Assignment 4

P1: In the on-off keying version of an ASK system, symbol 1 is represented by transmitting a sinusoidal carrier of amplitude $\sqrt{2E_b/T_b}$, where E_b is the signal energy per bit and T_b is the bit duration. Symbol 0 is represented by switching off the carrier. Assume that symbols 1 and 0 occur with equal probability.

For an AWGN channel, determine the average probability of error for this ASK system under the following scenario:

Coherent reception.

P2- The binary sequence 1100100010 is applied to the DPSK transmitter

(a) Sketch the resulting waveform at the transmitter output.

(b) Applying this waveform to the DPSK receiver, show that, in the absence of noise, the original binary sequence is reconstructed at the receiver output.

P3: A PSK signal is applied to a correlator supplied with a phase reference that lies within φ radians of the exact carrier phase. Determine the effect of the phase error φ on the average probability of error of the system.

P4: The signal component of a coherent PSK system is defined by

$$s(t) = A_c k \sin(2\pi f_c t) \pm A_c \sqrt{1 - k^2} \cos(2\pi f_c t)$$

where $0 \leq t \leq T_b$, and the plus sign corresponds to symbol 1 and the minus sign corresponds to symbol 0. The first term represents a carrier component included for the purpose of synchronizing the receiver to the transmitter.

(a) Draw a signal-space diagram for the scheme described here; what observations can you make about this diagram?

(b) Show that, in the presence of additive white Gaussian noise of zero mean and power spectral density $N_0/2$, the average probability of error is

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}(1 - k^2)}\right)$$

where

$$E_b = \frac{1}{2} A_c^2 T_b$$

(c) Suppose that 10 percent of the transmitted signal power is allocated to the carrier component. Determine the E_b/N_0 required to realize a probability of error equal to 10^{-4}.

(d) Compare this value of E_b/N_0 with that required for a conventional PSK system with the same probability of error.

P5: — Determine the transmission bandwidth reduction and average signal energy of 256-QAM, compared to 64-QAM.

P6: —Two passband data transmission systems are to be compared. One system uses 16-PSK, and the other uses 16-QAM. Both systems are required to produce an average probability of symbol error equal to 10^{-3}. Compare the signal-to-noise ratio requirements of these two systems.

P7: An FSK system transmits binary data at the rate of 2.5×10^6 bits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} W/Hz is added to the signal. In the absence of noise, the amplitude of the received sinusoidal wave for digit 1 or 0 is 1 mV. Determine the average probability of symbol error for the following system configurations:

(a) Coherent binary FSK
(b) Coherent MSK
(c) Noncoherent binary FSK

P8: (a) In a coherent FSK system, the signals $s_1(t)$ and $s_2(t)$ representing symbols 1 and 0, respectively, are defined by

$$s_1(t),\ s_2(t) = A_c \cos\left[2\pi\left(f_c \pm \frac{\Delta f}{2}\right)t\right], \qquad 0 \le t \le T_b$$

Assuming that $f_c > \Delta f$, show that the correlation coefficient of the signals $s_1(t)$ and $s_2(t)$ is approximately given by

$$\rho = \frac{\int_0^{T_b} s_1(t)s_2(t)\,dt}{\int_0^{T_b} s_1^2(t)\,dt} \simeq \text{sinc}(2\Delta f T_b)$$

(b) What is the minimum value of frequency shift Δf for which the signals $s_1(t)$ and $s_2(t)$ are orthogonal?
(c) What is the value of Δf that minimizes the average probability of symbol error?
(d) For the value of Δf obtained in part (c), determine the increase in E_b/N_0 required so that this coherent FSK system has the same noise performance as a coherent binary PSK system.

P9: (a) Sketch the waveforms of the in-phase and quadrature components of the MSK signal in response to the input binary sequence 1100100010.
(b) Sketch the MSK waveform itself for the binary sequence specified in part (a).

P10 – The values of E_b/N_0 required to realize an average probability of symbol error $P_e = 10^{-4}$ using coherent binary PSK and coherent FSK (conventional) systems are given in the text. Using the approximation

$$Q(u) \simeq \frac{1}{\sqrt{2\pi}\,u} e^{-u^2/2}$$

determine the separation in the values of E_b/N_0 for $P_e = 10^{-4}$, using

(a) Coherent binary PSK and DPSK.
(b) Coherent binary PSK and QPSK.
(c) Coherent binary FSK (conventional) and noncoherent binary FSK.
(d) Coherent binary FSK (conventional) and coherent MSK.