Solution to Assignment 5

1) a)

0	0	0	0	0	0	0
1	1	1	0	0	0	1
0	1	1	0	0	1	0
1	0	0	0	0	1	1
1	1	0	0	1	0	0
0	0	1	0	1	0	1
1	0	1	0	1	1	0
0	1	0	0	1	1	1
1	0	1	1	0	0	0
0	1	0	1	0	0	1
1	1	0	1	0	1	0
0	0	1	1	0	1	1
0	1	1	1	1	0	0
1	0	0	1	1	0	1
0	0	0	1	1	1	0
1	1	1	1	1	1	1

b) The codewords can be partitioned into four sets:

I) {0000000},

II) {1110001, 1111000, 0111100, 0011110, 0001111, 1000111, 1100011},

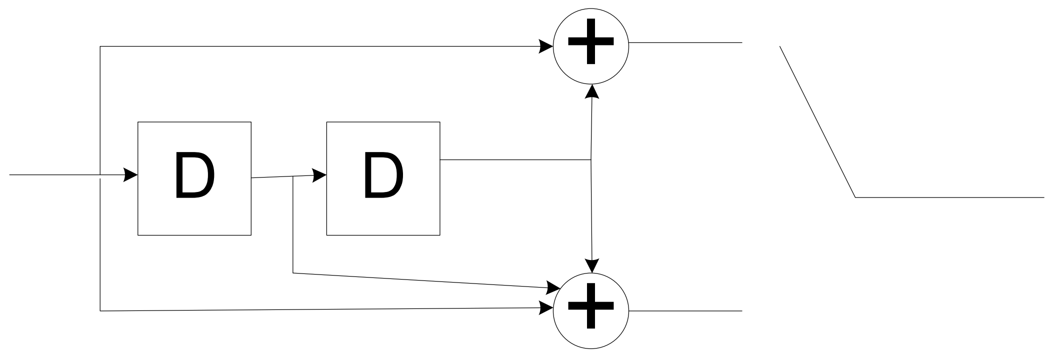
III) {0110010, 0011001, 1001100, 0100110, 0010011, 1001001, 1100100} and

IV) {1111111}.

Patterns in each set are generated by shifting one pattern in the set repeatedly.

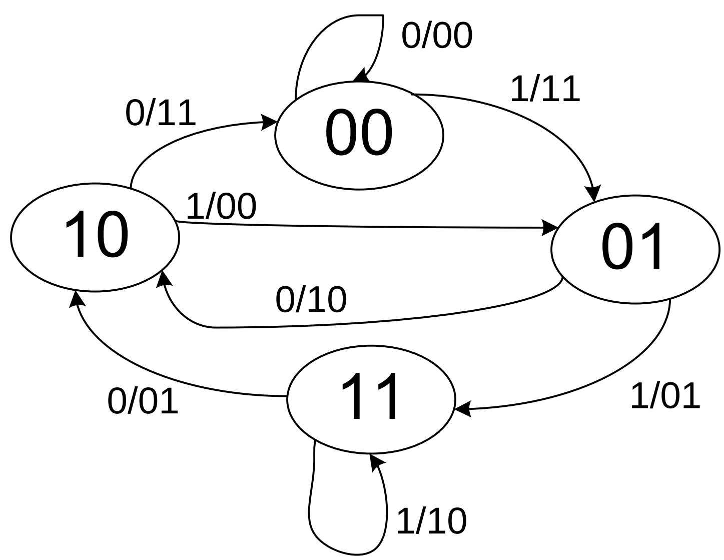
$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

2) a)



b)

Taking the output of the upper XOR as LSB and reading the output of the flip-flops from right to left (the LSB is the output of the left most flip-flop) as the state, we have:



c) 00,00,11,10,00,10,00.

3) $n = 2^6 - 1 = 63$.

$$\frac{n-k}{2} = 6 \quad \Rightarrow k = 51.$$

Length of a codeword is $6 * 63 = 378$ bits.

4) a) The fact that the code can be generated using a polynomial is an indication of it being cyclic. Take a codeword c(x)=a(x)g(x). Its cyclic shift would be c'(x)=xc(x)g(x) modulo ($x^8 + 1$).

That is, c'(x) is either equal to xc(x)g(x) or $xc(x)g(x)+x^8+1$ in both these cases c'(x) is divisible by g(x) and is therefore a codeword.

The code is not perfect since it has $2^{n-k}=2^{8-5}=2^3=8$ syndromes. It can correct only 7 error patterns (because one syndrome, the all 0 indicates no error). But there are 8 single errors. So, it cannot correct all single errors.

b) It does not have any error correcting capability. But it can detect a single error. You may check the minimum distance. It is two.

c) The probability of undetected error is the probability that we have more than one error in 8 bits. It is one minus the probability that we have one or zero errors.

$$P_u=1-(1-p)^8-8p(1-p)^7=0.068.$$

5)

a)

$E_b=\frac{P_r}{R}$. So, $\frac{E_b}{N_0}=\frac{P_r/N_0}{R}=\frac{10^{7.6}}{4\times10^6}=9.95$ or $\frac{E_b}{N_0}=9.98\ dB$

$$P_b\approx\frac{M}{2}Q\left(\sqrt{\frac{E_b}{N_0}\log_2 M}\right)=\frac{4}{2}Q\left(\sqrt{9.95\times2}\right)=1.9\times10^{-5}$$

Letting,

$$10^{-9}=\frac{M}{2}Q\left(\sqrt{\frac{E_b}{N_0}\log_2 M}\right)=\frac{4}{2}Q\left(\sqrt{\frac{E_b}{N_0}2}\right),$$

We get $\frac{E_b}{N_0}=19.1$ or in dB, $\frac{E_b}{N_0}=12.81\ dB$. So, the coding gain needed is 12.81-9.98=2.83 dB.

b) After coding for each code bit,

$$\frac{E_c}{N_0}=9.98\times\frac{106}{127}=8.33$$

So, probability of error before decoding is,

$$p = \frac{M}{2} Q\left(\sqrt{\frac{E_b}{N_0}\log_2 M}\right) = 2 \times Q\left(\sqrt{8.33 \times 2}\right) = 4.8 \times 10^{-5}.$$

The bit error rate after decoding can be approximated as,

$$P_b \approx \sum_{4}^{127} \frac{i+3}{127} p^i (1-p)^{127-i} \approx \frac{4+3}{127} p^4 (1-p)^{123} = 1.68 \times 10^{-19}.$$

c) The code is an over kill as it performs much better than expected. This can be considered as a margin for bad conditions or one can use a less strong (higher rate) code) or reduce the power.

6) $N = 2^5 - 1 = 31$. $\frac{N-K}{2} = t$, i.e., $\frac{31-K}{2} = 3$. So, K=25. The rate is $\frac{25}{31} = .806$ and the block length is $31 \times 5 = 155$.

7)

a) The constraint length is K=3.

b) $g_1(x) = 1 + x^2 + x^4$ and $g_2(x) = 1 + x + x^3 + x^4$.

c) …, 00, 00, 11, 01, 11, 00, 01, 10, 10, 11.

8)