# Chapter 9

## 9.1

(a) $H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6}$. Refer to fig 9.1-1
(b) $H(z) = 1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 2z^{-4} + z^{-5}$. Refer to fig 9.1-2

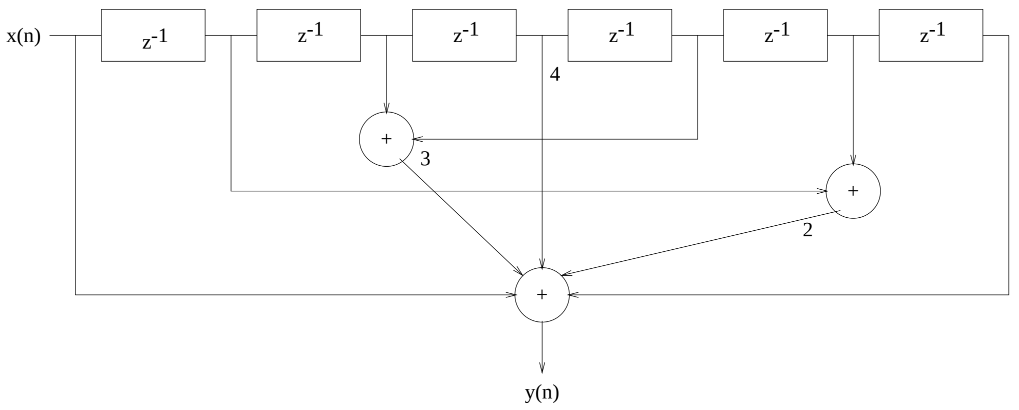


Figure 9.1-1:

## 9.2

Refer to fig 9.2-1

$$\begin{aligned}
A_4(z) = H(z) &= 1 + 2.88z^{-1} + 3.4048z^{-2} + 1.74z^{-3} + 0.4z^{-4} \\
B_4(z) &= 0.4 + 1.74z^{-1} + 3.4048z^{-2} + 2.88z^{-3} + z^{-4} \\
\text{Hence, } K_4 &= 0.4 \\
A_3(z) &= \frac{A_4(z) - k_4 B_4(z)}{1 - k_4^2} \\
&= 1 + 2.6z^{-1} + 2.432z^{-2} + 0.7z^{-3}
\end{aligned}$$

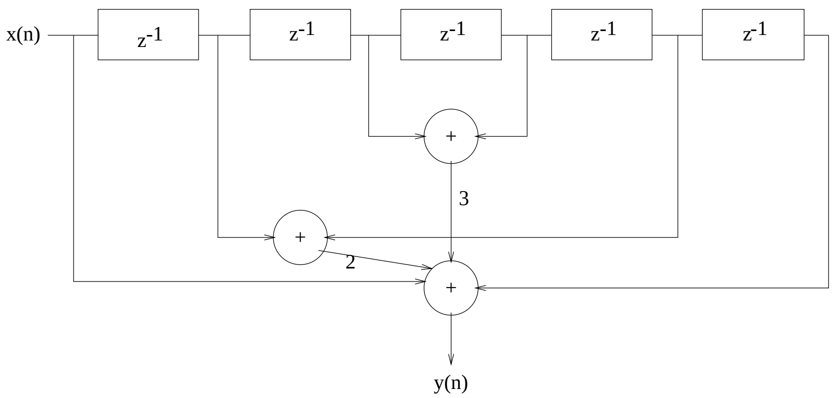


Figure 9.1-2:

$$
\begin{aligned}
B_3(z) &= 0.7 + 2.432z^{-1} + 2.6z^{-2} + z^{-3} \\
\text{Hence, } K_3 &= 0.7 \\
A_2(z) &= \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2} \\
&= 1 + 1.76z^{-1} + 1.2z^{-2} \\
B_2(z) &= 1.2 + 1.76z^{-1} + z^{-2} \\
\text{Then, } K_2 &= 1.2 \\
A_1(z) &= \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2} \\
&= 1 + 0.8z^{-1} \\
\text{Therefore, } K_1 &= 0.8
\end{aligned}
$$

Since $K_2 > 1$, the system is not minimum phase.

## 9.3

$$
\begin{aligned}
V(z) &= X(z) + \frac{1}{2}z^{-1}V(z) \\
v(n) &= x(n) + \frac{1}{2}v(n-1) \\
Y(z) &= 2[3X(z) + V(z)] + 2z^{-1}V(z) \\
H(z) &= \frac{Y(z)}{X(z)} \\
&= \frac{8 - z^{-1}}{1 - 0.5z^{-1}} \\
h(n) &= 8(0.5)^n u(n) - (0.5)^{n-1}u(n-1)
\end{aligned}
$$

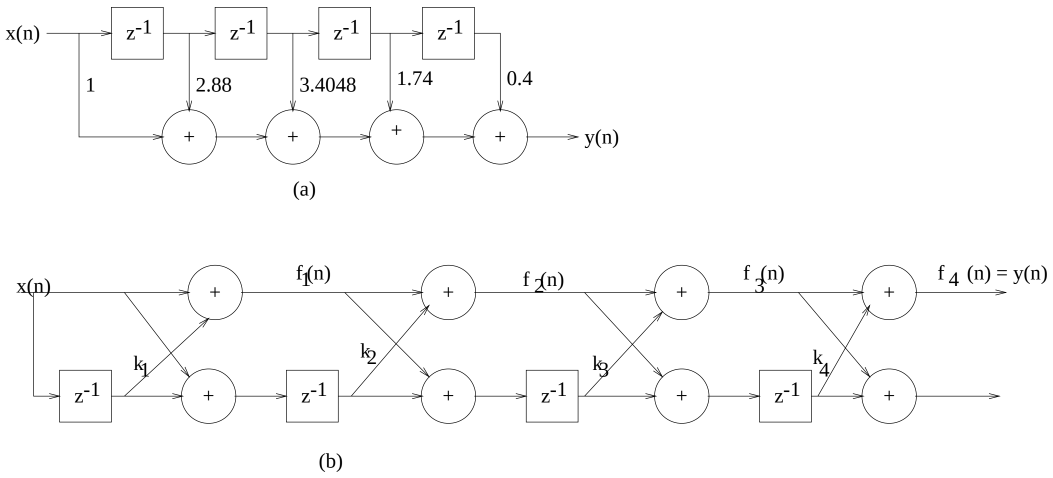


Figure 9.2-1: (a) Direct form. (b) Lattice form

## 9.4

$$\begin{aligned} H(z) &= 5 + \frac{3z^{1}}{1+\frac{1}{3}z^{-1}} + \frac{1+2z^{1}}{1-\frac{1}{2}z^{-1}} \\ h(n) &= 5\delta(n) + 3(-\frac{1}{3})^{n-1}u(n-1) + (\frac{1}{2})^{n}u(n) + 2(\frac{1}{2})^{n-1}u(n-1) \end{aligned}$$

## 9.5

$$\begin{aligned} H(z) &= \frac{6+\frac{9}{2}z^{1}-\frac{5}{3}z^{-2}}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})} \\ &= \frac{6+\frac{9}{2}z^{1}-\frac{5}{3}z^{-2}}{1-\frac{1}{6}z^{1}-\frac{1}{6}z^{-2}} \end{aligned}$$

Refer to fig 9.5-1

## 9.6

$$\begin{aligned} \text{For the first system, } H(z) &= \frac{1}{1-b_1z^{-1}} + \frac{1}{1-b_2z^{-1}} \\ H(z) &= \frac{1-(b_1+b_2)z^{-1}}{(1-b_1z^{-1})(1-b_2z^{-1})} \\ \text{For the second system, } H(z) &= \frac{c_0+c_1z^{-1}}{(1-d_1z^{-1})(1-a_2z^{-1})} \\ \text{clearly, } c_0 &= 1 \end{aligned}$$

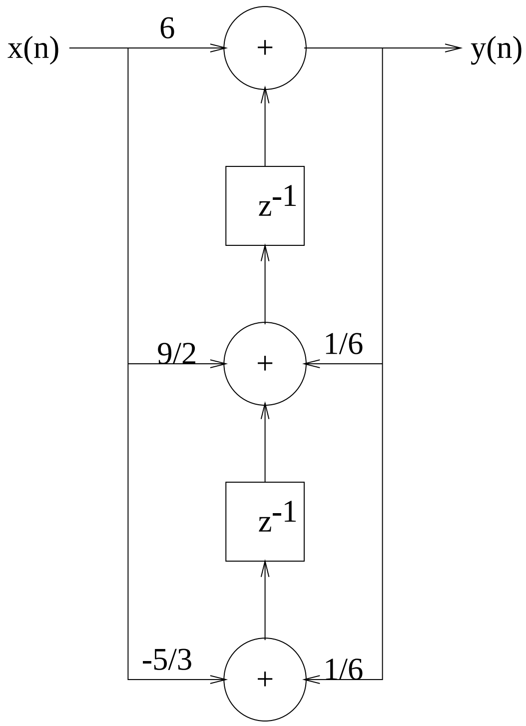


Figure 9.5-1:

$$
\begin{aligned}
c_1 &= -(b_1 + b_2) \\
d_1 &= b_1 \\
a_2 &= b_2
\end{aligned}
$$

## 9.7

(a)

$$
\begin{aligned}
y(n) &= a_1 y(n-1) + a_2 y(n-2) + b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) \\
H(z) &= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H(z) &= \frac{1 + 2z^{-1} + z^{-2}}{1 + 1.5z^{-1} + 0.9z^{-2}} \\
\text{Zeros at } z &= -1, -1 \\
\text{Poles at } z &= -0.75 \pm j0.58
\end{aligned}
$$

Since the poles are inside the unit circle, the system is stable.

$$
\begin{aligned}
H(z) &= \frac{1 + 2z^{-1} + z^{-2}}{1 + z^{-1} - 2z^{-2}} \\
\text{Zeros at } z &= -1, -1 \\
\text{Poles at } z &= 2, -1
\end{aligned}
$$

The system is unstable.
(c)

$$\begin{aligned}
x(n) &= cos(\frac{\pi}{3}n) \\
H(z) &= \frac{1}{1+z^{-1}-0.99z^{-2}} \\
H(w) &= \frac{1}{1+e^{-jw}-0.99e^{-j2w}} \\
H(\frac{\pi}{3}) &= 100e^{-j\frac{\pi}{3}} \\
\text{Hence, } y(n) &= 100cos(\frac{\pi}{3}n-\frac{\pi}{3})
\end{aligned}$$

# 9.8

$$\begin{aligned}
y(n) &= \frac{1}{4}y(n-2)+x(n) \\
H(z) &= \frac{1}{1-\frac{1}{4}z^{-2}}
\end{aligned}$$

(a)

$$\begin{aligned}
h(n) &= \frac{1}{2}\left[(\frac{1}{2})^n+(-\frac{1}{2})^n\right]u(n) \\
H(z) &= \frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}}+\frac{\frac{1}{2}}{1+\frac{1}{2}z^{-1}}
\end{aligned}$$

(b)

$$\begin{aligned}
x(n) &= \left[(\frac{1}{2})^n+(-\frac{1}{2})^n\right]u(n) \\
X(z) &= \frac{1}{1-\frac{1}{2}z^{-1}}+\frac{1}{1+\frac{1}{2}z^{-1}} \\
X(z) &= \frac{2}{1-\frac{1}{4}z^{-2}} \\
Y(z) &= X(z)H(z) \\
&= \frac{1}{1+\frac{1}{2}z^{-1}}+\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{-\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})^2}+\frac{\frac{1}{2}z^{-1}}{(1+\frac{1}{2}z^{-1})^2} \\
y(n) &= \left[(\frac{1}{2})^n+(-\frac{1}{2})^n-n(\frac{1}{2})^n+n(-\frac{1}{2})^n\right]u(n)
\end{aligned}$$

(c)Refer to fig 9.8-1
(d)

$$\begin{aligned}
H(w) &= \frac{1}{1-\frac{1}{4}e^{-j2w}} \\
&= \frac{4}{\sqrt{17-8cos2w}}\angle -tan^{-1}\frac{sin2w}{4-cos2w}
\end{aligned}$$

Refer to fig 9.8-2.

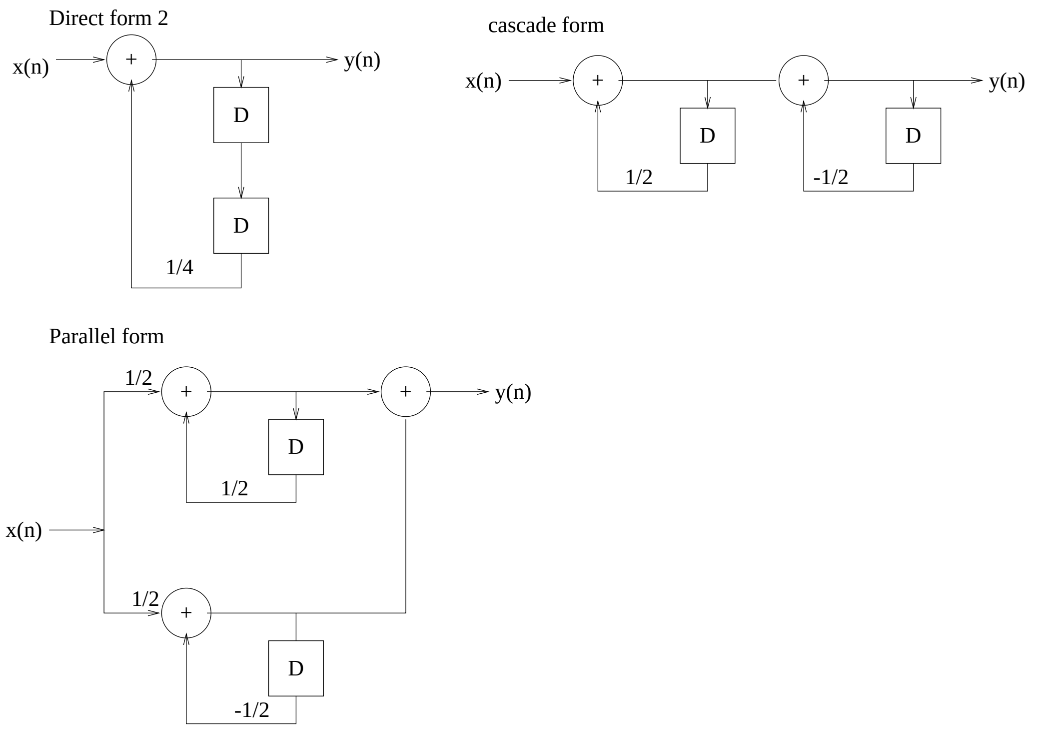


Figure 9.8-1:

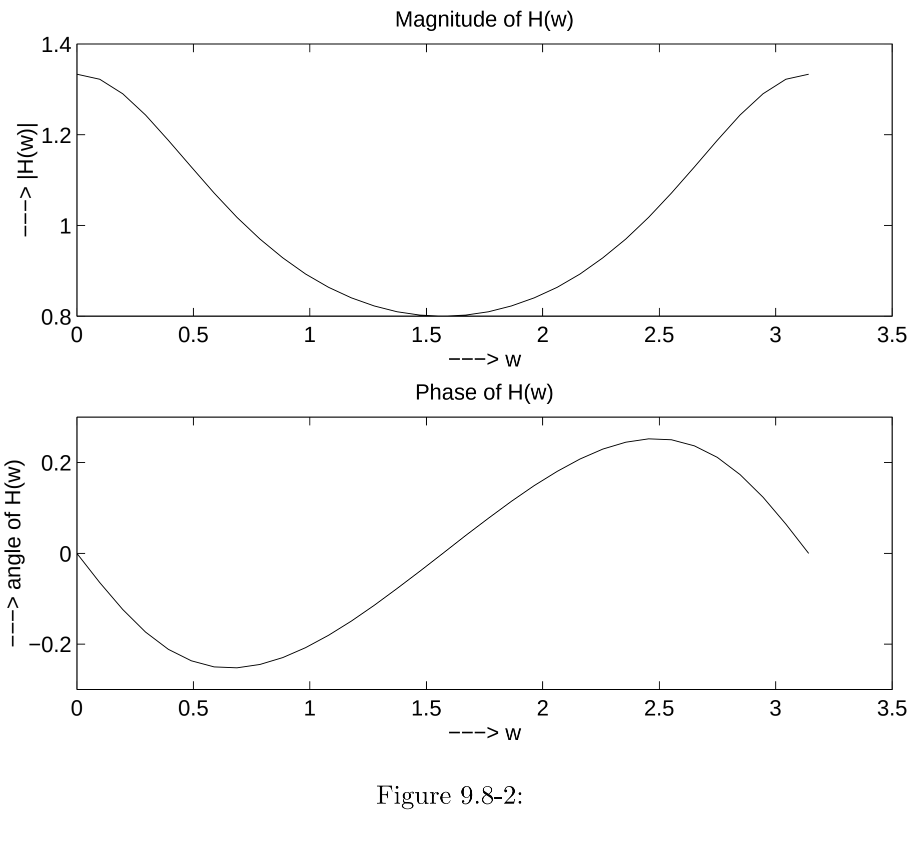


Figure 9.8-2:

## 9.9

(a)

$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$
$$= \frac{1+\frac{1}{3}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$
$$= \frac{\frac{10}{3}}{1-\frac{1}{2}z^{-1}} + \frac{-\frac{7}{3}}{1-\frac{1}{4}z^{-1}}$$

Refer to fig 9.9-1

(b)

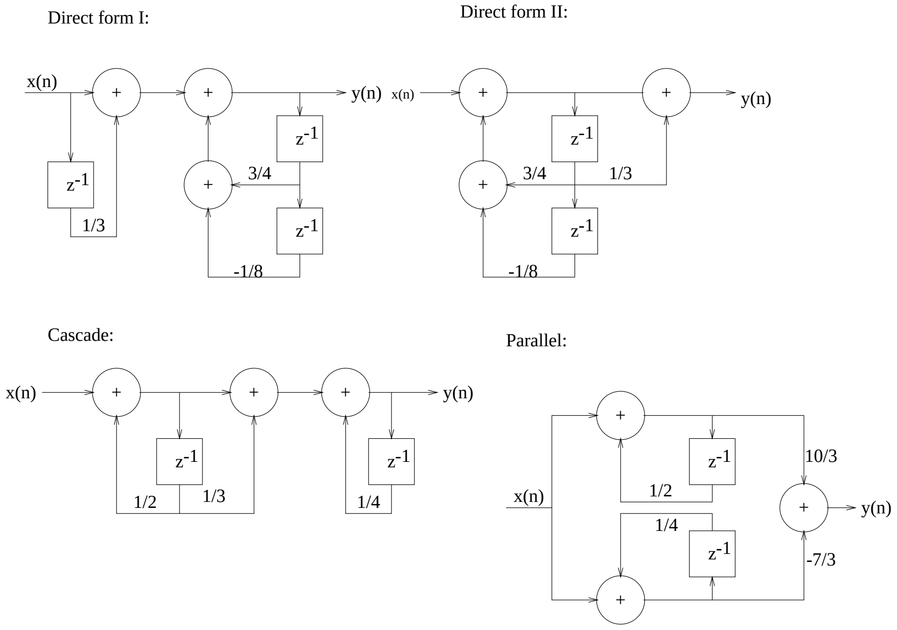


Figure 9.9-1:

$$H(z) = \frac{0.7(1-0.36z^{-2})}{1+0.1z^{-1}-0.72z^{-2}}$$
$$= \frac{0.7(1-0.6z^{-1})(1+0.6z^{-1})}{(1+0.9z^{-1})(1-0.8z^{-1})}$$
$$= 0.35 - \frac{0.1647}{1+0.9z^{-1}} - \frac{0.1853}{1-0.8z^{-1}}$$

Refer to fig 9.9-2

(c)

$$H(z) = \frac{3(1+1.2z^{-1}+0.2z^{-2})}{1+0.1z^{-1}-0.2z^{-2}}$$

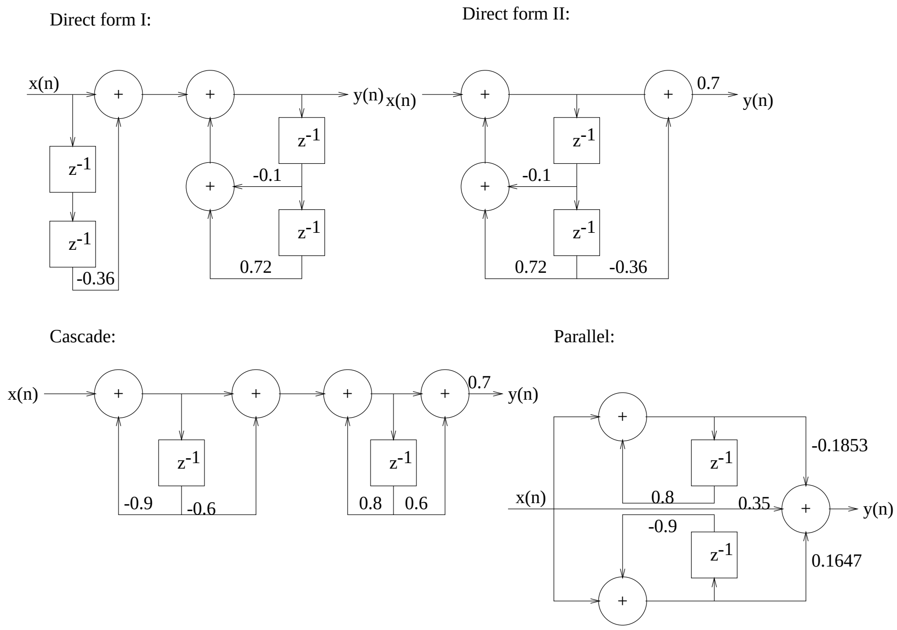


Figure 9.9-2:

$$
\begin{aligned}
&= \frac{3(1+0.2z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})} \\
&= -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}
\end{aligned}
$$

Refer to fig 9.9-3

(d)

$$
\begin{aligned}
H(z) &= \frac{2(1-z^{-1})(1+\sqrt{2}z^{-1}+z^{-2})}{(1+0.5z^{-1})(1-0.9z^{-1}+0.8z^{-2})} \\
&= \frac{2+(2\sqrt{2}-2)z^{-1}+(2-2\sqrt{2})z^{-2}-2z^{-3})}{1-0.4z^{-1}+0.36z^{-2}+0.405z^{-3}} \\
&= \frac{A}{1+0.5z^{-1}} + \frac{B+Cz^{-1}}{1-0.9z^{-1}+0.8z^{-1}}
\end{aligned}
$$

Refer to fig 9.9-4

(e)

$$
\begin{aligned}
H(z) &= \frac{1+z^{-1}}{1-\frac{1}{2}z^{-1}-\frac{1}{4}z^{-2}} \\
&= \frac{1+z^{-1}}{(1-0.81z^{-1})(1+0.31z^{-1})} \\
&= \frac{1.62}{1-0.81z^{-1}} + \frac{-0.62}{1+0.31z^{-1}}
\end{aligned}
$$

Refer to fig 9.9-5

(f) $H(z) = \frac{1-z^{-1}+z^{-2}}{1-z^{-1}+0.5z^{-2}} \Rightarrow$ Complex valued poles and zeros.Refer to fig 9.9-6 All the above

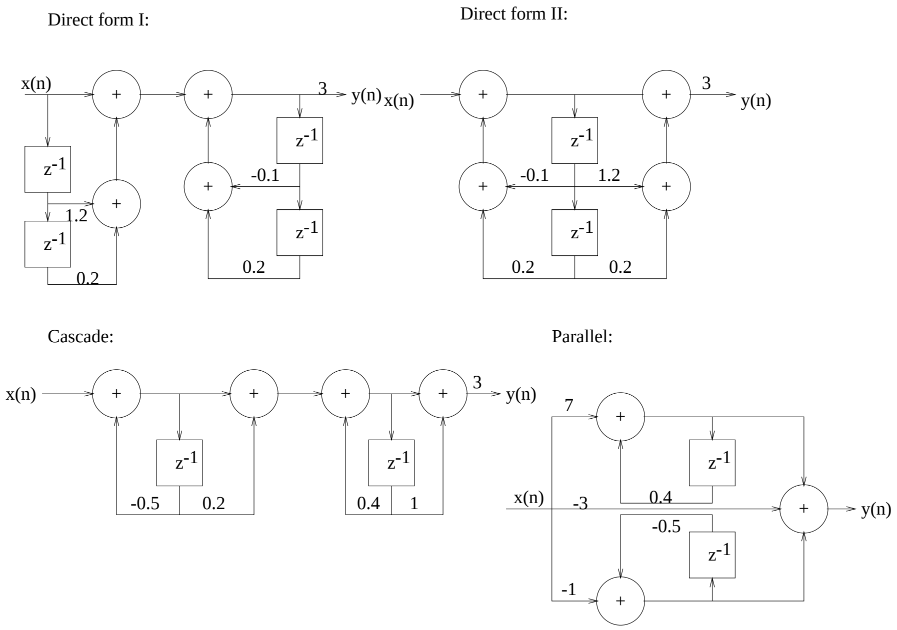


Figure 9.9-3:

systems are stable.

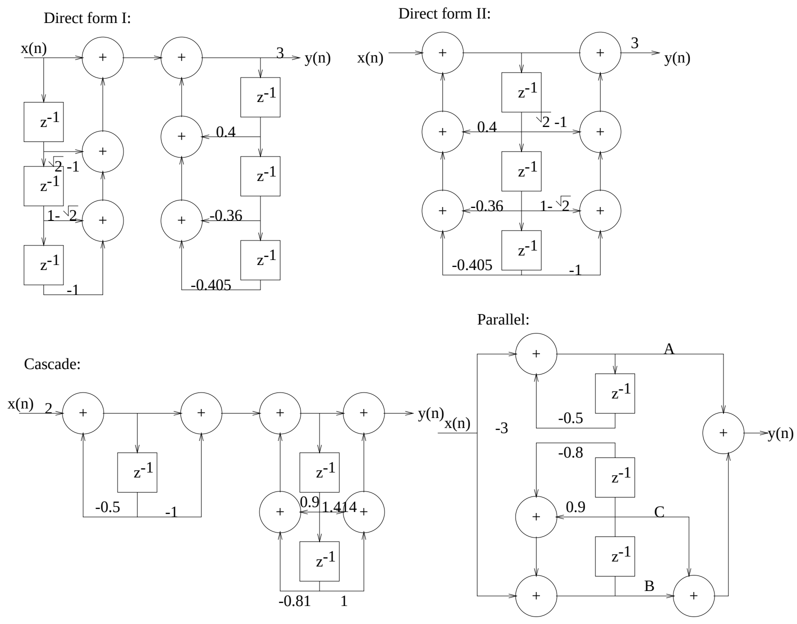


Figure 9.9-4:

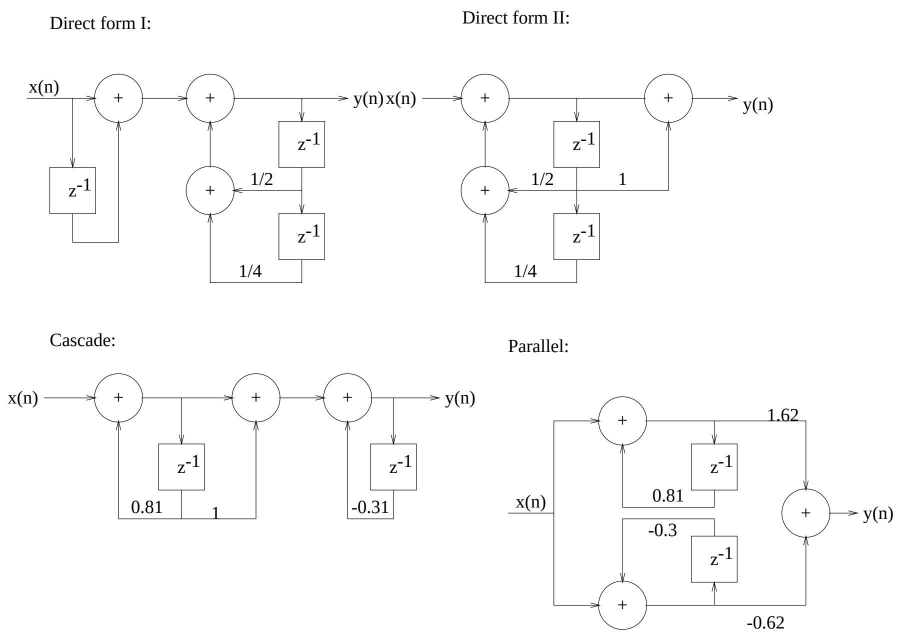


Figure 9.9-5:

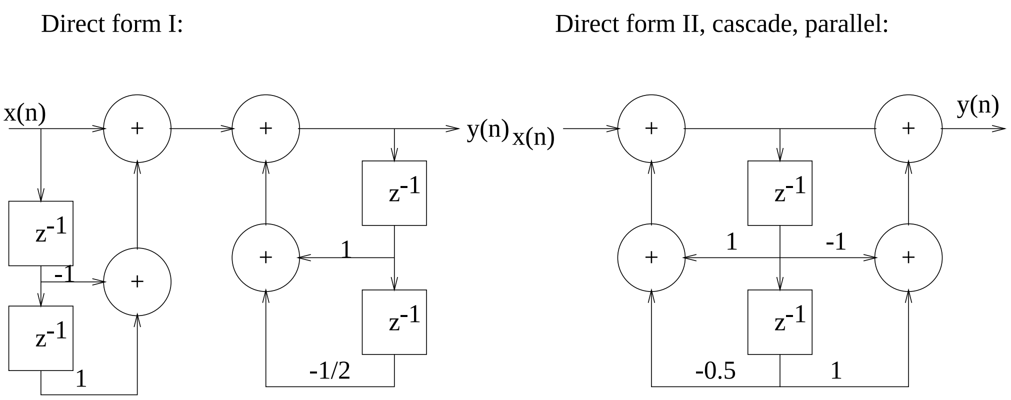
Direct form I:
Direct form II, cascade, parallel:
x(n)
y(n)
z-1
-1
1
-1/2
-0.5

Figure 9.9-6:

## 9.10

Refer to fig 9.10-1

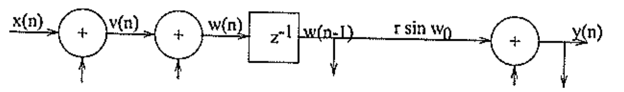


Figure 9.10-1:

$$
\begin{aligned}
H(z) &= \frac{1}{1 - 2rcosw_0z^{-1} + r^2z^{-2}} \\
(1) \quad V(z) &= X(z) - rsinw_0z^{-1}Y(z) \\
(2) \quad W(z) &= V(z) - rcosw_0z^{-1}W(z) \\
(3) \quad Y(z) &= rcosw_0z^{-1}Y(z) - rsinw_0z^{-1}W(z)
\end{aligned}
$$

By combining (1) and (2) we obtain

$$
(4) \quad W(z) = \frac{1}{1 - rcosw_0z^{-1}}X(z) - \frac{rsinw_0z^{-1}}{1 - rcosw_0z^{-1}}Y(z)
$$

Use (4) to eliminate $W(z)$ in (3). Thus,

$$
\begin{aligned}
Y(z)[(1 - rcosw_0z^{-1})^2 + r^2sin^2w_0z^{-2}] &= X(z) \\
Y(z)[1 - 2rcosw_0z^{-1} + (r^2cos^2w_0 + r^2sin^2w_0)z^{-2}] &= X(z) \\
\frac{Y(z)}{X(z)} &= \frac{1}{1 - 2rcosw_0z^{-1} + r^2z^{-2}}
\end{aligned}
$$

## 9.15

$$
\begin{aligned}
H(z) = A_2(z) &= 1 + 2z^{-1} + \frac{1}{3}z^{-2} \\
B_2(z) &= \frac{1}{3} + 2z^{-1} + z^{-2} \\
k_2 &= \frac{1}{3} \\
A_1(z) &= \frac{A_2(z) - k_2B_2(z)}{1 - k_2^2} \\
&= 1 + \frac{3}{2}z^{-1} \\
k_1 &= \frac{3}{2}
\end{aligned}
$$

## 9.16

(a)

$$
\begin{bmatrix} A_1(z) \\ B_1(z) \end{bmatrix} = \begin{bmatrix} 1 & k_1 \\ k_1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}z^{-1} \\ \frac{1}{2} + z^{-1} \end{bmatrix}
$$

$$
\begin{bmatrix} A_2(z) \\ B_2(z) \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{3} \\ -\frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} A_1(z) \\ z^{-1}B_1(z) \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{3}z^{-1} - \frac{1}{3}z^{-2} \\ -\frac{1}{3} + \frac{1}{3}z^{-1} + z^{-2} \end{bmatrix}
$$

$$\begin{bmatrix} A_3(z) \\ B_3(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A_2(z) \\ z^{-1}B_2(z) \end{bmatrix}$$

$$H_1(z) = A_3(z) = 1 + z^{-3} \Rightarrow$$

zeros at $z = -1, e^{\pm j\frac{\pi}{3}}$

(b)

$$\begin{aligned} H_2(z) &= A_2(z) - z^{-1}B_2(z) \\ &= 1 + \frac{2}{3}z^{-1} - \frac{2}{3}z^{-2} - z^{-3} \\ \text{The zeros are } z &= 1, \frac{-5 \pm j\sqrt{11}}{6} \end{aligned}$$

(c) If the magnitude of the last coefficient $|k_N| = 1$, i.e., $k_N = \pm 1$, all the zeros lie on the unit circle.

(d) Refer to fig 9.16-1. We observe that the filters are linear phase filters with phase jumps at

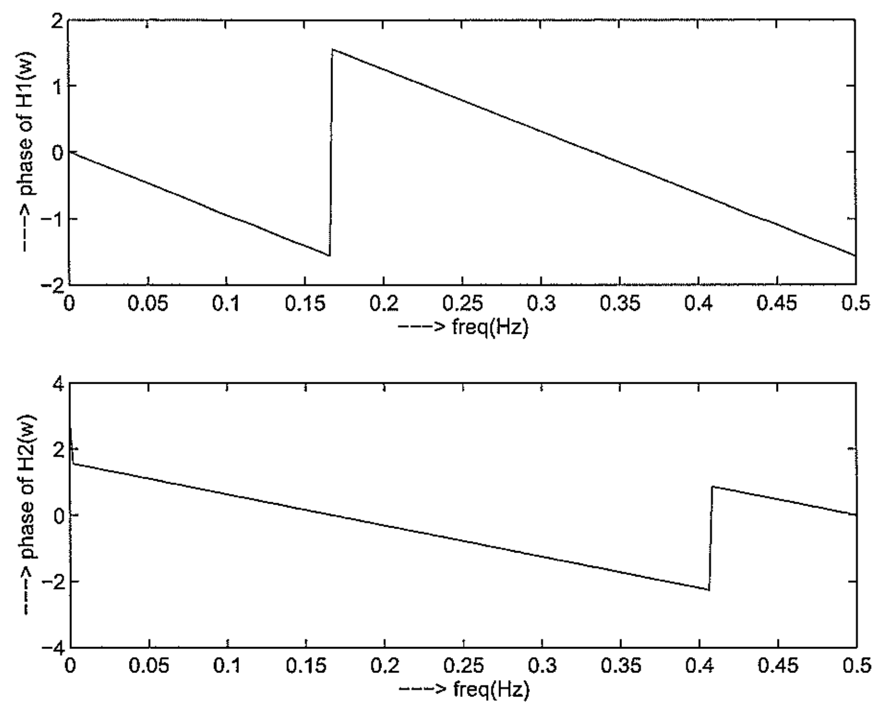


Figure 9.16-1:

the zeros of $H(z)$.

## 9.18

(a)

$$
\begin{aligned}
H(z) &= \frac{C_3(z)}{A_3(z)} \\
A_3(z) &= 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3} \\
B_3(z) &= 0.5 - 0.8z^{-1} + 0.9z^{-2} + z^{-3} \\
k_3 &= 0.5 \\
A_2(z) &= \frac{A_3(z) - k_3 B_3(z)}{1 - k_3^2} \\
&= 1 + 1.73z^{-1} - 1.67z^{-2} \\
B_2(z) &= -1.67 + 1.73z^{-1} + z^{-2} \\
k_2 &= -1.67 \\
A_1(z) &= \frac{A_2(z) - k_2 B_2(z)}{1 - k_2^2} \\
&= 1 + 1.62z^{-1} \\
B_1(z) &= 1.62 + z^{-1} \\
k_1 &= 1.62 \\
C_3(z) &= 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} \\
D_3(z) &= 2 + 3z^{-1} + 2z^{-2} + z^{-3} \\
k_3 &= 2 \\
C_2(z) &= \frac{C_3(z) - k_3 D_3(z)}{1 - k_3^2} \\
&= 1 + \frac{4}{3}z^{-1} + \frac{1}{3}z^{-2} \\
D_2(z) &= \frac{1}{3} + \frac{4}{3}z^{-1} + z^{-2} \\
k_2 &= \frac{1}{3} \\
C_1(z) &= \frac{C_2(z) - k_2 D_2(z)}{1 - k_2^2} \\
&= 1 + \frac{3}{4}z^{-1} \\
D_1(z) &= \frac{3}{4} + z^{-1} \\
k_1 &= \frac{3}{4} \\
C_3(z) &= v_0 + v_1 D_1(z) + v_2 D_2(z) + v_3 D_3(z) \\
&= 1 + 2z^{-1} + 3z^{-2} + 2z^{-3}
\end{aligned}
$$

From the equations, we obtain

$$
\begin{aligned}
v_0 &= -\frac{107}{48} \\
v_1 &= -\frac{13}{4} \\
v_2 &= -1 \\
v_3 &= 2
\end{aligned}
$$

The equivalent lattice-ladder structure is: Refer to fig 9.18-1

(b) $A_3(z) = 1 + 0.9z^{-1} - 0.8z^{-2} + 0.5z^{-3}$, $|k_1| > 1$ and $|k_2| > 1 \Rightarrow$ the system is unstable.

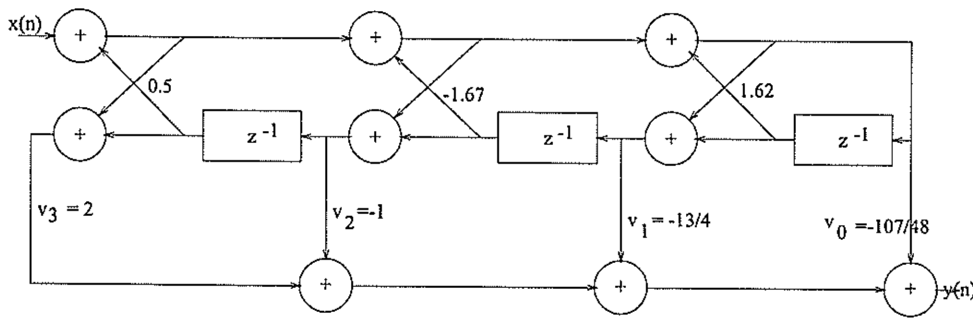


Figure 9.18-1:

## 9.35

$$y(n) = Q[0.1\delta(n)] + Q[\alpha y(n-1)]$$

(a)

$$\begin{aligned} y(n) &= Q[0.1\delta(n)] + Q[0.5y(n-1)] \\ y(0) &= Q[0.1] = \frac{1}{8} \end{aligned}$$

$$\begin{aligned} y(1) &= Q[\frac{1}{16}] = 0 \\ y(2) = y(3) = y(4) &= 0 \end{aligned}$$

no limit cycle

(b)

$$\begin{aligned} y(n) &= Q[0.1\delta(n)] + Q[0.75y(n-1)] \\ y(0) &= Q[0.1] = \frac{1}{8} \\ y(1) &= Q[\frac{3}{32}] = \frac{1}{8} \\ y(2) &= Q[\frac{3}{32}] = \frac{1}{8} \\ y(3) = y(4) &= \frac{1}{8} \end{aligned}$$

limit cycle occurs

## 9.38

(a)

$$h_1(n) = (\frac{1}{2})^n u(n)$$

$$\begin{aligned}
h_2(n) &= (\frac{1}{4})^n u(n) \\
h(n) &= [2(\frac{1}{2})^n - (\frac{1}{4})^n] u(n) \\
\sigma_q^2 &= 2\sigma_{e1}^2 \sum_{n=0}^{\infty} h_1^2(n) + 2\sigma_{e2}^2 \sum_{n=0}^{\infty} h_2^2(n) \\
&= \frac{64}{35}\sigma_{e1}^2 + \frac{16}{15}\sigma_{e2}^2
\end{aligned}$$

(b)

$$\begin{aligned}
\sigma_q^2 &= \sigma_{e1}^2 \sum_n h^2(n) + \sigma_{e2}^2 \sum_n h_1^2(n) \\
&= \frac{64}{35}\sigma_{e1}^2 + \frac{4}{3}\sigma_{e2}^2
\end{aligned}$$