Solution to Assignment 2

Problem: 1

(a) $g(t) = \text{sinc}(200t)$

This sinc pulse corresponds to a bandwidth W = 100 Hz. Hence, the Nyquist rate is 200 Hz, and the Nyquist interval is 1/200 seconds.

(b) $g(t) = \text{sinc}^2(200t)$

This signal may be viewed as the product of the sinc pulse sinc(200t) with itself. Since multiplication in the time domain corresponds to convolution in the frequency domain, we find that the signal g(t) has a bandwidth equal to twice that of the sinc pulse sin(200t), that is, 200 Hz. The Nyquist rate of g(t) is therefore 400 Hz, and the Nyquist interval is 1/400 seconds.

(c) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

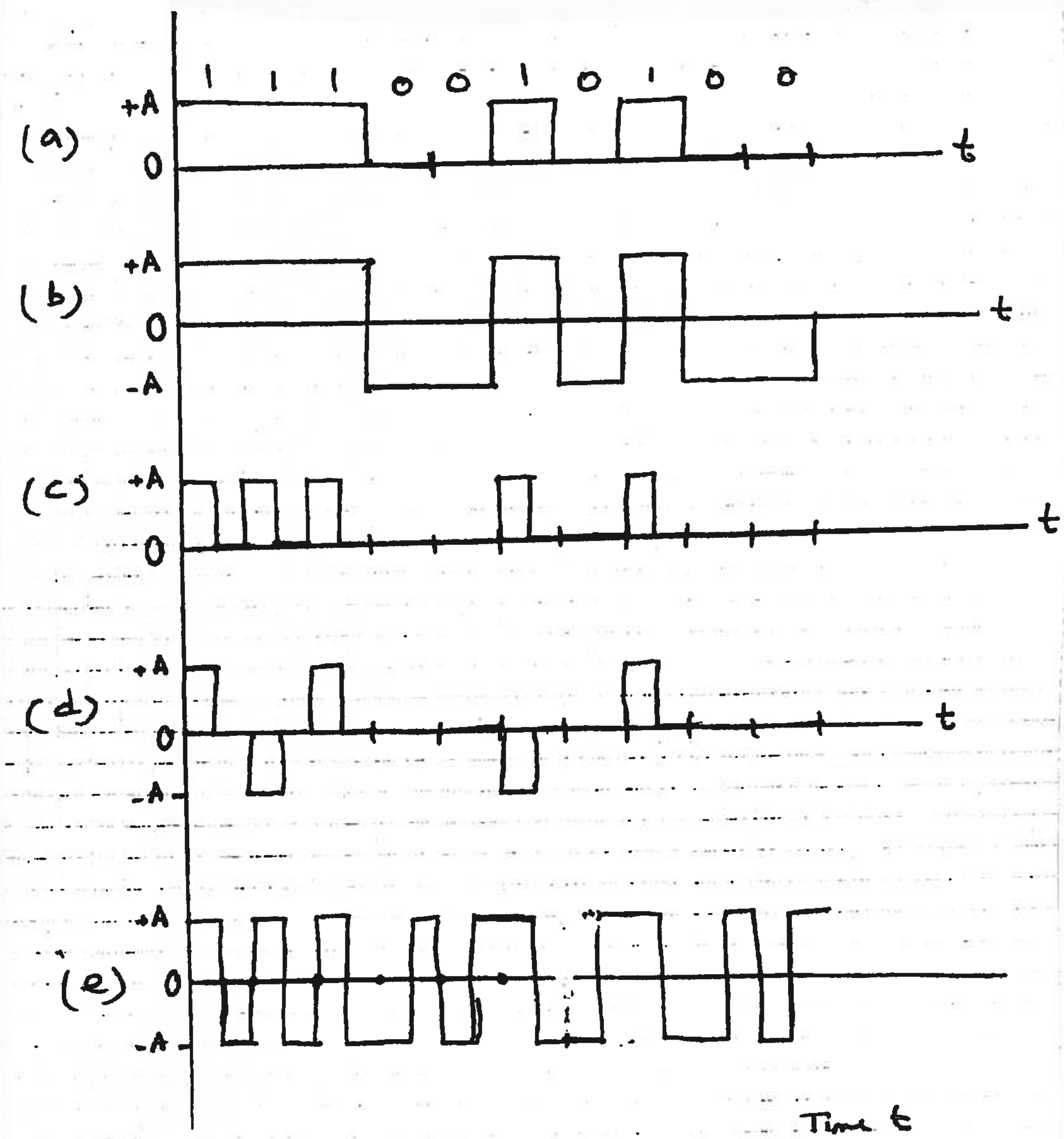
The bandwidth of g(t) is determined by the highest frequency component of sinc(200t) or $\text{sinc}^2(200t)$, whichever one is the largest. With the bandwidth (i.e., highest frequency component of) the sinc pulse sinc(200 t) equal to 100 Hz and that of the squared sinc pulse $\text{sinc}^2(200t)$ equal to 200 Hz, it follows that the bandwidth of g(t) is 200 Hz. Correspondingly, the Nyquist rate of g(t) is 400 Hz, and its Nyquist interval is 1/400 seconds.

Problem: 2

(a) The sampling interval is T_s = 125 μs. There are 24 channels and 1 sync pulse, so the time alloted to each channel is $T_c = T_s/25$ = 5 μs. The pulse duration is 1 μs, so the time between pulses is 4 μs.

(b) If sampled at the nyquist rate, 6.8 kHz, then T_s = 147 μs, T_c = 6.68 μs, and the time between pulses is 5.68 μs.

Problem 3.



Problem 4

Suppose that baseband signal m(t) is modeled as the sample function of a Gaussian random process of zero mean, and that the amplitude range of m(t) at the quantizer input extends from $-4A_{rms}$ to $4A_{rms}$. We find that samples of the signal m(t) will fall outside the amplitude range $8A_{rms}$ with a probability of overload that is less than 1 in 10^4. If we further assume the use of a binary code with each code word having a length n, so that the number of quantizing levels is 2", we find that the resulting quantizer step size is

$$\delta = \frac{8A_{rms}}{2^R} \quad (1)$$

Substituting Eq. (1) to the formula for the output signal-to-quantization noise ratio, we get

$$(SNR)_o = \frac{3}{16}(2^{2R}) \quad (2)$$

Expressing the signal-to-noise ratio in decibels:

$$10\log_0(SNR)_o = 6R - 7.2 \quad (3)$$

This formula states that each bit in the code word of a PCM system contributes 6dB to the signal-to-noise ratio. It gives a good description of the noise performance of a PCM system, provided that the following conditions are satisfied:

1. The system operates with an average signal power above the error threshold, so that the effect of transmission noise is made negligible, and performance is thereby limited essentially by quantizing noise alone.
2. The quantizing error is uniformly distributed.
3. The quantization is fine enough (say $R > 6$) to prevent signal-correlated patterns in the quantizing error waveform.
4. The quantizer is aligned with the amplitude range from $-4A_{rms}$ to $4A_{rms}$.

In general, conditions (1) through (3) are true of toll quality voice signals. However, when demands on voice quality are not severe, we may use a coarse quantizer corresponding to $R \leq 6$. In such a case, degradation in system performance is reflected not only by a lower signal-to-noise ratio, but also by an undesirable presence of signal-dependent patterns in the waveform of quantizing error.

Problem 5

(a) The probability p_1 of any binary symbol being inverted by transmission through the system is usually quite small, so that the probability of error after n regenerations in the system is very nearly equal to $n\ p_1$. For very large n, the probability of more than one inversion must be taken into account. Let p_n denote the probability that a binary symbol is in error after transmission through the complete system. Then, p_n is also the probability of an odd number of errors, since an even number of errors restores the original value. Counting zero as an even number, the probability of an even number of errors is $1-p_n$. Hence

$$p_{n+1} = p_n(1-p_1)+(1-p_n)p_1$$
$$= (1-2p_1)p_n+p_1$$

This is a linear difference equation of the first order. Its solution is

$$p_n = \frac{1}{2}\,[1-(1-2p_1)^n]$$

(b) If p_1 is very small and n is not too large, then

$$(1-2p_1)^n \approx 1-2p_1n$$

and

$$p_n \approx \frac{1}{2}[1-(1-2p_1n)]$$
$$= p_1n$$