$= \cos(\omega_c - \omega_m)t$$

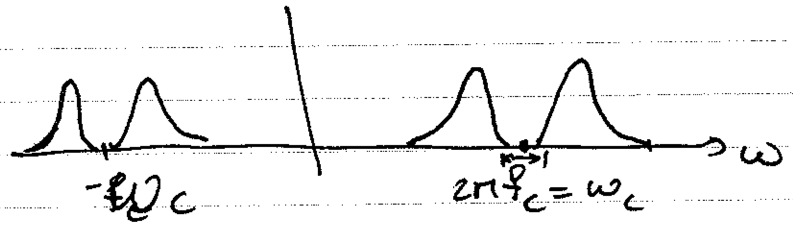


Generation of SSB Signals:

1) Sideband filtering

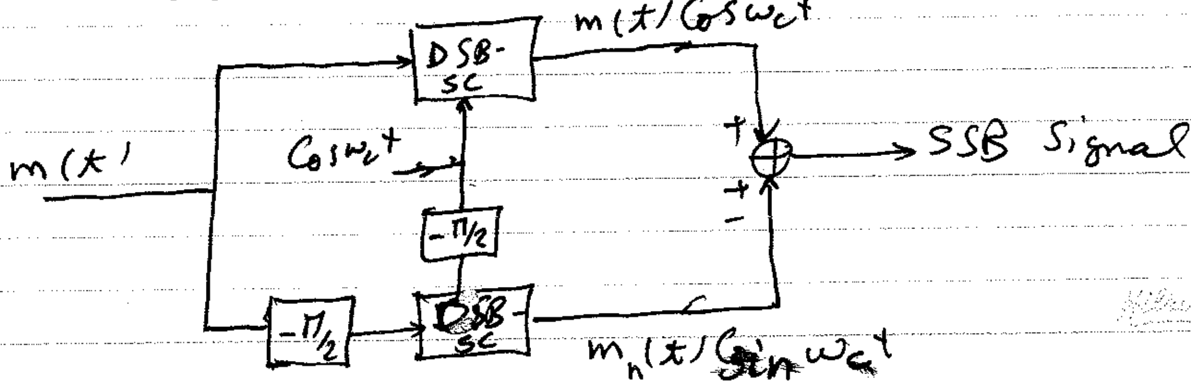


- difficulty of realizing sharp filters
  ⇒ necessity of having guardband between sidebands ⇒ need to have no components around zero
  example of voice 0 to 300 Hz. unimportant
  ⇒ So, we have 600 Hz. Guardband.



2) Phase-Shift method

$$\phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h \sin\omega_c t$$



Demodulation of SSB

$$\phi_{SSB}(t) = m(t)\cos w_c t \pm m_h(t)\sin w_c t$$

$$\phi_{SSB}(t)\cos w_c t = \frac{1}{2}m(t)[1+\cos 2w_c t] \pm \frac{1}{2}m_h(t)\sin 2w_c t$$

after LPF we get $\frac{1}{2}m(t)$

Envelope Detection of SSB

$$\phi_{SSB+C}(t) = A\cos w_c t + [m(t)\cos w_c t \mp m_h(t)\sin w_c t]$$

$$= [A + m(t)]\cos w_c t + m_h(t)\sin w_c t$$

$$= E(t)\cos(w_c t + \theta)$$

$$E(t) = \left\{(A+m(t))^2 + m_h^2(t)\right\}^{1/2}$$

$$= A\left[1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right]$$

if $A \gg |m(t)|$ then $\frac{m^2(t)}{A^2}$ and $\frac{m_h^2(t)}{A^2}$

can be ignored out

$$E(t) \simeq A\left[1 + \frac{m(t)}{A}\right] = A + m(t)$$

~~if m(t)~~ since $\frac{m(t)}{A} \ll 1$

Carrier Acquisition

$\phi_{DSB-SC}(t) = m(t)\cos w_c t$ DSB-SC

$$e(t) = m(t)\cos w_c t \cos[(w_c + \Delta w)t + \delta]$$

$$= m(t)\{\cos[(\Delta w)t + \delta] + \cos[(2w_c + \Delta w)t + \delta]\}$$

if $\Delta w = 0$ and $\delta = 0$

i.e., no frequency or phase offset,

then the output of the LPF is:

$$e_0(t) = m(t)$$

otherwise

$$e_0(t) = m(t)\cos[(\Delta w)t + \delta]$$

if $\Delta w = 0$

then $e_0(t) = m(t)\cos\delta$

i.e., a constant attenuation. $e_0(t)$

is max. when $\delta = 0$ and 0 when $\delta = \pm\frac{\pi}{2}$.

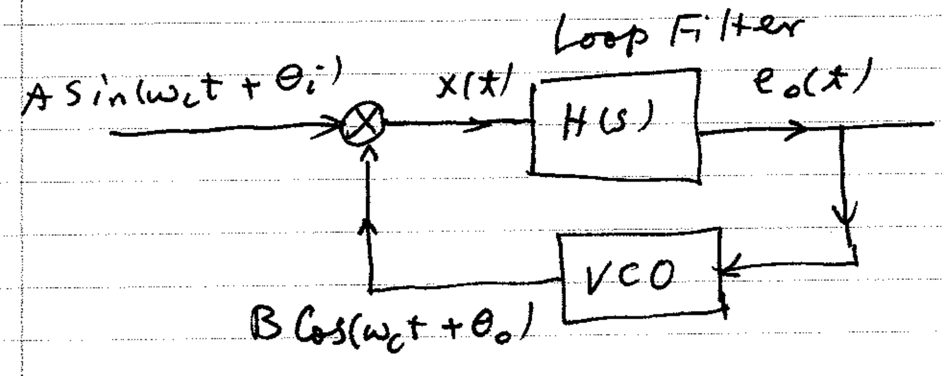
when $\delta = 0$ & $\Delta w \neq 0$

$$e_0(t) = m(t)\cos(\Delta w)t$$

i.e., the output is distorted by being

modulated with a low frequency sinusoid.

Phase Locked Loop (PLL)



VCO : Voltage Controlled Oscillator

has the frequency

$$\omega(t) = \omega_c + c\, e_o(t)$$

where ω_c is the free running frequency.

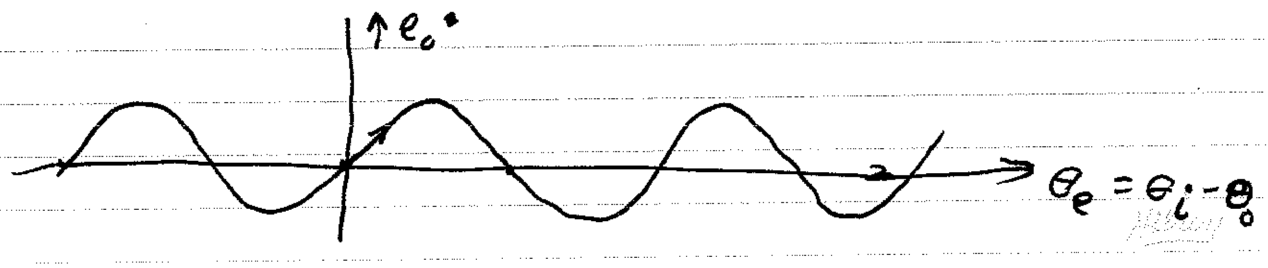
$$x(t) = A\,B \cos(\omega_c t + \theta_i)\cos(\omega_c t + \theta_o)$$

$$= \frac{AB}{2}\left[\sin(\theta_i - \theta_o) + \sin(2\omega_c t + \theta_i + \theta_o)\right]$$

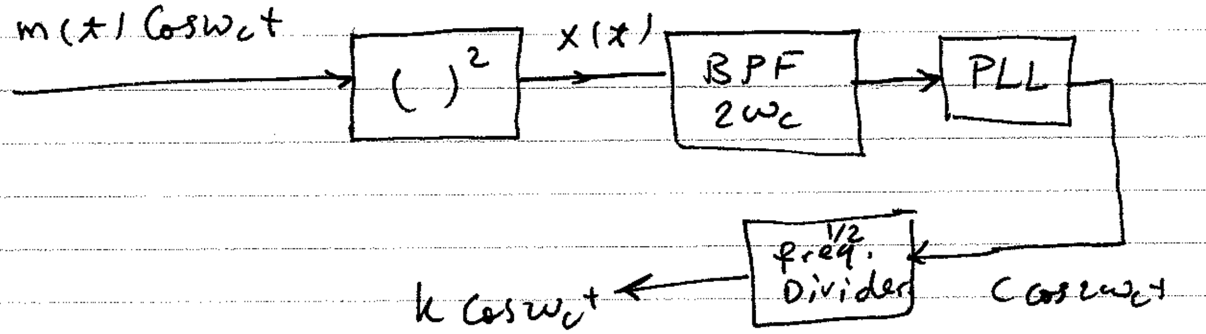
$$e_o(t) = \frac{AB}{2}\sin(\theta_i - \theta_o)$$

or

$$e_o(t) = \frac{AB}{2}\sin\theta_e \quad \text{where } \theta_e = \theta_i - \theta_o$$



Carrier Acquisition in DSB-SC



$$x(t) = \{m(t)\cos\omega_c t\}^2 = \frac{1}{2}m^2(t) + \frac{1}{2}m^2(t)\cos 2\omega_c t$$

let $\frac{1}{2}m^2(t) = k + \phi(t)$

wher k is the average value of $\frac{1}{2}m^2(t)$

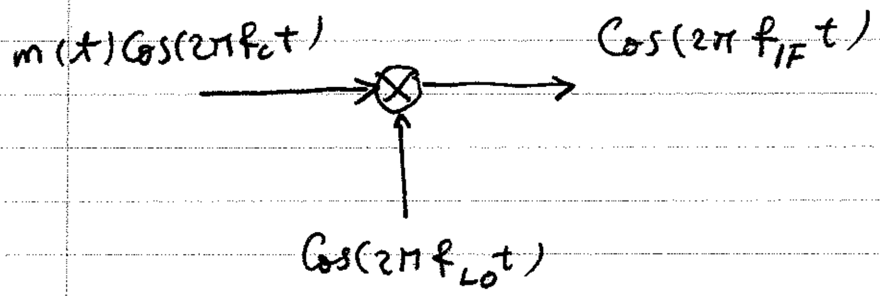
then:

$$x(t) = \frac{1}{2}m^2(t) + \underbrace{k\cos 2\omega_c t}_{\downarrow} + \phi(t)\cos 2\omega_c t$$

A narrow BPF filters out $\frac{1}{2}m^2(t)$ and most of $\phi(t)\cos 2\omega_c t$ and gives $k\cos 2\omega_c t$

Superheterodyne AM receiver

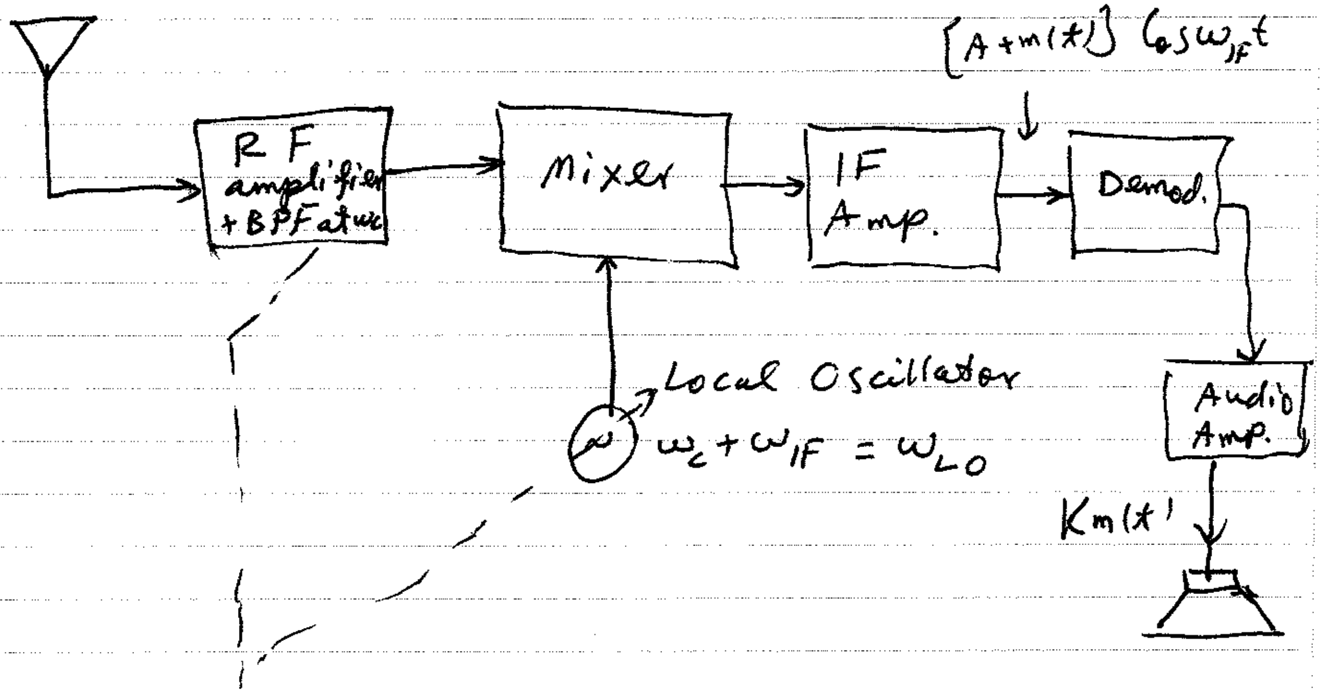
$f_{IF} = 455$ kHz.



$$f_{IF} = f_{LO} - f_c \quad \text{or} \quad f_{LO} = f_c + f_{IF}$$

The reason for ~~sup~~ heterodyning:

Difficulty of designing 10 kHz filters (also AMPs) at all high frequencies.



The broadcast BW for AM is

from 550 to 1600 KHz. $\Rightarrow$ 105 stations

So you can have f_{LO} ~~from~~

from 95 ~~KHz to~~ 2055 KHz

or

from

1005 KHz. to 2055 KHz

Image station:

Assume we like to receive a station at

$$f_c = 1000 \text{ kHz}.$$

we use

$$f_{LO} = 1455 \text{ kHz}$$

since

$$f_{LO} - f_c = 455$$

but at the same Time we get also

$$f_{c'} = 1910 \text{ kHz}$$

since

$$f_{c'} - f_{LO} = 1910 - 1455 = 455 \text{ kHz}.$$

Image rejection is done by RF filter.

Lecture 5:

Angle Modulation

$$\varphi(t) = A\cos(\omega_c t + \theta)$$

or in general

$$\varphi(t) = A\cos\theta(t)$$

where $\theta(t)$ is total phase or the generalized angle.

Instantaneous frequency is defined as

$$\omega_i(t) = \frac{d\theta(t)}{dt} \quad \text{or} \quad f_i(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

or

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha$$

For AM we have:

$$\varphi(t) = A\, m(t) \cos[\omega_c t]$$

so $\theta(t) = \omega_c t$ and $\omega_i(t) = \omega_c$ = constant

For Phase Modulation (PM), we have

$$\theta(t) = \omega_c t + k_p m(t)$$

that is, phase changes with (in proportion to) the message, $m(t)$.

That is

$$\varphi_{PM}(t) = A \cos[\omega_c t + k_p m(t)]$$

Then, for PM, the frequency is:

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \omega_c + k_p \dot{m}(t)$$

So, the frequency varies with the derivative of the message.

For frequency modulation, FM, we have

$$\omega_i(t) = \omega_c + k_f m(t)$$

that is, here, frequency varies (is modulated) by the message; The phase is:

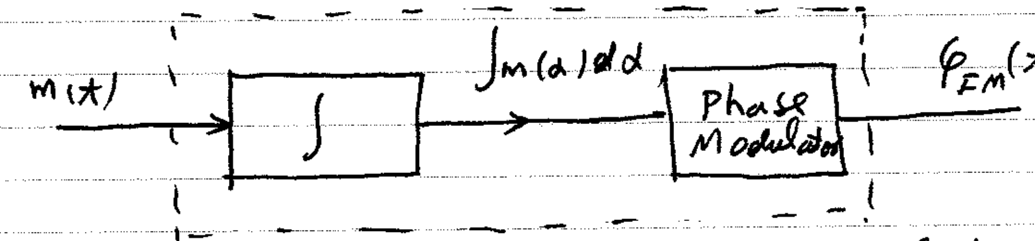
$$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f m(\alpha)] d\alpha$$

$$= \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha$$

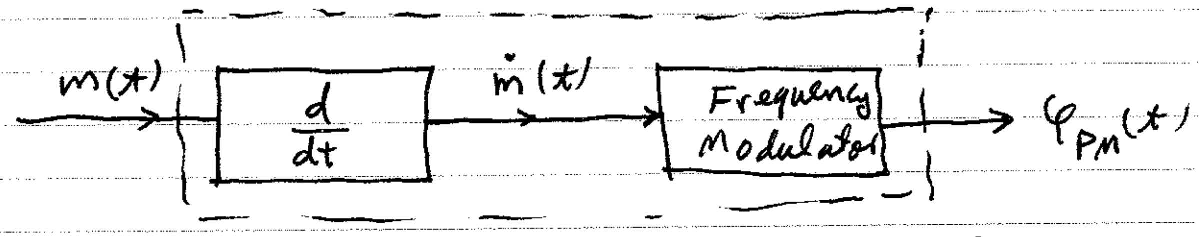
and, therefore

$$\varphi_{FM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right]$$

Note that FM and PM are quite similar: FM is PM with the modulating signal being the integral of the message and PM is FM with the modulating signal being derivative of the message.



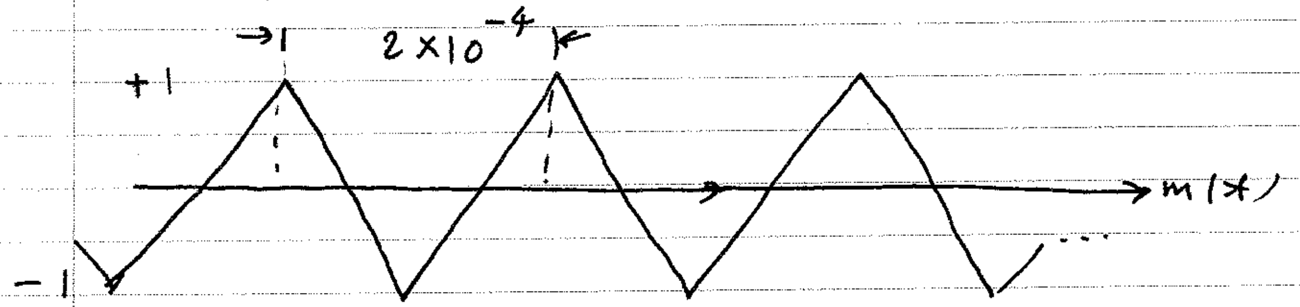
generating FM using PM modulator



generating PM signal using FM modulator

Example:

Sketch FM and PM wave for the modulating signal $m(t)$ shown below. $k_f = 2\pi \times 10^5$ and $k_p = 10\pi$ and $f_c = 100$ MHz



$$f_0 = \frac{1}{2\times10^{-4}} = 5\times10^{3} \quad \text{or} \quad \omega_0 = 10\pi\times10^{3}$$

For FM:

$$\omega_i = \omega_c + k_f\, m(t)$$

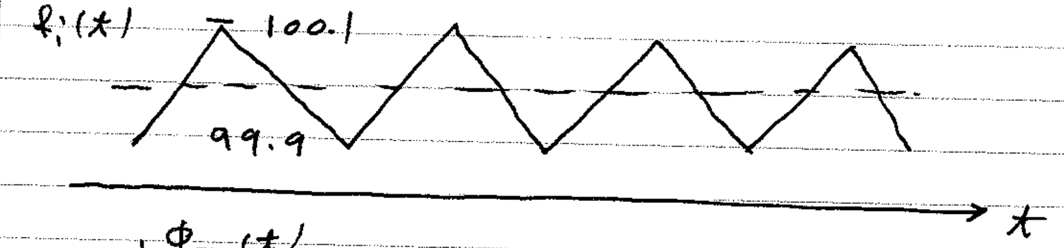
or

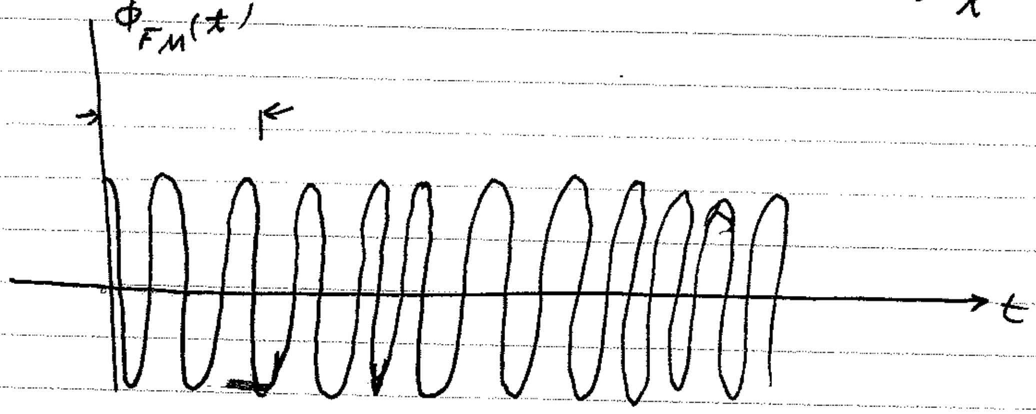
$$f_i = f_c + \frac{k_f}{2\pi}\, m(t)$$

$$f_i = 10^8 + 10^5 m(t)$$

$$f_{i,min} = 10^8 - 10^5 = 99.9 \text{ MHz}$$

$$f_{i,max} = 10^8 + 10^5 = 100.1 \text{ MHz}.$$

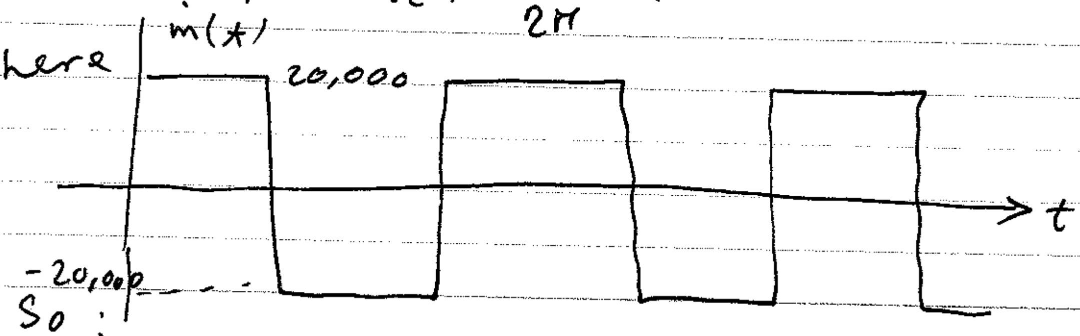




for PM

$$f_i(t) = f_c + \frac{k_p}{2\pi} \dot{m}(t)$$

where



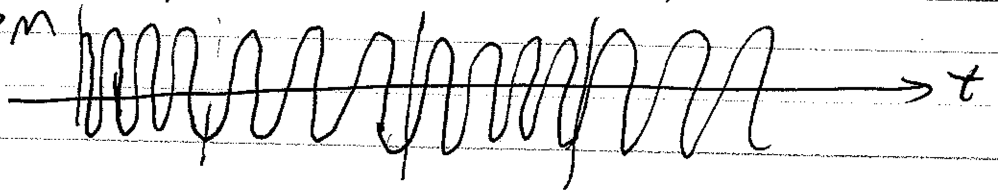
So:

$$f_i(t) = 10^5 + 5\dot{m}(t)$$

$$f_{i,min} = 10^8 - 10^5 = 99.9 \text{ MHz}$$

$$f_{i,max} = 10^8 + 10^5 = 100.1 \text{ MHz}$$

φ_{PM}



Bandwidth requirement of FM signals:

$$\varphi_{FM}(t) = A \cos\left[w_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

let $\int_{-\infty}^{t} m(\alpha)\, d\alpha = a(t)$

then

$$\varphi_{FM}(t) = A \cos[w_c t + k_f a(t)]$$

to make presentation easier let:

$$\varphi_{FM}(t) = Re\, A e^{j[w_c t + k_f a(t)]}$$

$$A e^{j[w_c t + k_f a(t)]} = A e^{j w_c t}\left[1 + j k_f a(t) - \frac{k_f^2}{2!} a^2(t) \cdots\right]$$

So

$$\varphi_{FM}(t) = A\left[\cos w_c t - k_f a(t) \sin w_c - \frac{k_f^2}{2!} a^2(t) \cos w_c t + \frac{k_f^3}{3!} a^3(t) \sin w_c t + \cdots\right]$$

So, the spectrum of FM signal has the carrier plus the spectra of $a(t), a^2(t), a^3(t), \cdots$ centered around w_c.

Narrowband FM (NBFM)

$$\varphi_{FM}(t) = A\cos[\omega_c t + k_f a(t)] = A\cos\omega_c t \cos(k_f a(t)) - A\sin\omega_c t \sin(k_f a(t))$$

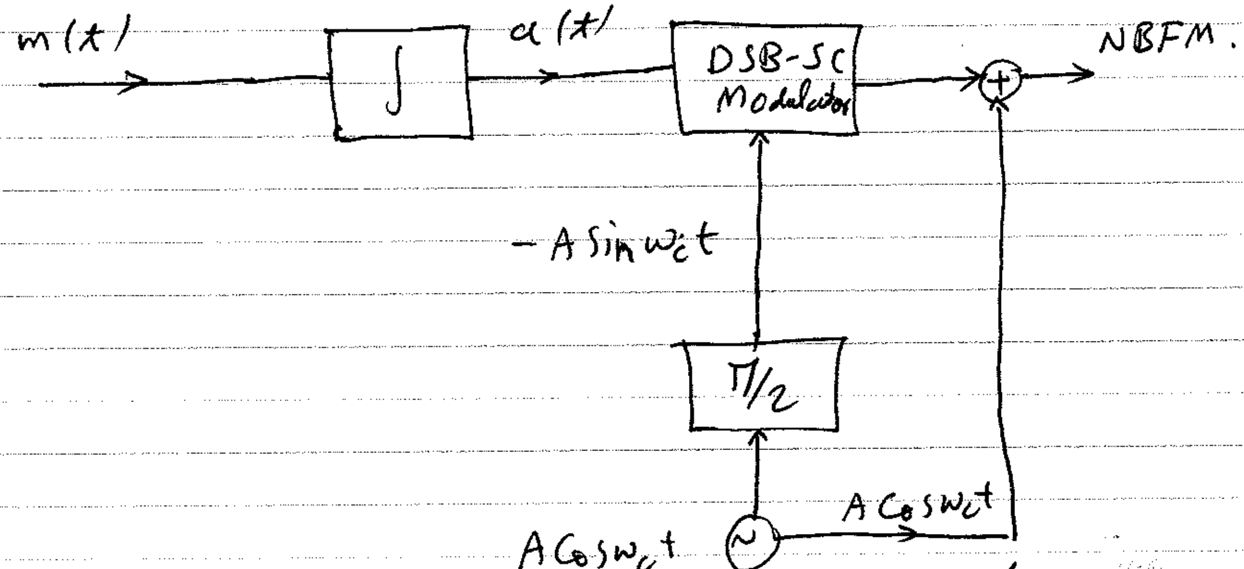
if $|k_f a(t)| \ll 1$

then
$$\sin(k_f a(t)) \simeq k_f a(t)$$

and
$$\cos(k_f a(t)) \simeq 1$$

and
$$\varphi_{FMB}(t) \cong A[\cos\omega_c t - k_f a(t) \sin\omega_c t]$$

and modulation looks like (but, is not) AM modulation.



Here: $BW_{NBFM} = 2B$ where B is the BW of $m(t)$

For FM, in general, i.e., for wide Band FM it is difficult to find the effective required Bandwidth. So, we find it for a tone being modulated, i.e., when

$$m(t) = \alpha \cos \omega_m t$$

then

$$a(t) = \frac{\alpha}{\omega_m} \sin \omega_m t$$

then
$$\varphi_{FM}(t) = A \cos[\omega_c t + \frac{\alpha k_f}{\omega_m} \sin \omega_m t]$$
$$= Re\, A e^{j\omega_c t} \cdot e^{j\frac{\alpha k_f}{\omega_m} \sin \omega_m t}$$

we ~~call~~ denote

$$\beta = \frac{\alpha k_f}{\omega_m}$$

this is called the modulation index.

Note that αk_f is the peak frequency diviation of FM signal, since

$$\omega_i(t) = \omega_c t + k_f \alpha \cos \omega_m t$$

so
$$\beta = \frac{\alpha k_f}{\omega_m} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

Now

$$\varphi_{FM}(t) = \mathrm{Re}\, A e^{j\omega_c t} \cdot \left[e^{j\beta \sin \omega_m t} \right]$$

$e^{j\beta \sin \omega_m t}$ is periodic with period $\frac{2\pi}{\omega_m}$

since letting $t \to t + \frac{2\pi}{\omega_m}$, we get

$$e^{j\beta \sin (t + \frac{2\pi}{\omega_m})\omega_m} = e^{j\beta \sin \omega_m t}.$$

So, $e^{j\beta \sin \omega_m t}$ can be represented as

a Fourier Series:

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} C_n e^{j\omega_m t}$$

where

$$C_n = \frac{\omega_m}{2\pi} \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} \cdot \cancel{dt}\, e^{-jn\omega_m t} \, dt$$

let $x = \omega_m t$ to get

$$C_n = \underbrace{\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx}$$

The integral is called the Bessel Function of first kind and nth order and is denoted as $J_n(\beta)$.

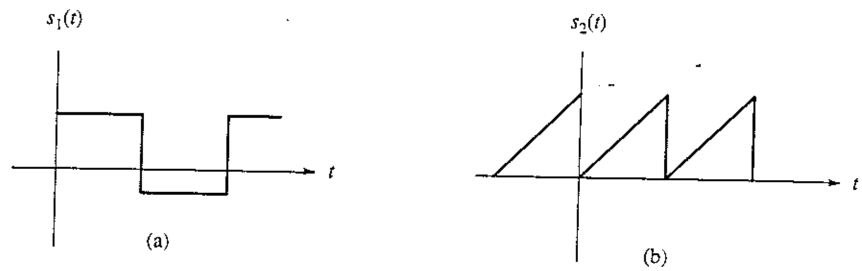
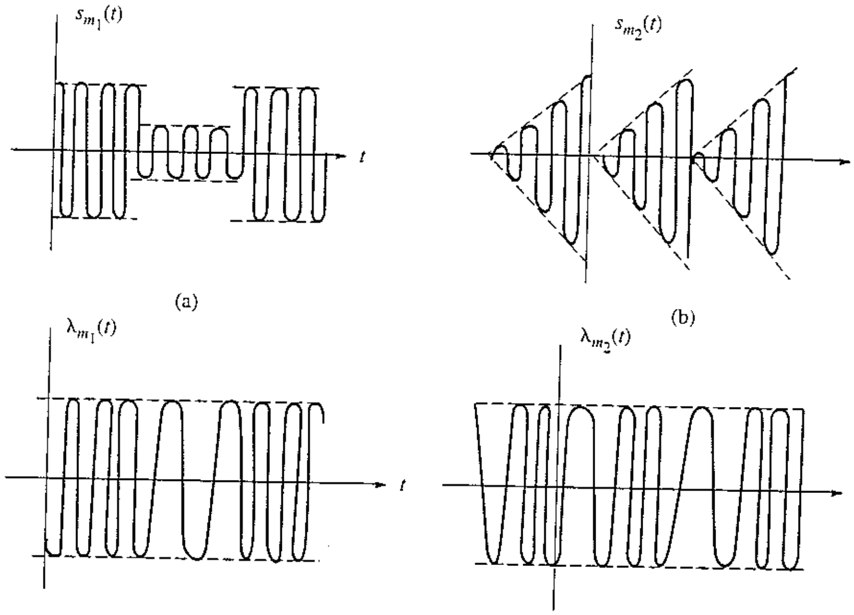
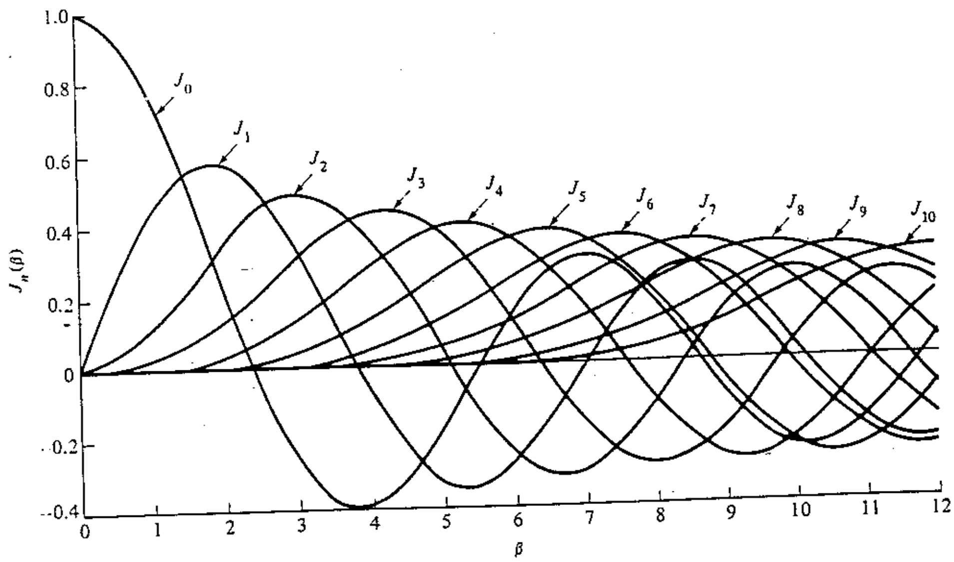


Figure 7.4 Waveforms for Example 7.3.





Plot of Bessel function of the first kind, $J_n(\beta)$.

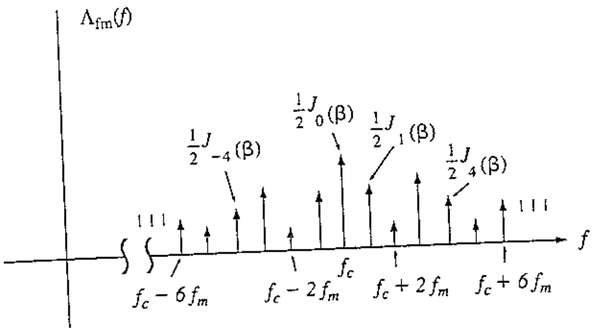
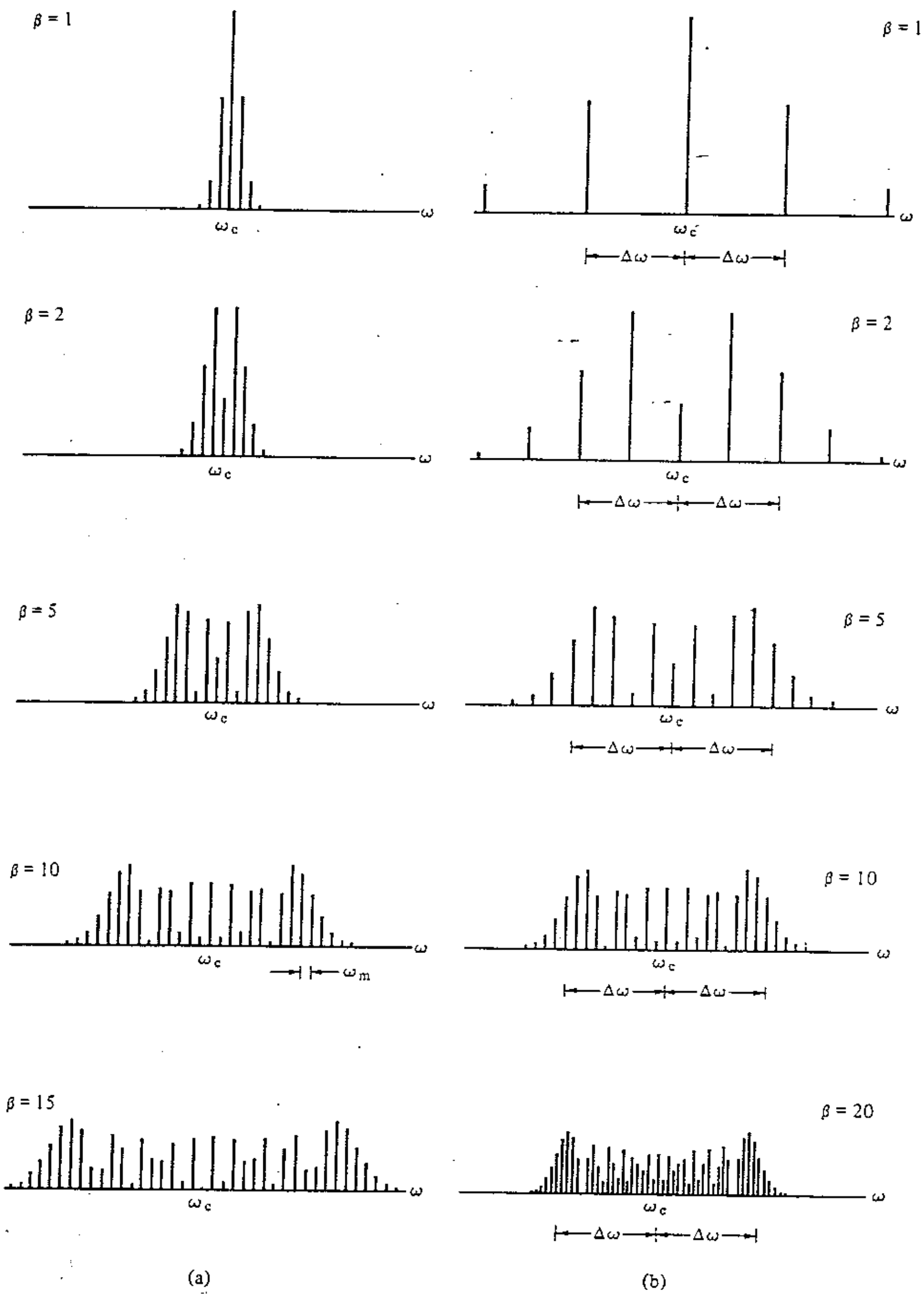


Figure 7.11 Fourier transform of FM with sinusoidal information signal.



Magnitude line spectra for FM waveforms with sinusoidal modulation: (a) for constant ω_m; (b) for constant $\Delta\omega$.

So:

$$e^{j\beta \sin n\omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\omega_m t}$$

so, we get

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m) t$$

From plot of $J_n(\beta)$ or from its tabulated values, one can observe that $J_n(\beta)$ is negligible for $n > \beta + 1$

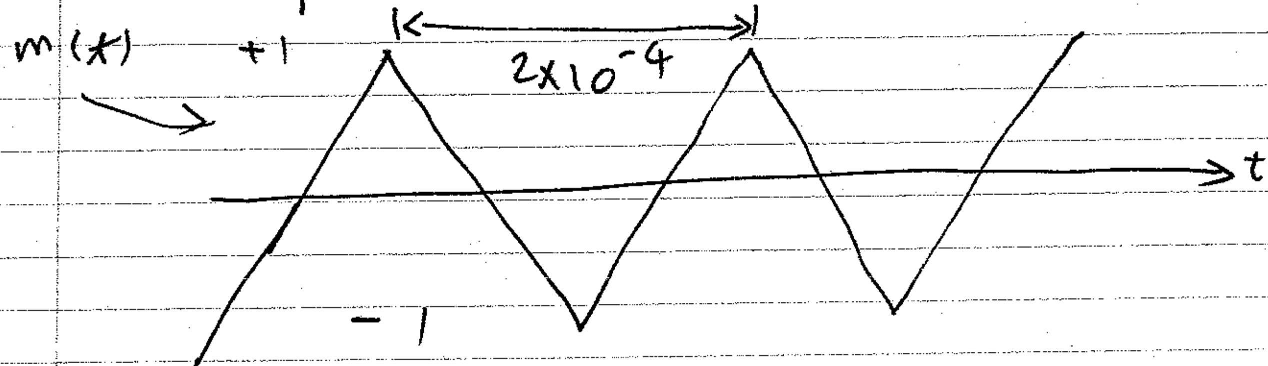
So:

$$B_{FM} = 2n f_m = 2(\beta+1) f_m = 2(\beta+1) B$$

$$= 2(\Delta f + B)$$

in general

Example: a) Find the approximate bandwidth of an FM signal with $m(t)$ given below and $k_f = 2\pi \times 10^5$.



b) repeat if $m(t)$ is amplified by 2.

a)

$$m(t) = \sum_n C_n \cos n\omega_0 t \qquad \omega_0 = \frac{2\pi}{2\times 10^{-4}} = 10^4 \pi$$

$$C_n = \begin{cases} \dfrac{8}{\pi^2 n^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases} \qquad C_1 = 0.81, \quad C_3 = \frac{0.81}{9}, \quad C_5 = \frac{0.81}{25}$$

Power in Fundamental frequency is $\frac{64}{2\pi^4} = P_1$

in the third harmonic is $\frac{64}{2\pi^4 \times 9^2} \approx 1.21\%\ P_1$

and in fifth harmonic $\frac{64}{2\pi^4 \times 25^2} \approx$ 0.16% of P_1

So, we can ignore anything beyond third harmonic and assume $B = 15\,kHz$.

$$\Delta f = \frac{k_f m_p}{2\pi} = \frac{1}{2\pi}(2\pi \times 10^5)(1) = 100\,kHz$$

So

$$B_{FM} = 2(\Delta F + B) = 230$$

we could also do

$$\beta = \frac{\Delta f}{B} = \frac{100}{15}$$

$$B_{FM} = 2B(\beta + 1) = 15 \times 2\left(\frac{100}{5} + 1\right) = 230\,kHz.$$

---

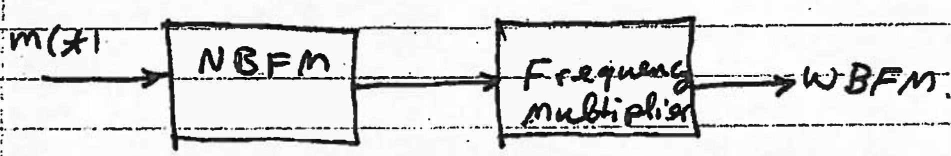
b) $m_p = 2$, $B = 15\,kHz$

$$\Delta f = \frac{1}{2\pi} \cdot k_f m_p = \frac{1}{2\pi}(2\pi \times 10^5) \times 2 = 200\,kHz$$

$$B_{FM} = 2(\Delta f + B) = 430\,kHz.$$

Generation of FM waves:

Indirect FM modulation (Armstrong's method)



Example: Assume that we wish to modulate a signal to carrier frequency of 91.2 MHZ with $\Delta f = 75$ kHZ. We begin by modulating to a frequency of 200 kHZ with a $\Delta f = 25$ Hz. (to make $\beta \ll 1$). Assume that $B = 15$ kHZ. (and range is 50 HZ to 15 kHZ)

Then $\beta = \dfrac{\Delta f}{f_m} = \dfrac{25}{50} = 0.5$ even for the worst case of $f_m = 50$ HZ.

Then, in order to get $\Delta f = 75$ kHZ, we need to multiply the frequency by $\dfrac{75000}{25} = 3000$.

This can be done using two frequency multipliers: one by 64 and the second by 48.

This results in a multiplication by

$$64 \times 48 = 3072.$$

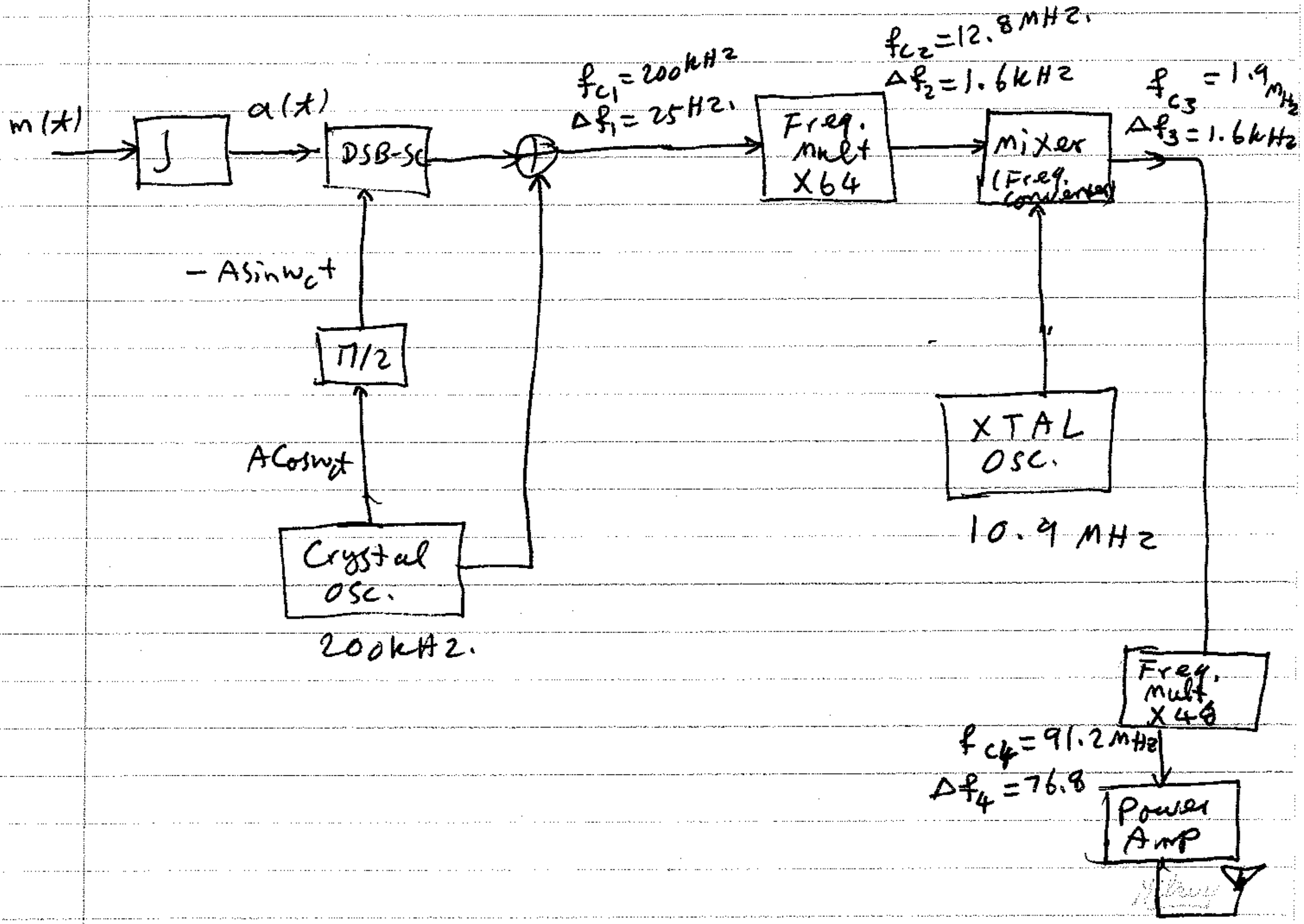
This gives $\Delta f = 76.8$ kHz which is close to 75 kHz.

But the carrier frequency will be

$$200 \times 64 \times 48 \approx 600 \text{ MHz}.$$

But, we needed $f_c = 91.2$ MHz.

So, in order to get this: after the first multiplier, we translate the carrier by 10.9 MHz to 1.9 MHz.



Direct Generation of FM Signal

$$\omega_i(t) = \omega_c + k_f\, m(t)$$

$$m(t) \longrightarrow \boxed{VCO} \longrightarrow \text{FM signal}$$

VCO is an LC circuit with

$$C = C_0 - k\,m(t)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{LC_0\left[1 - \frac{k\,m(t)}{C_0}\right]}}$$

$$\omega_0 = \frac{1}{\sqrt{LC_0}} \cdot \frac{1}{\left[1 - \frac{k\,m(t)}{C_0}\right]^{1/2}}$$

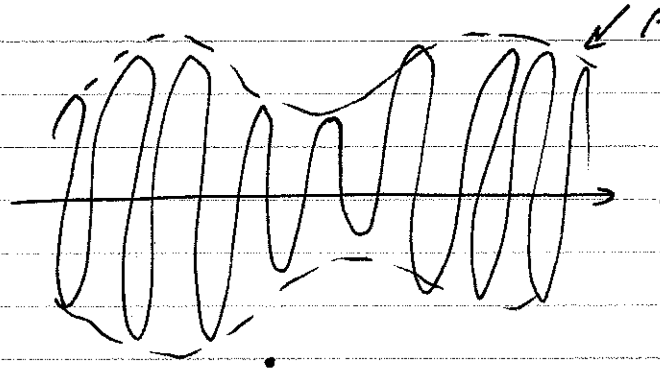
$$\omega_0 \simeq \frac{1}{\sqrt{LC_0}}\left[1 + \frac{k\,m(t)}{2C_0}\right] \quad \text{if} \quad \frac{k\,m(t)}{C_0} \ll 1.$$

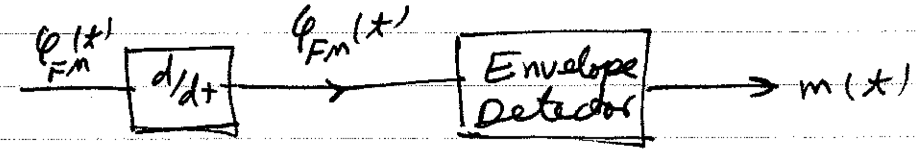
The reason for use of indirect FM is that direct FM lacks frequency stability.

Demodulation of FM

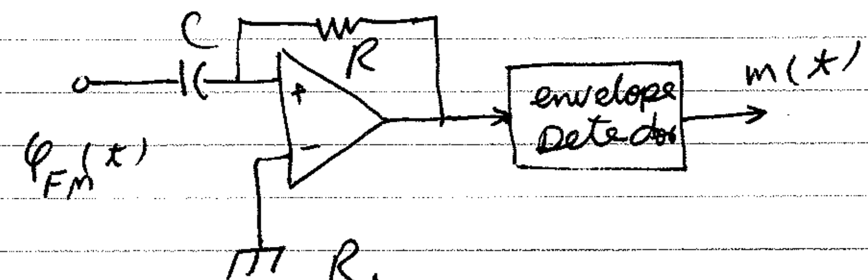
$$\varphi_{FM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

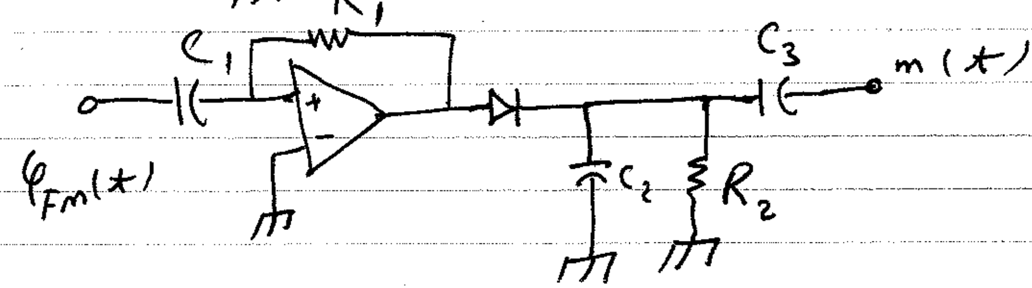
$$\frac{d}{dt}\varphi_{FM}(t) = -A\left[\omega_c + k_f m(t)\right] \sin\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$





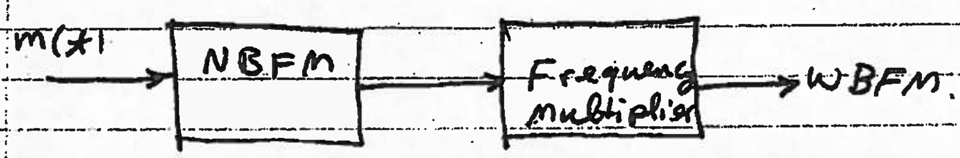
$H(\omega) = j\omega$





Generation of FM waves:

Indirect FM modulation (Armstrong's method)



Example: Assume that we wish to modulate a signal to carrier frequency of 91.2 MHZ with $\Delta f = 75$ kHZ. We begin by modulating to a frequency of 200 kHZ with a $\Delta f = 25$ Hz. (to make $\beta \ll 1$). Assume that $B = 15$ kHZ. (and range is 50 HZ to 15 kHZ)

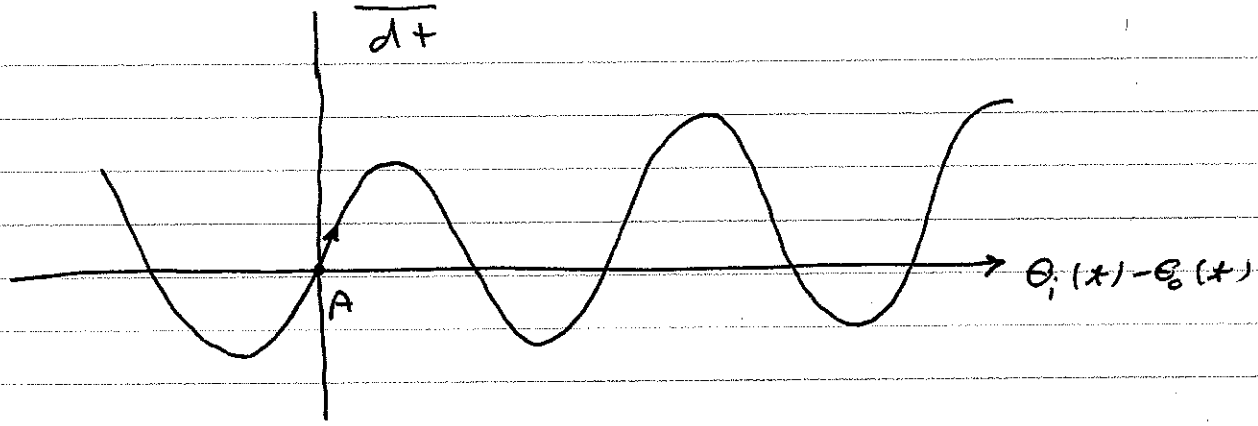
Then $\beta = \frac{\Delta f}{f_m} = \frac{25}{50} = 0.5$ even for the worst case of $f_m = 50$ HZ.

Then, in order to get $\Delta f = 75$ kHZ, we need to multiply the frequency by $\frac{75000}{25} = 3000$.

This can be done using two frequency multipliers: one by 64 and the second by 48.

This results in a multiplication by

$$64 \times 48 = 3072.$$



at point A if $\theta_i(t)-\theta_o(t)$ increases (either as a result of an increase in $\theta_i(t)$ or a decrease in $\theta_o(t)$, former being usually the case) then $\frac{d\theta_o(t)}{dt}$ becomes (more) positive, i.e., increasing $\theta_o(t)$, and as a result decreasing $\theta_i(t)-\theta_o(t)$. The net result would be to push the operating point back to A.

If $\theta_i(t)-\theta_o(t)$ becomes negative (as a result of a decrease in $\theta_i(t)$) ~~then~~ $\frac{d\theta_o(t)}{dt}$ becomes negative and $\theta_o(t)$ decreases and the operating point is pushed up to A.

So, in the steady-state, we have

$$\theta_i(t)-\theta_o(t)\simeq 0$$

or

$$\theta_o(t)\simeq\theta_i(t)$$

But

$$\theta_i(t) = k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

since

$$\varphi_{FM}(t) = A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

therefore:

$$\theta_o(t) = k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

and

$$\frac{d}{dt}\theta_o(t) = c\, e_o(t) = k_f\, m(t)$$

or

$$\boxed{e_o(t) = \frac{k_f}{c}\, m(t)}$$

Pre-emphasis - Deemphasis (PDE)

The interference in FM increases as frequency. In order to protect higher frequencies, we may want to amplify higher frequency components (more than lower frequency ones) before transmission (Emphasize them) and at the receiver attenuate them (less amplify them). This way, we suppress the noise (since noise has not been emphasized while signal has been).

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The preemphasis transfer function is

$$H_p(\omega) = K\frac{j\omega + \omega_1}{j\omega + \omega_2} \tag{5.39a}$$

where K, the gain, is set at a value of ω_2/ω_1. Thus,

$$H_p(\omega) = \left(\frac{\omega_2}{\omega_1}\right)\frac{j\omega + \omega_1}{j\omega + \omega_2} \tag{5.39b}$$

For $\omega \ll \omega_1$,

$$H_p(\omega) \simeq 1 \tag{5.39c}$$

For frequencies $\omega_1 \ll \omega \ll \omega_2$,

$$H_p(\omega) \simeq \frac{j\omega}{\omega_1} \tag{5.39d}$$

Thus, the preemphasizer acts as a differentiator at intermediate frequencies (2.1 to 15 kHz), which effectively makes the scheme PM over these frequencies. This means that FM with PDE

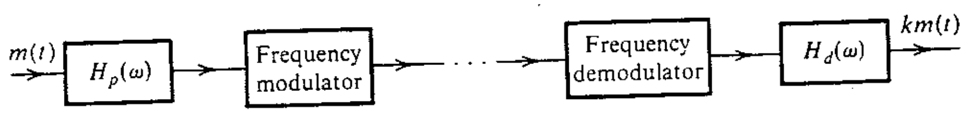

Figure 5.17 Preemphasis-deemphasis in an FM system.

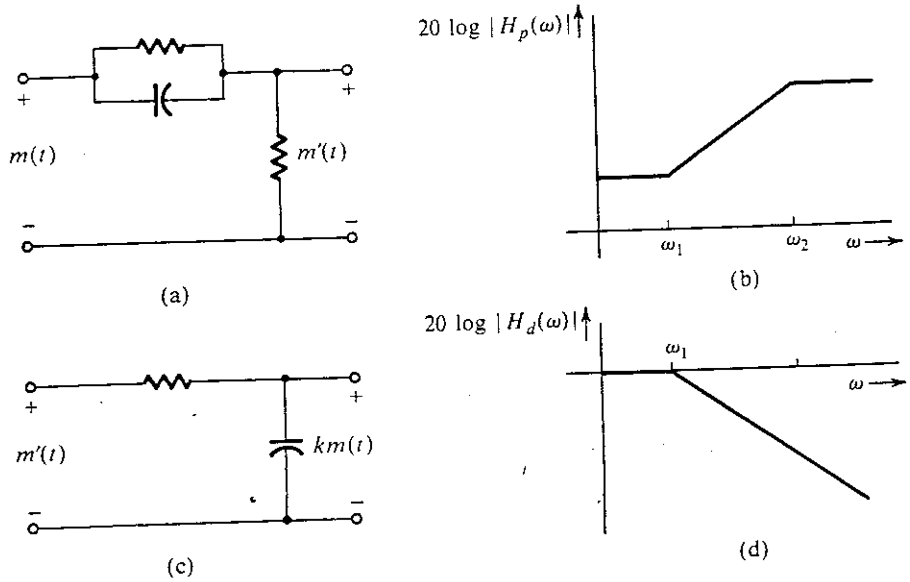

Figure 5.18 (a) Preemphasis filter. (b) Its frequency response. (c) Deemphasis filter. (d) Its frequency response

is FM over the modulating-signal frequency range of 0 to 2.1 kHz and is nearly PM over the range of 2.1 to 15 kHz as desired.

The deemphasis filter $H_d(\omega)$ is given by

$$H_d(\omega) = \frac{\omega_1}{j\omega + \omega_1}$$

Note that for $\omega \ll \omega_2$, $H_p(\omega) \simeq (j\omega + \omega_1)/\omega_1$. Hence, $H_p(\omega)H_d(\omega) \simeq 1$ over the baseband of 0 to 15 kHz.

FM broadcast

The frequency range assigned to FM broadcast is 88 to 108 MHz. with a separation of 200 kHz and a peak frequency deviation of $\Delta f = 75$ kHz.

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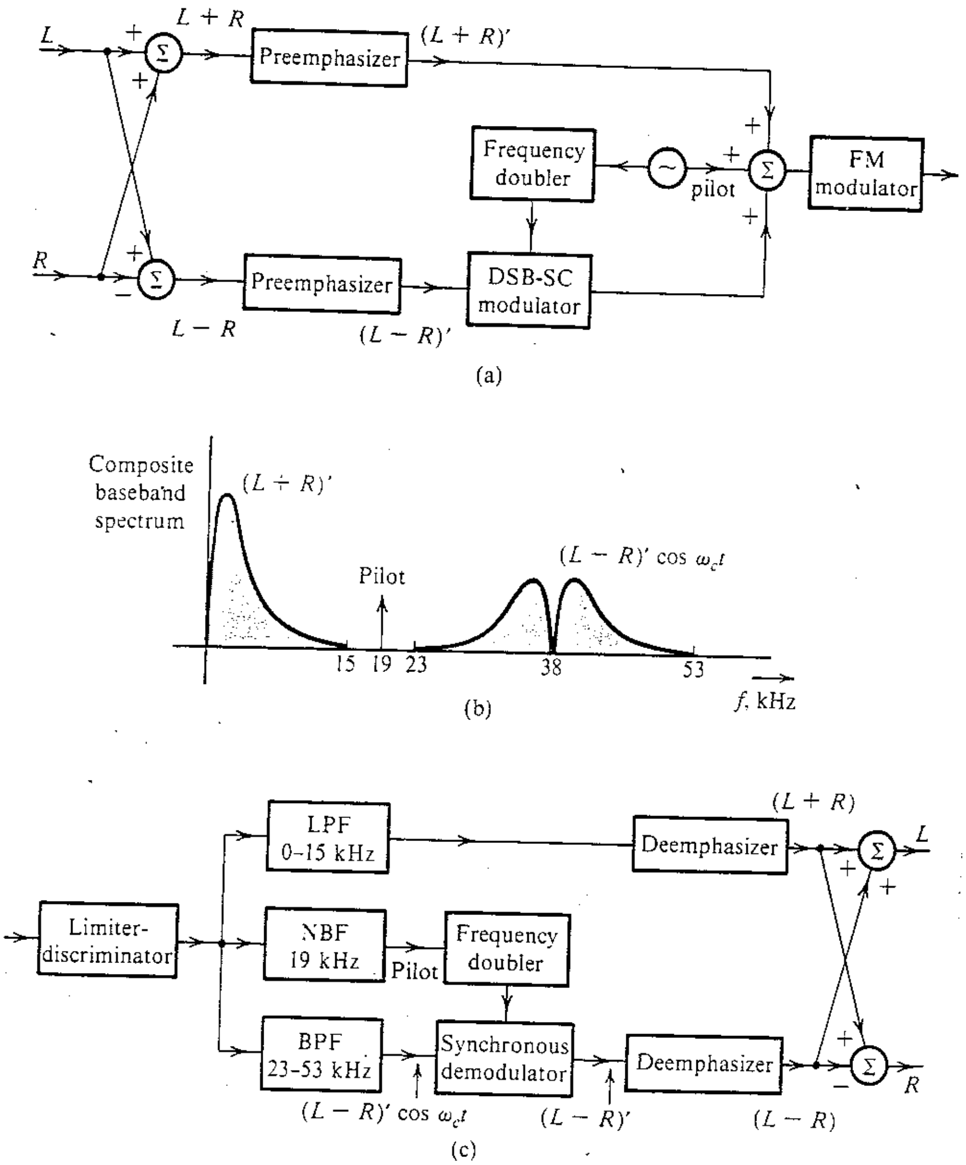

Figure 5.19 (a) FM stereo transmitter. (b) Spectrum of a baseband stereo signal. (c) FM stereo receiver.