ELEC6831/ELEC462

ASSIGNMENT 1

1. Let X be a uniform random variable in the interval $[-2, 2]$. Find and plot $P[|X| > x]$.

2. Let X be a Gaussian random variable with $m = 5$ and $\sigma^2 = 16$.
   (a) Find $P[X > 4]$, $P[X \geq 7]$, $P[6.72 < X < 10.16]$, $P[2 < X < 7]$, $P[6 \leq X \leq 8]$.
   (b) $P[X < a] = 0.8869$, find a.
   (c) $P[X > b] = 0.11131$, find b.
   (d) $P[13 < X \leq c] = 0.0123$, find c.

3. Show that the Q-function for the Gaussian random variable satisfies $Q(-x) = 1 - Q(x)$.

4. A binary transmission system transmits a signal X (-1 to send a "0" bit; $+1$ to send a "1" bit). The received signal is $Y = X + N$ where noise N has a zero-mean Gaussian distribution with variance σ^2. Assume that "0" bits are three times as likely as "1" bits.
   (a) Find the conditional pdf of Y given the input value: $f_Y(y \mid X = +1)$ and $f_Y(y \mid X = -1)$.
   (b) The receiver decides a "0" was transmitted if the observed value of y satisfies

$$f_Y(y \mid X = -1)P[X = -1] > f_Y(y \mid X = +1)P[X = +1]$$

   and it decides a "1" was transmitted otherwise. Use the results from part a to show that this decision rule is equivalent to: If $y < T$ decide "0"; if $y \geq T$ decide "1".
   (c) What is the probability that the receiver makes an error given that a $+1$ was transmitted? a -1 was transmitted? Assume $\sigma^2 = 1/16$.
   (d) What is the overall probability of error?

5. Let X and Y have joint pdf:

$$f_{X,Y}(x, y) = k(x + y) \qquad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

   (a) Find k.
   (b) Find the joint cdf of (X, Y).
   (c) Find the marginal pdf of X and of Y.
   (d) Find $P[X < Y]$, $P[Y < X^2]$, $P[X + Y > 0.5]$.

6. Let $X(t)$ be a zero-mean Gaussian random process with autocovariance function given by

$$C_X(t_1, t_2) = 4e^{-2|t_1 - t_2|}. \qquad C_X(t_1, t_2) = 4e^{-2|t_1 - t_2|}$$

   Find the joint pdf of $X(t)$ and $X(t + s)$.

7. Let $X(t)$ and $Y(t)$ be independent Gaussian random processes with zero means and the same covariance function $C(t_1, t_2)$. Define the "amplitude-modulated signal" by

$$Z(t) = X(t) \cos \omega t + Y(t) \sin \omega t.$$

   (a) Find the mean and autocovariance of $Z(t)$.
   (b) Find the pdf of $Z(t)$.

8. The input into a filter is zero-mean white noise with noise power density $N_0/2$. The filter has transfer function

$$H(f) = \frac{1}{1 + j2\pi f}.$$

   (a) Find $S_{Y,X}(f)$ and $R_{Y,X}(\tau)$.
   (b) Find $S_Y(f)$ and $R_Y(\tau)$.
   (c) What is the average power of the output?

9. Let $Y(t) = h(t) * X(t)$ and $Z(t) = X(t) - Y(t)$ as shown in Fig. P10.5.

(a) Find $S_Z(f)$ in terms of $S_X(f)$.

(b) Find $E[Z^2(t)]$.

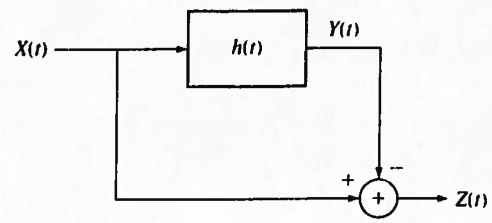

FIGURE P10.5

10. Let $Y(t)$ be the output of a linear system with impulse response $h(t)$ and input $X(t) + N(t)$. Let $Z(t) = X(t) - Y(t)$.

(a) Find $R_{X,Y}(\tau)$ and $R_Z(\tau)$.

(b) Find $S_Z(f)$.

(c) Find $S_Z(f)$ if $X(t)$ and $N(t)$ are independent random processes.