Lecture 4

Interference and System capacity:

Two major types of interferences are

- Co-Channel interference (CCI)

and

- Adjacent Channel interferences (ACI).

CCI is the interference from other mobiles in other cells using the same frequency. Interference unlike noise, cannot be remedied by increasin power as increasing one's power increases CCI to other users.

CCI can mainly be reduced by spatially separating co-channel cells.

Take a cellular system with cells of radius R. Let the distance between co-chann cells be D. The ratio

$$Q = \frac{D}{R}$$ is called co-channel re-use ratio

The higher Q the better quality.

However, for hexagonal geometry:

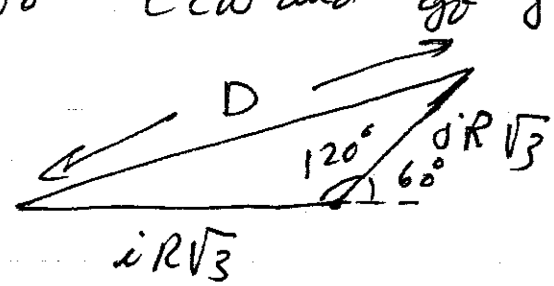
$$Q = \frac{D}{R} = \sqrt{3N}$$

So, larger Q requires larger N, i.e., worse frequency re-use

Proof of $\frac{D}{R} = \sqrt{3N}$

1) Go i steps of each $R\sqrt{3}$ (center to center)

2) turn 60° CCW and go j steps of $R\sqrt{3}$



$$D^2 = i^2R^2 \cdot 3 + j^2R^2 \cdot 3 - 2(iR\sqrt{3})(jR\sqrt{3})\cos 120^\circ$$

$$D^2 = 3R^2(i^2+j^2+ij) = 3R^2N$$

$$\Rightarrow Q = \frac{D}{R} = \sqrt{3N}$$

~~a+4+~~6

i, j	N	Q
1, 1	3	3
1, 2	7	4.58
2, 2	12	6
2, 3	19	7.55

Let the number of co-channel cells be i_0.

Then SIR is:

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{i_0} I_i}$$

$$P_r = P_0 \left(\frac{d}{d_0}\right)^{-n}$$

or

$$P_r = P_0 - 10n \log \frac{d}{d_0} \quad (\text{in dB})$$

where P_0 is the ~~cd~~ power received at a reference point at distance d_0 from transmitter.

Let the distance from ith interferer to be D_i then:

$$\frac{S}{I} = \frac{R^{-n}}{\sum_{i=1}^{i_0} D_i^{-n}}$$

assuming that all co-channel cells are at the same distance D, then

$$\frac{S}{I} = \frac{R^{-n}}{i_0 D^{-n}} = \frac{(D/R)^n}{i_0} = \frac{(\sqrt{3N})^n}{i_0}$$

Find SIR

Example: ~~Assume~~ in a system with $N=7$ frequency re-use. Assume $n=4$. ~~we required 18 dB of~~ ~~SIR~~ Taking the 6 closest co-channels, i.e., $i_0=6$, we get

$$\frac{S}{I} = \frac{\left(\sqrt{3\times 7}\right)^4}{6} = 75.3 = 18.66 \text{ dB}.$$

Adjacent Channel Interference (ACI)

ACI is the effect of channels in adjacent frequencies not being sufficiently filtered. In such cases, a transmitter closer to base station may be received wit ~~at~~ a higher power at the base station than the desired user. Assuming a path loss exponent of n, the received signal will be inversely proportional to d^n. So, if the desired user is at a distance d_0 and an interferer is at a distance d. The powers of the desired user, i.e., signal S and the interference by the undesired user, I are

$$S \propto \frac{P}{d_0^n} \quad \text{and} \quad I \propto \frac{P}{d^n}$$

given that they transmit with the same power. So, the sign-to-interference ratio would be

$$\frac{S}{I} = \left(\frac{d}{d_0}\right)^n$$

Now, assume that the interferer is 20 times closer to the base station and $n = 4$. Then,

$$\frac{S}{I} = \left(\frac{1}{20}\right)^4 \rightarrow \frac{S}{I} = -52 \text{ dB}.$$

If the filter limiting each mobiles transmission band has a slope of 20 dB/octave, one needs 6 channels separation between two frequencies to make desired and un-desired users to have the same power at the receiver, i.e., to mak SIR equal to 0 dB.

Trunking and Grade of Services

Trunking allows a large number of users with a limited number of channels. In a trunked system, a limited number of channels are pooled and given to subscribers per demand. After termination of the call the channel returns to the pool and is available for use by the ~~user~~ subscribers in the group (cell in the cellular system).

Grade of Service (GOS) is a measure of subscribers to get a channel in the busiest hours. GOS is usually given as the probability that a call is blocked (blocking probability) or probability that a call waits for service more that a pre-define queuing time.

Following parameters are used in traffic calculation:

Request rate (or arrival rate): The average number of call requests per unit of time (hour, min. or sec. It is denoted by λ and has unit of sec^{-1} or min^{-1} etc.

Holding time: The average ~~rate~~ duration of a call. It is denoted as H. Alternatively, one may define departure rate $\mu = \frac{1}{H}$, which is the rate of calls being terminated.

Traffic intensity: ~~...~~ The average channel occupancy. It is a dimensionless quantity and is given in Erlangs. An Erlang is a single channel being utilized all the time.

GOS: Blocking probability, i.e., probability that a call is being blocked (Erlang B) or a call being delayed beyond a certain amount of time (Erlang C).

Assume that we have U users each with ~~an~~ a request rate of λ and holding time H. Traffic intensity per subscriber is,

$$A_u = \lambda H = \frac{\lambda}{\mu}$$

The total offered ~~load~~ traffic is:

$$A = U A_u = U \lambda H$$

If there are C channels then traffice intensity per channel is,

$$A_c = U A_u / C$$

When ~~there~~ a call arrives, if there is a channel available, the call is served, else either the call is rejected (blocked) or queued waiting for a channel to become available.

$$GOS = P[\text{blocking}] = \frac{A^C / C!}{\sum_{k=0}^{C} \frac{A^k}{k!}}$$

This is called Erlang B formula.

(proof of this formal can be found in Appendix)

The arrival is according to Poisson distribution.

The probababilty that number of calls arriving in τ seconds is n:

$$P_r(n) = \frac{e^{-\lambda\tau}}{n!} (\lambda\tau)^n \quad n = 0, 1, 2, \ldots$$

So, Probability of 0, 1, 2, ... is

$$P(0) = e^{-\lambda\tau}, \quad P(1) = \lambda\tau e^{-\lambda\tau}, \quad P(2) = \frac{e^{-\lambda\tau}}{2} (\lambda\tau)^2$$

etc.

Let $e^{-\lambda\tau} = 1 - \lambda\tau + \frac{(\lambda\tau)^2}{2} - \frac{(\lambda\tau)^3}{6} + \cdots$, we get

$P(0) = 1 - \lambda\tau$ + terms of degree 2 and higher

$P(1) = \lambda\tau$ + terms of degree 2 and higher

$P(2)$ = terms of degree 2 and higher.

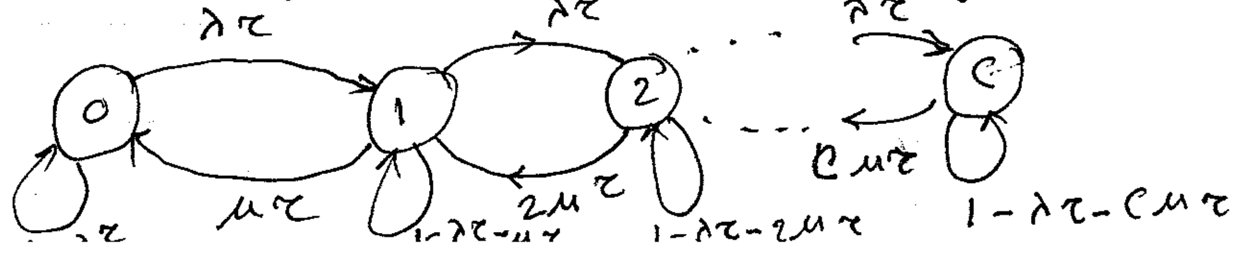
If τ is very small, only $P(0)$ and $P(1)$ are non-negligible, i.e.,

$P(0) \simeq 1 - \lambda\tau$

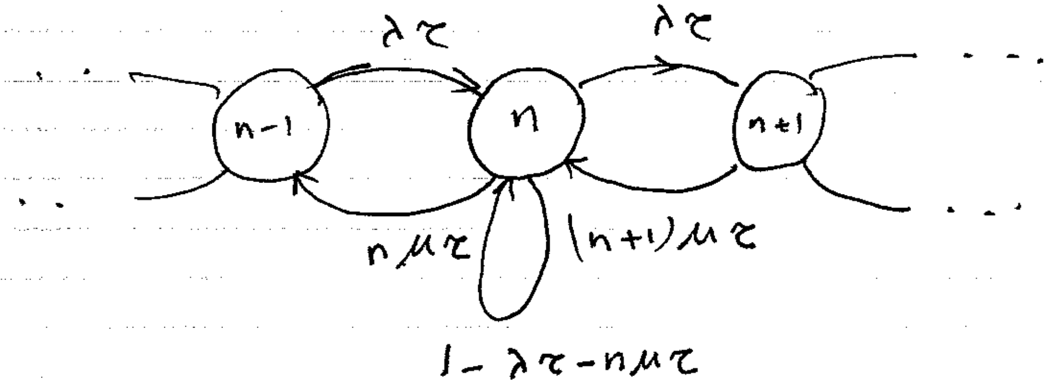
$P(1) \simeq \lambda\tau$

$P(2) = P(3) = \cdots = 0$

For the departure, the departure rate for calls in the system is $\mu = \frac{1}{H}$ per each call in the system. So, when there are n calls in the system, a call leaves the system with probability $n\mu\tau$ and no call leaves with probability $1 - n\mu\tau$. Probability of more than one departure is negligible if τ is small. The behaviour of the system is shown by the following state diagram:



Now Consider state $n < C$



In order for the system to be in balance, we need that

$$P(n) = \lambda\tau P(n-1) + (n+1)\mu\tau P(n+1)$$
$$+ (1 - \lambda\tau - n\mu\tau) P(n)$$

or

$$\lambda\tau P(n) - (n+1)\mu\tau P(n+1) = \lambda\tau P(n-1) - n\mu\tau P(n$$

or

$$\lambda P(n) - (n+1)\mu P(n+1) = \lambda P(n-1) - n\mu P(n)$$

So, $\lambda P(n) - (n+1)\mu P(n+1)$ should be constant, i.e., independent of n. But for $n=0$, we have

$$P(0) = P(0)(1-\lambda\tau) + P(1)\mu\tau$$

or $\lambda P(0) - \mu P(1) = 0$.

So, $\lambda P(n) - (n+1)\mu P(n+1) = 0 \quad \text{all } n \leq c$

or $$\frac{P(n+1)}{P(n)} = \frac{\lambda}{(n+1)\mu}$$

So:

$$P(1) = \frac{\lambda}{\mu} P(0)$$

$$P(2) = \frac{\lambda}{2\mu} P(1) = \frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2 P(0)$$

$$P(3) = \frac{\lambda}{3 \times 2\mu} P(2) = \frac{1}{3!}\left(\frac{\lambda}{\mu}\right)^3 P(0)$$

$$\vdots$$

$$P(n) = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P(0)$$

$$\vdots$$

$$P(c) = \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c P(0)$$

But
$$\sum_{i=0}^{c} P(i) = 1$$

So,
$$\sum_{n=0}^{c} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P(0) = 1$$

So, $$P(0) = \frac{1}{\sum_{n=0}^{c} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n}$$

Table 3.4 Capacity of an Erlang B System

Number of Channels C	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

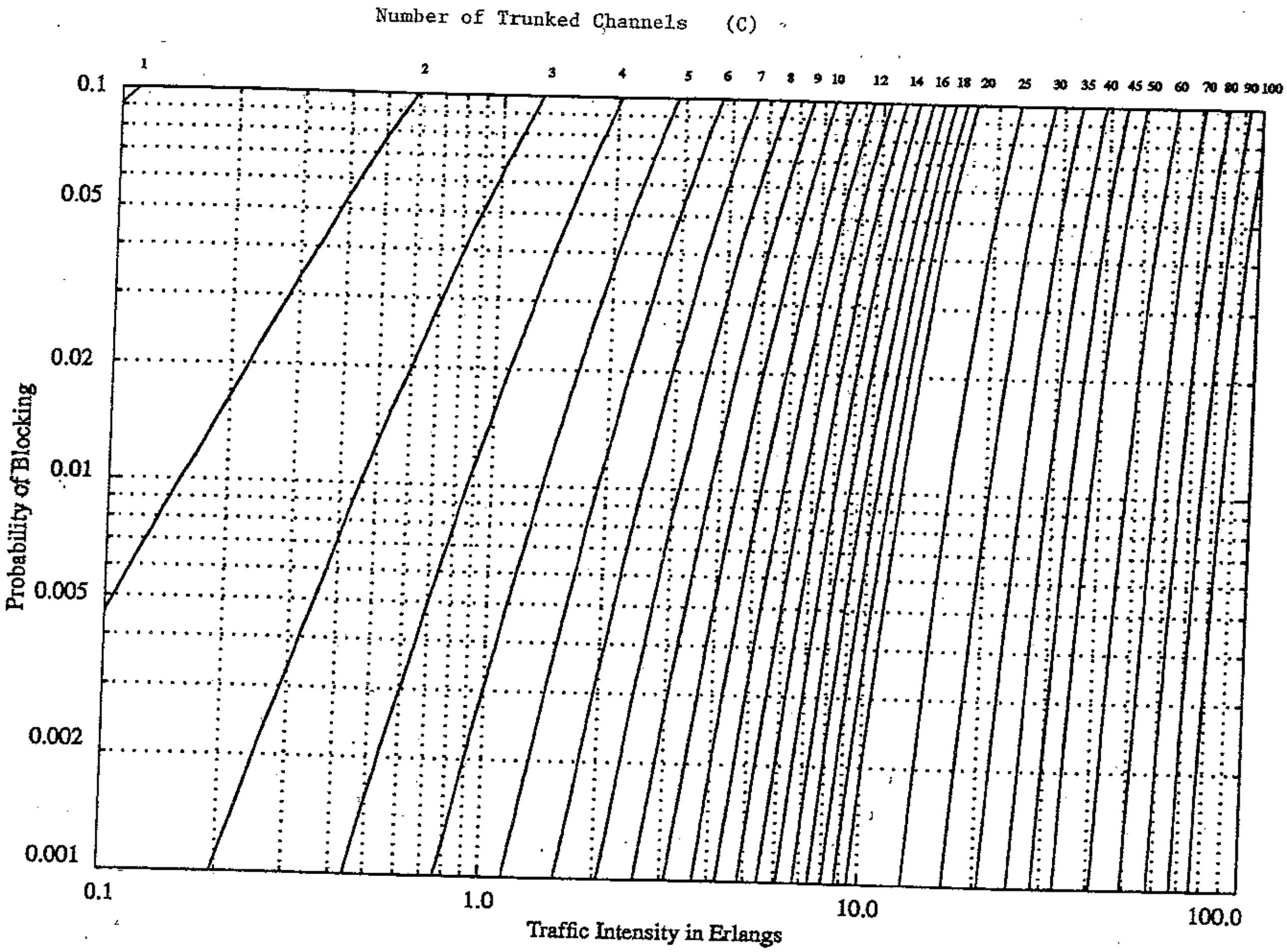


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

$$P(\text{blocking}) = P(C) = \frac{\left(\frac{\lambda}{\mu}\right)^C \frac{1}{C!}}{\sum_{n=0}^{C} \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}}$$

But $A = \lambda H = \frac{\lambda}{\mu}$

So:

$$P(\text{blocking}) = \frac{A^C \frac{1}{C!}}{\sum_{n=0}^{C} A^n \frac{1}{n!}}$$

Example: A Cellular System ~~uses~~ has 490 voice channels and uses 7-cell re-use. What is the total number of ~~cells that~~ subscribers that it can support if the area of the city is 10,000 km^2, Cell radius is 3 km. and a blocking probability of 1% is required. Assume that each user generates 1 call per hour and each call last 3 minutes on the average.

Solutions: Number of channels per cell is

$$\frac{490}{7} = 70$$

for GOS of 1% and $C = 70$, we have $A = 56.1$

$$A_u = \lambda H = 1 \times \frac{3}{60} = \frac{1}{20} = 0.05 \text{ Erlangs}$$

$$U = \frac{A}{A_u} = \frac{56.1}{0.05} = 1122 \quad \text{users / cell}$$

Area of a cell $= 2.5981(3)^2 = 23.38\ km^2$

$$\# \text{ of cells} = \frac{10{,}000}{23.38} \simeq 427$$

Total System Capacity $= 427 * 1122 = 479{,}094$

In order not to block calls ~~too~~ often, one can queue calls and wait for a channel to be free.

For a system like this, the probability that a call is not accepted in the first instance is

$$P[\text{delay} > 0] = \frac{A^C}{A^C + C!(1-\frac{A}{C})\sum_{k=0}^{C-1}\frac{A^k}{k!}}$$

This is called Erlang C formula.

For a given delay tolerance t, we have

$$P[\text{delay} > t] = P[\text{delay} > 0]\, P[\text{delay} > t \mid \text{delay} > 0]$$

$$= P[\text{delay} > 0] \exp[-\frac{(C-A)t}{H}]$$

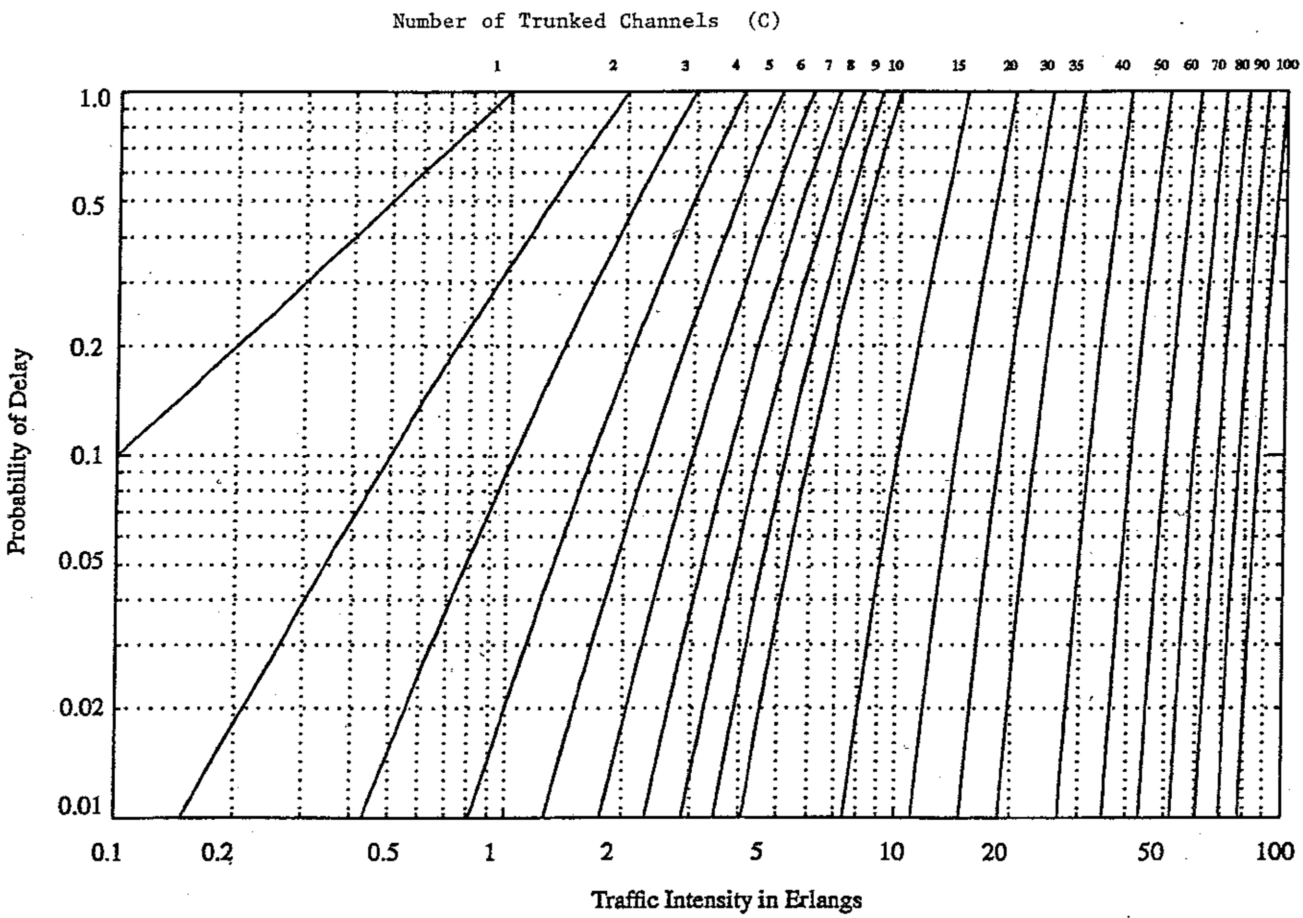


Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

and the average delay is

$$D = P[\text{delay} > 0] \frac{H}{C-A}.$$

Example: A cellular System is assigned 700 voice channels and uses 7-cell reuse. The calls that cannot be served immediately are queued and dropped (blocked) only if the delay is more than 10 seconds. The System has 1 million customers in a coverage area of 2000 km^2 and the cell radius is 1.5 km^2. If the load per user is 0.0275 and $\lambda = 2^r$ calls/Hr. Find

a) The average number of subscribers per cluster.

b) The probabability that a call is delayed.

c) The probability that a call is blocked.

a) the number of cells $= \dfrac{2000}{2.5981 * (1.5)^2} \cong 342$

U = Customers/Cell $= \dfrac{10^6}{342} \simeq$ ~~2924~~ 2924

2924*7

b)

$A = 2924 * 0.0275 = 80.4$

$C = \frac{700}{7} = 100$

For $C=100$ and $A=80.4$ we get,

$$P[\text{delay}>0]=0.02$$

c) $A_u = \lambda H$

$$0.0275 = 2H \Rightarrow H = 0.01375 \text{ hr.} = 49.5 \text{ sec}$$

$$P[\text{delay}>10 \text{ sec.}] = 0.02 * e^{-(100-80.4)\frac{10}{49.5}}$$

$$= 0.00038 = 0.038\% \simeq 3.8\times10^{-3}$$