# Chapter 7

## 7.1

Since $x(n)$ is real, the real part of the DFT is even, imaginary part odd. Thus, the remaining points are $\{0.125 + j0.0518, 0, 0.125 + j0.3018\}$

## 7.2

(a)

$$\begin{aligned}
\tilde{x}_2(l) &= x_2(l), \quad 0 \le l \le N-1 \\
&= x_2(l+N), \quad -(N-1) \le l \le -1 \\
\tilde{x}_2(l) &= sin(\frac{3\pi}{8}l), \quad 0 \le l \le 7 \\
&= sin(\frac{3\pi}{8}(l+8)), \quad -7 \le l \le -1 \\
&= sin(\frac{3\pi}{8}|l|), \quad |l| \le 7 \\
\text{Therefore, } x_1(n) \, \textcircled{8} \, x_2(n) &= \sum_{m=0}^{3} \tilde{x}_2(n-m) \\
&= sin(\frac{3\pi}{8}|n|) + sin(\frac{3\pi}{8}|n-1|) + \ldots + sin(\frac{3\pi}{8}|n-3|) \\
&= \{1.25, 2.55, 2.55, 1.25, 0.25, -1.06, -1.06, 0.25\}
\end{aligned}$$

(b)

$$\begin{aligned}
\tilde{x}_2(n) &= cos(\frac{3\pi}{8}n), \quad 0 \le l \le 7 \\
&= -cos(\frac{3\pi}{8}n), \quad -7 \le l \le -1 \\
&= [2u(n)-1]\, cos(\frac{3\pi}{8}n), \quad |n| \le 7 \\
\text{Therefore, } x_1(n) \, \textcircled{8} \, x_2(n) &= \sum_{m=0}^{3} \left(\frac{1}{4}\right)^m \tilde{x}_2(n-m) \\
&= \{0.96, 0.62, -0.55, -1.06, -0.26, -0.86, 0.92, -0.15\}
\end{aligned}$$

(c)

$$\text{for (a) } X_1(k) = \sum_{n=0}^{7} x_1(n) e^{-j\frac{\pi}{4}kn}$$

$$= \{4, 1 - j2.4142, 0, 1 - j0.4142, 0, 1 + j0.4142, 0, 1 + j2.4142\}$$

similarly,

$$X_2(k) = \{1.4966, 2.8478, -2.4142, -0.8478, -0.6682, -0.8478, -2.4142, 2.8478\}$$

$$\text{DFT of } x_1(n) \textcircled{8} x_2(n) = X_1(k)X_2(k)$$

$$= \{5.9864, 2.8478 - j6.8751, 0, -0.8478 + j0.3512, 0, -0.8478 - j0.3512, 0, 2.8478 + j6.8751\}$$

For sequences of part (b)

$$X_1(k) = \{1.3333, 1.1612 - j0.2493, 0.9412 - j0.2353, 0.8310 - j0.1248, 0.8, 0.8310 + j0.1248, 0.9412 + j0.2353, 1.1612 + j0.2493\}$$

$$X_2(k) = \{1.0, 1.0 + j2.1796, 1.0 - j2.6131, 1.0 - j0.6488, 1.0, 1.0 + j0.6488, 1.0 + j2.6131, 1.0 - j2.1796\}$$

Consequently,

$$\text{DFT of } x_1(n) \textcircled{8} x_2(n) = X_1(k)X_2(k)$$

$$= \{1.3333, 1.7046 + j2.2815, 0.3263 - j2.6947, 0.75 - j0.664, 0.8, 0.75 + j0.664, 0.3263 + j2.6947, 1.7046 - j2.2815\}$$

## 7.3

$\hat{x}(k)$ may be viewed as the product of $X(k)$ with

$$F(k) = \begin{cases} 1, & 0 \le k \le k_c,\ N - k_c \le k \le N - 1 \\ 0, & k_c < k < N - k_c \end{cases}$$

$F(k)$ represents an ideal lowpass filter removing frequency components from $(k_c + 1)\frac{2\pi}{N}$ to $\pi$. Hence $\hat{x}(n)$ is a lowpass version of $x(n)$.

## 7.4

(a)

$$\begin{aligned} x_1(n) &= \frac{1}{2}\left(e^{j\frac{2\pi}{N}n} + e^{-j\frac{2\pi}{N}n}\right) \\ X_1(k) &= \frac{N}{2}[\delta(k-1) + \delta(k+1)] \\ \text{also } X_2(k) &= \frac{N}{2j}[\delta(k-1) - \delta(k+1)] \\ \text{So } X_3(k) &= X_1(k)X_2(k) \\ &= \frac{N^2}{4j}[\delta(k-1) - \delta(k+1)] \\ \text{and } x_3(n) &= \frac{N}{2}sin(\frac{2\pi}{N}n) \end{aligned}$$

(b)

$$\begin{aligned} \tilde{R}_{xy}(k) &= X_1(k)X_2^*(k) \\ &= \frac{N^2}{4j}[\delta(k-1) - \delta(k+1)] \\ \Rightarrow \quad \tilde{r}_{xy}(n) &= -\frac{N}{2}sin(\frac{2\pi}{N}n) \end{aligned}$$

## 7.8

$$\begin{aligned} y(n) &= x_1(n) \textcircled{4} x_2(n) \\ &= \sum_{m=0}^{3} x_1(m)_{\text{mod}4} x_2(n-m)_{\text{mod}4} \\ &= \{17, 19, 22, 19\} \end{aligned}$$

## 7.13

(a)

$$\begin{aligned} X_1(k) &= \sum_{n=0}^{N-1} x(n) W_N^{kn} \\ X_3(k) &= \sum_{n=0}^{3N-1} x(n) W_{3N}^{kn} \\ &= \sum_{n=0}^{N-1} x(n) W_{3N}^{kn} + \sum_{n=N}^{2N-1} x(n) W_{3N}^{kn} + \sum_{n=2N}^{3N-1} x(n) W_{3N}^{kn} \\ &= \sum_{n=0}^{N-1} x(n) W_N^{n\frac{k}{3}} + \sum_{n=0}^{N-1} x(n) W_3^k W_N^{n\frac{k}{3}} + \sum_{n=0}^{N-1} x(n) W_3^{2k} W_N^{n\frac{k}{3}} \\ &= \sum_{n=0}^{N-1} x(n) \left[1 + W_3^k + W_3^{2k}\right] W_N^{n\frac{k}{3}} \\ &= (1 + W_3^k + W_3^{2k}) X_1(k) \end{aligned}$$

(b)

$$\begin{aligned} X_1(k) &= 2 + W_2^k \\ X_3(k) &= 2 + W_6^k + 2W_6^{2k} + W_6^{3k} + 2W_6^{4k} + W_6^{5k} \\ &= (2 + W_2^{\frac{k}{3}}) + W_6^{2k}(2 + W_2^{\frac{k}{3}}) + W_6^{4k}(2 + W_2^{\frac{k}{3}}) \\ &= (1 + W_3^k + W_3^{2k}) X_1(\frac{k}{3}) \end{aligned}$$

## 7.17

$$\begin{aligned} X(k) &= \sum_{n=0}^{7} x(n) e^{-j\frac{2\pi}{8}kn} \\ &= \{6, -0.7071 - j1.7071, 1 - j, 0.7071 + j0.2929, 0, 0.7071 - j0.2929, 1 + j, \\ &\quad -0.7071 + j1.7071\} \\ |X(k)| &= \{6, 1.8478, 1.4142, 0.7654, 0, 0.7654, 1.4142, 1.8478\} \\ \angle X(k) &= \left\{0, -1.9635, \frac{-\pi}{4}, 0.3927, 0, -0.3927, \frac{\pi}{4}, 1.9635\right\} \end{aligned}$$

## 7.18

$$
\begin{aligned}
x(n) &= \sum_{i=-\infty}^{\infty} \delta(n-iN) \\
y(n) &= \sum_{m} h(m)x(n-m) \\
&= \sum_{m} h(m)\left[\sum_{i} \delta(n-m-iN)\right] \\
&= \sum_{i} h(n-iN)
\end{aligned}
$$

Therefore, $y(.)$ is a periodic sequence with period N. So

$$
\begin{aligned}
Y(k) &= \sum_{n=0}^{N-1} y(n)W_N^{kn} \\
&= H(w)|_{w=\frac{2\pi}{N}k} \\
Y(k) &= H(\frac{2\pi k}{N}) \qquad k=0,1,\ldots,N-1
\end{aligned}
$$

## 7.22

$$
\begin{aligned}
x(n) &= \frac{1}{2}e^{j\frac{2\pi}{N}n}+\frac{1}{2}e^{-j\frac{2\pi}{N}n}, \qquad 0\le n\le N, \qquad N=10 \\
X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn} \\
&= \sum_{n=0}^{N-1} \frac{1}{2}e^{-j\frac{2\pi}{N}(k-1)n}+\sum_{n=0}^{N-1} \frac{1}{2}e^{-j\frac{2\pi}{N}(k+1)n} \\
&= 5\delta(k-1)+5\delta(k-9), \qquad 0\le k\le 9
\end{aligned}
$$

## 7.23

(a) $X(k)=\sum_{n=0}^{N-1}\delta(n)e^{-j\frac{2\pi}{N}kn}=1, \quad 0\le k\le N-1$

(b)

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} \delta(n-n_0)e^{-j\frac{2\pi}{N}kn} \\
&= e^{-j\frac{2\pi}{N}kn_0}, \qquad 0\le k\le N-1
\end{aligned}
$$

(c)

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}kn} \\
&= \sum_{n=0}^{N-1} (ae^{-j\frac{2\pi}{N}k})^n \\
&= \frac{1-a^N}{1-ae^{-j\frac{2\pi}{N}k}}
\end{aligned}
$$

(d)

$$
\begin{aligned}
X(k) &= \sum_{n=0}^{\frac{N}{2}-1} e^{-j\frac{2\pi}{N}kn} \\
&= \frac{1-e^{-j\frac{2\pi}{N}\frac{N}{2}k}}{1-e^{-j\frac{2\pi}{N}k}} \\
&= \frac{1-(-1)^k}{1-e^{-j\frac{2\pi}{N}k}}
\end{aligned}
$$

(e)

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}nk_0} e^{-j\frac{2\pi}{N}kn} \\ &= \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-k_0)n} \\ &= N\delta(k-k_0) \end{aligned}$$

(f)

$$\begin{aligned} x(n) &= \frac{1}{2}e^{j\frac{2\pi}{N}nk_0} + \frac{1}{2}e^{-j\frac{2\pi}{N}nk_0} \\ \text{From (e) we obtain } X(k) &= \frac{N}{2}\left[\delta(k-k_0) + \delta(k-N+k_0)\right] \end{aligned}$$

(g)

$$\begin{aligned} x(n) &= \frac{1}{2j}e^{j\frac{2\pi}{N}nk_0} - \frac{1}{2j}e^{-j\frac{2\pi}{N}nk_0} \\ \text{Hence } X(k) &= \frac{N}{2j}\left[\delta(k-k_0) - \delta(k-N+k_0)\right] \end{aligned}$$

(h)

$$\begin{aligned} X(k) &= \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} (\text{ assume N odd }) \\ &= 1 + e^{-j\frac{2\pi}{N}2k} + e^{-j\frac{2\pi}{N}4k} + \ldots + e^{-j\frac{2\pi}{N}(n-1)k} \\ &= \frac{1-(e^{-j\frac{2\pi}{N}2k})^{\frac{N+1}{2}}}{1-e^{-j\frac{2\pi}{N}2k}} \\ &= \frac{1-e^{-j\frac{2\pi}{N}k}}{1-e^{-j\frac{4\pi}{N}k}} \\ &= \frac{1}{1-e^{-j\frac{2\pi}{N}k}} \end{aligned}$$