A nonreturn-to-zero data stream (of amplitude levels ± 1) is passed through a low-pass filter whose impulse response is defined by the Gaussian function

$$h(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2 t^2}{\alpha^2}\right)$$

where α is a design parameter defined in terms of the filter's 3-dB bandwidth by

$$\alpha = \sqrt{\frac{\log 2}{2}} \, \frac{1}{W}$$

(a) Show that the transfer function of the filter is defined by

$$H(f) = \exp(-\alpha^2 f^2)$$

Hence demonstrate that the 3-dB bandwidth of the filter is indeed equal to W. You may use Table A6.3 on Fourier-transform pairs.

- (b) Show that the response of the filter to a rectangular pulse of unit amplitude and duration T centered on the origin is defined by Equation (6.135).
- P2 Plot the waveform of a GMSK modulator produced in response to the binary sequence 1101000, assuming the use of a gain-bandwidth product $WT_b = 0.3$. Compare your result with that of Example 6.5.
- P3 The binary sequence 1100100010 is applied to the DPSK transmitter of Figure 6.43a.
 - (a) Sketch the resulting waveform at the transmitter output.
 - (b) Applying this waveform to the DPSK receiver of Figure 6.43b, show that, in the absence of noise, the original binary sequence is reconstructed at the receiver output.
- The values of E_b/N_0 required to realize an average probability of symbol error $P_c = 10^{-4}$ using coherent binary PSK and coherent FSK (conventional) systems are equal to 7.2 and 13.5, respectively. Using the approximation

$$\operatorname{erfc}(u) = \frac{1}{\sqrt{\pi u}} \exp(-u^2)$$

determine the separation in the values of E_b/N_0 for $P_e = 10^{-4}$, using

- (a) Coherent binary PSK and DPSK.
- (b) Coherent binary PSK and QPSK.
- (c) Coherent binary FSK (conventional) and noncoherent binary FSK.
- (d) Coherent binary FSK (conventional) and coherent MSK.
- 75 The noise equivalent bandwidth of a bandpass signal is defined as the value of bandwidth that satisfies the relation

$$2BS(f_c) = P/2$$

where 2B is the noise equivalent bandwidth centered around the midband frequency f_c , $S(f_c)$ is the maximum value of the power spectral density of the signal at $f = f_c$, and P is the average power of the signal. Show that the noise equivalent bandwidths of binary PSK, QPSK, and MSK are as follows:

Type of Modulation	Noise Bandwidth/Bit Rate
Binary PSK	1.0
QPSK	0.5
MSK	0.62

Note: You may use the definite integrals in Table A6.10. A discussion of noise equivalent bandwidth is presented in Appendix 2.