Chapter 6

6.1

(a) Fourier transform of $dx_a(t)/dt$ is $\hat{X}_a(F) = j2\pi F X_a(F)$, then $F_s \geq 2B$
(b) Fourier transform of $x_a^2(t)$ is $\hat{X}_a(F) = X_a(F) * X_a(F)$, then $F_s \geq 4B$
(c) Fourier transform of $x_a(2t)$ is $\hat{X}_a(F) = 2X_a(F/2)$, then $F_s \geq 4B$
(d) Fourier transform of $x_a(t)\cos(6\pi Bt)$ is $\hat{X}_a(F) = \frac{1}{2}X_a(F+3B) + \frac{1}{2}X_a(F-3B)$ resulting in $F_L = 2B$ and $F_H = 4B$. Hence, $F_s = 2B$
(d) Fourier transform of $x_a(t)\cos(7\pi Bt)$ is $\hat{X}_a(F) = \frac{1}{2}X_a(F+3.5B) + \frac{1}{2}X_a(F-3.5B)$ resulting in $F_L = 5B/2$ and $F_H = 9B/2$. Hence, $k_{max} = \lfloor \frac{F_H}{B} \rfloor = 2$ and $F_s = 2F_H/k_{max} = 9B/2$

6.2

(a) $F_s = 1/T \geq 2B \Rightarrow A = T, F_c = B$.

(b) $X_a(F) = 0$ for $|F| \geq 3B$. $F_s = 1/T \geq 6B \Rightarrow A = T, F_c = 3B$.

(c) $X_a(F) = 0$ for $|F| \geq 5B$. $F_s = 1/T \geq 10B \Rightarrow A = T, F_c = 5B$.

6.3

$$x_a(t) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} e^{j2\pi kt/T_p} \tag{6.1}$$

Since filter cut-off frequency, $F_c = 102.5$, then terms with $|n|/T_p > F_c$ will be filtered resulting

$$\begin{aligned} y_a(t) &= \sum_{k=-10}^{10} \left(\frac{1}{2}\right)^{|k|} e^{j2\pi kt/T_p} \\ Y_a(F) &= \sum_{k=-10}^{10} \left(\frac{1}{2}\right)^{|k|} \delta(F - k/T_p) \end{aligned}$$

Sampling this signal with $Fs = 1/T = 1/0.005 = 200 = 20/T_p$ results in aliasing

$$\begin{aligned} Y(F) &= \frac{1}{3} \sum_{n=-\infty}^{\infty} X_a(F - nFs) \\ &= \frac{1}{3} \sum_{n=-\infty}^{\infty} \left(\sum_{k=-9}^{9} \left(\frac{1}{2}\right)^{|k|} \delta(F - k/T_p - nF_s) + \left(\frac{1}{2}\right)^{9} \delta(F - 10/T_p - nF_s) \right) \end{aligned}$$

6.9

(a)

$$
\begin{aligned}
x_a(t) &= e^{-j2\pi F_0 t} \\
X_a(F) &= \int_0^{\infty} x_a(t) e^{-j2\pi F t} dt \\
&= \int_0^{\infty} e^{-j2\pi F_0 t} e^{-j2\pi t} dt \\
&= \int_0^{\infty} e^{-j2\pi (F+F_0) t} dt \\
&= \frac{e^{-j2\pi(F+F_0)t}}{-j2\pi(F+F_0)} \Big|_0^{\infty} \\
X_a(F) &= \frac{1}{j2\pi(F+F_0)}
\end{aligned}
$$

(b)

$$
\begin{aligned}
x(n) &= e^{-\frac{j2\pi F_0 n}{F_s}} \\
X(f) &= \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n} \\
&= \sum_{n=0}^{\infty} e^{-\frac{j2\pi F_0 n}{F_s}} e^{-j2\pi f n} \\
&= \sum_{n=0}^{\infty} e^{-j2\pi(F+\frac{F_0}{F_s})n} \\
&= \frac{1}{1-e^{-j2\pi(F+\frac{F_0}{F_s})}}
\end{aligned}
$$

(c) Refer to fig 6.9-1
(d) Refer to fig 6.9-2
(e) Aliasing occurs at $F_s = 10\text{Hz}$.

6.10

Since $\frac{F_c+\frac{B}{2}}{B} = \frac{50+10}{20} = 3$ is an integer, then $F_s = 2B = 40Hz$

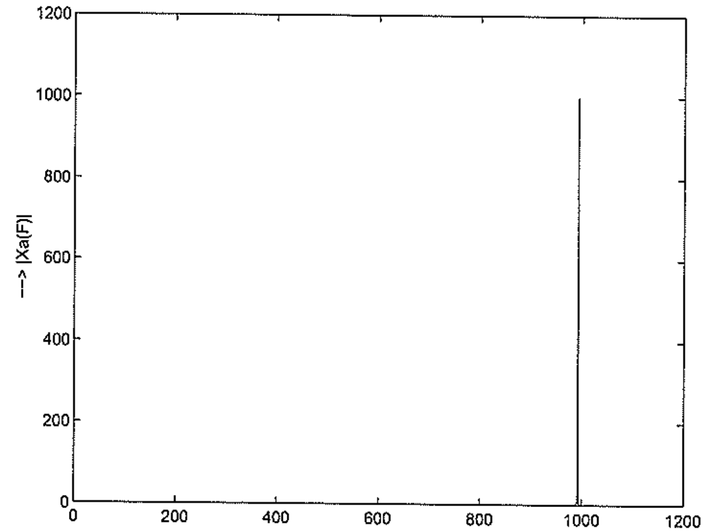

Figure 6.9-1:

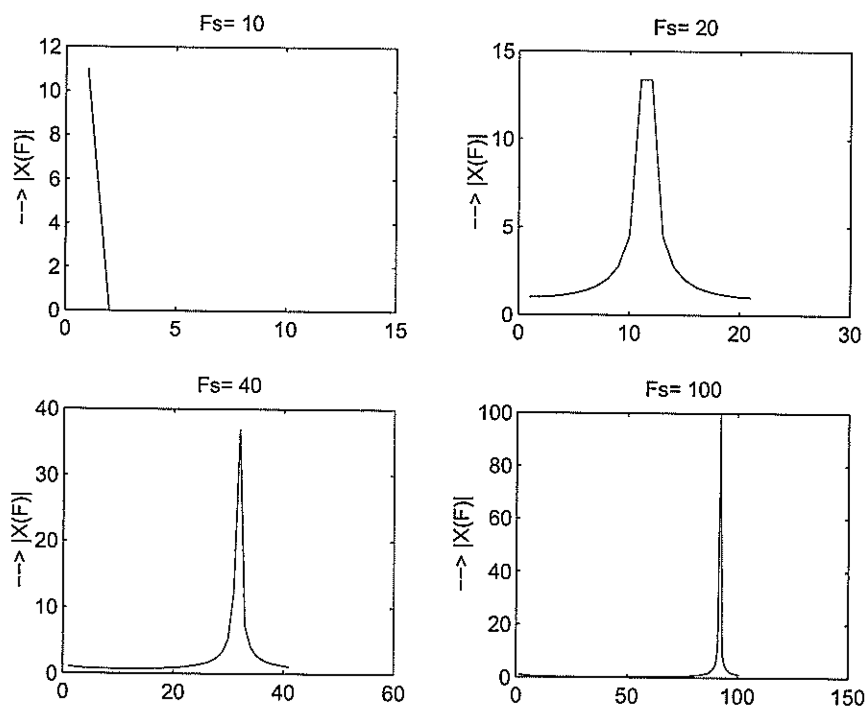

Figure 6.9-2:

6.11

$$\begin{aligned}
F_c &= 100 \\
B &= 12 \\
r &= \lceil \frac{F_c + \frac{B}{2}}{B} \rceil \\
&= \lceil \frac{106}{12} \rceil \\
&= \lceil 8.83 \rceil = 8 \\
B' &= \frac{F_c + \frac{B}{2}}{r} \\
&= \frac{106}{8} \\
&= \frac{53}{4} \\
F_s &= 2B' \\
&= \frac{53}{2} \text{ Hz}
\end{aligned}$$

6.15

(a)

$$\begin{aligned}
H(F) &= \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t} dt \\
&= \underbrace{\int_0^T \frac{t}{T} e^{-j2\pi f t} dt}_{A} + \underbrace{\int_T^{2T} 2e^{-j2\pi f t} dt}_{B} - \underbrace{\int_T^{2T} \frac{t}{T} e^{-j2\pi f t} dt}_{C}
\end{aligned}$$

Substituting $a = -j2\pi f$

$$\begin{aligned}
A(F) &= \frac{1}{T}\left[\frac{e^{aT}}{a^2}(aT-1) - \frac{1}{a^2}(-1)\right] \\
&= \underbrace{\frac{e^{aT}}{a}}_{A1} - \underbrace{\frac{e^{aT}}{Ta^2}}_{A2} + \underbrace{\frac{1}{Ta^2}}_{A3} \\
B(F) &= \frac{2}{a}\left[e^{a2T} - e^{aT}\right] \\
&= \frac{2e^{a3T/2}}{\pi f} \sin(\pi f T) \\
C(F) &= -\frac{1}{T}\left[\frac{e^{a2T}}{a^2}(a2T-1) - \frac{e^{aT}}{a^2}(aT-1)\right] \\
&= -\underbrace{\frac{e^{a2T}}{a}}_{C1} - \underbrace{\frac{e^{a2T}}{a}}_{C2} + \underbrace{\frac{e^{a2T}}{Ta^2}}_{C3} + \underbrace{\frac{e^{aT}}{a}}_{C4} - \underbrace{\frac{e^{aT}}{Ta^2}}_{C5}
\end{aligned}$$

$$\begin{aligned}
A1(F) + C1(F) &= -\frac{e^{a3T/2}}{\pi f} \sin(\pi f T) \\
A2(F) + C3(F) &= \frac{e^{a3T/2}}{Ta\pi f} \sin(\pi f T) \\
A3(F) + C5(F) &= -\frac{e^{aT/2}}{Ta\pi f} \sin(\pi f T) \\
C2(F) + c4(F) &= -\frac{e^{a3T/2}}{\pi f} \sin(\pi f T)
\end{aligned}$$

Then,

$$H(F) = \frac{e^{-j2\pi f T}}{T} \left(\frac{\sin(\pi f T)}{\pi f}\right)^2$$

6.18

Let P_d denote the power spectral density of the quantization noise. Then (a)

$$\begin{aligned} P_n &= \int_{-\frac{B}{F_s}}^{\frac{B}{F_s}} P_d df \\ &= \frac{2B}{F_s} P_d \\ &= \sigma_e^2 \\ \text{SQNR} &= 10log_{10}\frac{\sigma_x^2}{\sigma_e^2} \\ &= 10log_{10}\frac{\sigma_x^2 F_s}{2BP_d} \end{aligned}$$

$$= 10log_{10}\frac{\sigma_x^2 F_s}{2BP_d} + 10log_{10}F_s$$

Thus, SQNR will increase by 3dB if F_s is doubled.
(b) The most efficient way to double the sampling frequency is to use a sigma-delta modulator.