

8.7

$$\begin{aligned} \text{Let } M &= \frac{N}{2}, \quad L = 2. \text{ Then} \\ F(0, q) &= \sum_{n=0}^{\frac{N}{2}-1} x(0, m) W_{\frac{N}{2}}^{mq} \\ F(1, q) &= \sum_{n=0}^{\frac{N}{2}-1} x(1, m) W_{\frac{N}{2}}^{mq} \end{aligned}$$

which are the same as $F_1(k)$ and $F_2(k)$ in (8.1.26)

$$\begin{aligned} G(0, q) &= F(0, q) = F_1(k) \\ G(1, q) &= W_N^q F(1, q) = F_2(k) W_N^k \\ X(0, q) = x(k) &= G(0, q) + G(1, q) W_2^0 \\ &= F_1(k) + F_2(k) W_N^k \\ X(1, q) = x(k) &= G(0, q) + G(1, q) W_2^1 \\ &= F_1(k) - F_2(k) W_N^k \end{aligned}$$

8.8

$$W_8 = \frac{1}{\sqrt{2}}(1 - j)$$

Refer to Fig.8.1.9. The first stage of butterflies produces (2, 2, 2, 2, 0, 0, 0, 0). The twiddle factor multiplications do not change this sequence. The next stage produces (4, 4, 0, 0, 0, 0, 0, 0) which again remains unchanged by the twiddle factors. The last stage produces (8, 0, 0, 0, 0, 0, 0, 0). The bit reversal to permute the sequence into proper order unscrambles only zeros so the result remains (8, 0, 0, 0, 0, 0, 0, 0).

8.9

See Fig. 8.1.13.

8.10

Using (8.1.45), (8.1.46), and (8.1.47) the fig 8.10-1 is derived:

8.11

Using DIT following fig 8.1.6:

$$\begin{aligned} \text{1}^{\text{st}} \text{ stage outputs} &: \left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \dots, \frac{1}{2} \right\} \\ \text{2}^{\text{nd}} \text{ stage outputs} &: \left\{ 1, \frac{1}{2}(1 + W_8^2), 0, \frac{1}{2}(1 - W_8^2), 1, \frac{1}{2}(1 + W_8^2), 0, \frac{1}{2}(1 - W_8^2) \right\} \end{aligned}$$

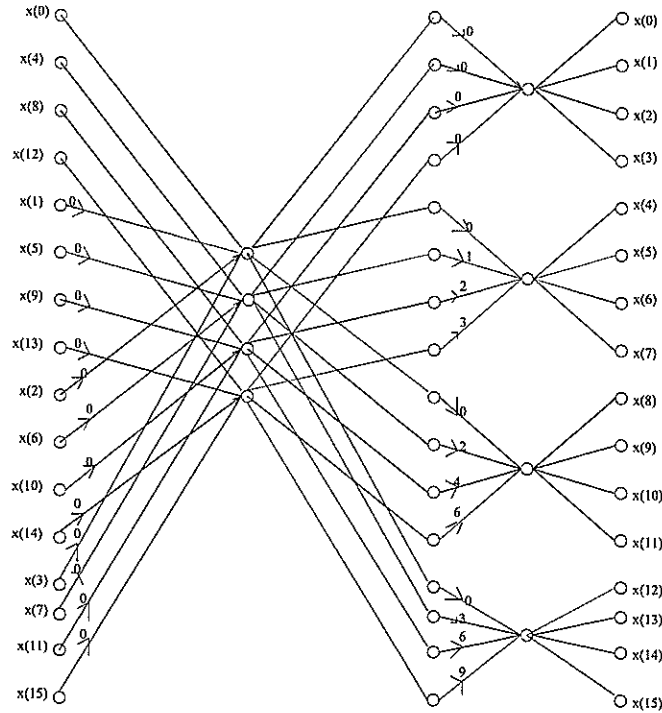


Figure 8.10-1:

$$3^{\text{rd}} \text{ stage outputs} : \left\{ 2, \frac{1}{2}(1 + W_8^1 + W_8^2 + W_8^3), 0, \frac{1}{2}(1 - W_8^2 + W_8^3 - W_8^5), 0, \right. \\ \left. \frac{1}{2}(1 - W_8^1 + W_8^2 - W_8^3), 0, \frac{1}{2}(1 - W_8^2 - W_8^3 + W_8^5) \right\}$$

Using DIF following fig 8.1.11:

$$1^{\text{st}} \text{ stage outputs} : \left\{ \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}W_8^1, \frac{1}{2}W_8^2, \frac{1}{2}W_8^3 \right\}$$

$$2^{\text{nd}} \text{ stage outputs} : \left\{ 1, 1, 0, 0, \frac{1}{2}(1 + W_8^2), 0, \frac{1}{2}(W_8^1 + W_8^3), \frac{1}{2}(1 - W_8^2), \frac{1}{2}(W_8^3 - W_8^5) \right\}$$

$$3^{\text{rd}} \text{ stage outputs} : \left\{ 2, 0, 0, 0, \frac{1}{2}(1 + W_8^1 + W_8^2 + W_8^3), \frac{1}{2}(1 - W_8^1 + W_8^2 - W_8^3), \right. \\ \left. \frac{1}{2}(1 - W_8^2 + W_8^3 - W_8^5), \frac{1}{2}(1 - W_8^2 - W_8^3 + W_8^5) \right\}$$