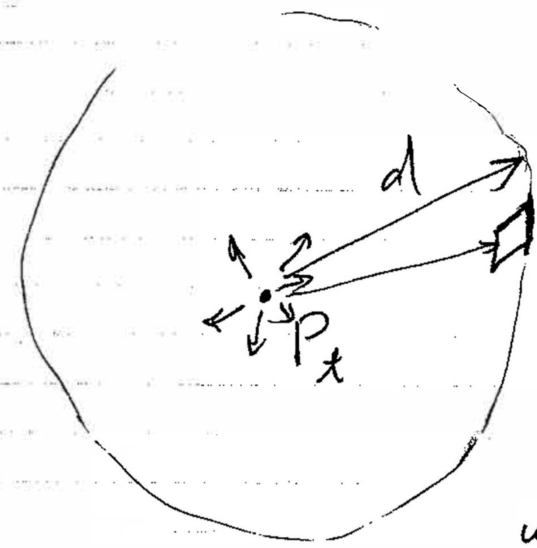
Mobile Radio propagation:

Large Scale path loss.

Line of Sight (LOS): Free Space Model

Take a transmitter with power P_t with an omnidirect. antenna. A receiver at a distance d receives a power flux density of $P_t / 4\pi d^2$ where $4\pi d^2$ is the area of the a sphere of radius d.



If an antenna with an effective are A_e, the received power will be:

$$P_r(d) = \frac{P_t}{4\pi d^2} A_e$$

The gain of an antenna is related to its effective area as:

$$G = \frac{4\pi A_e}{\lambda^2}$$

where $\lambda = \frac{c}{f}$ is the wavelength of the transmitte signal. Substituting A_e (in terms of G_r) in the expression for received power, we get

$$P_r(d) = \frac{P_t G_r \lambda^2}{(4\pi)^2 d^2}$$

If instead of an omnidirectional antenna, the transmitter sends information using an antenna with a gain G_t, then P_t is replaced by $G_t P_t = EIRP$, i.e.
Effective Isotropic Radiated Power.

So, for LOS propagation, we have

$$P_r(d) = \frac{P_t G_t G_r}{(4\pi d/\lambda)^2} = \frac{P_t G_t G_r}{L_s}$$

where $L_s = \left(\frac{4\pi d}{\lambda}\right)^2$ is the path loss.

If there are other losses such as rain fading or insertion loss in cabling and connections we have

$$P_r(d) = \frac{P_t G_t G_r}{L_s L_o}$$

where L_o is the product of all other losses.

To put this in dB, we have

$$P_r(d) \text{ in dBW} = P_t + G_t + G_r - L_s$$

where L_s is the path loss in dB, i.e.,

$$L_s = 10 \log\left(\frac{4\pi d}{\lambda}\right)^2$$

Example: A transmitter transmits at ~~a~~ ~~rate~~ with 50 W power at a frequency of 10 GHz. to a satellite at a Geostationary orbit (at an altitude of 36000 km.). Find the received power at the satellite if the Earth station antenna has a diameter of 2 m and an efficiency of $\eta = 0.65$ and the receiving antenna (at the satellite) has a gain of 45 dB.

Solution:

The gain of the transmitter antenna is $G_t = \frac{4\pi A_e}{\lambda^2}$

where $\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03$ m

$$A_e \cong \left(\frac{D}{2}\right)^2 \pi \times \eta = \left(\frac{2}{2}\right)^2 \times \pi \times 0.65 = 2.04$$

$$G_t = \frac{4\pi A_e}{\lambda^2} = \frac{2.04 \times 4 \times \pi}{(0.03)^2} = 28484 = 44.55 \text{ dB}$$

$$P_t = 10 \log 50 \simeq 17 \text{ dB W} = 47 \text{ dB}_m$$

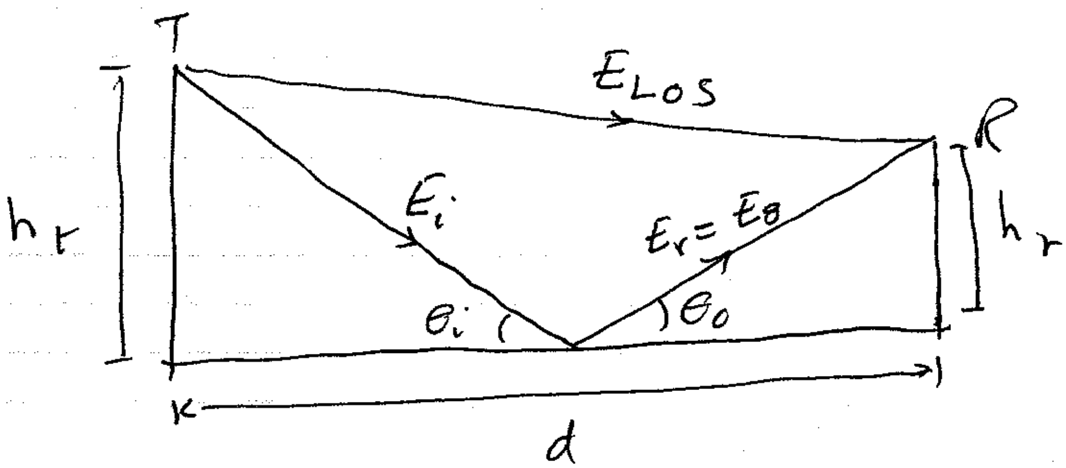
$$L_s = 10 \log \left(\frac{4\pi d}{\lambda}\right)^2 = 203.57$$

$$P_r(d) = P_t + G_t + G_r - L_s = 47 + 44.55 + 45 - 203.$$

$$P_r(d) = -67. \text{ dBm or } -97 \text{ dBW}$$

Two-ray reflection model:

In this model in addition to LOS path, there is a ray reflected from ground



If the distance d, between transmitter and receiver is much larger than the product of the antenna heights, i.e., if

$$d > \frac{20\pi h_t h_r}{3\lambda} \cong \frac{20 h_t h_r}{\lambda}$$

then the received signal power at a distance d is:

$$P_r(d) = \frac{P_t G_t G_r h_t^2 h_r^2}{d^4}$$

Example: A mobile transmit to a base station with a power 50mW ~~in~~ 900 MHz. band. The distance between the base station and the mobile is 10km. The base station antenna is at a height of 20m. and mobiles antenna is at a height of 1.5m. The mobiler's antenna has a gain of 3dB and base station's antenna has 10dB gain. Find the power received at the base station.

Solution:

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = \frac{1}{3}\ m$$

$d = 10,000$

and $\dfrac{20 h_t h_r}{\lambda} = \dfrac{20 \times 20 \times 1.5}{1/3} = 1800$

So,

$$d > \frac{20 h_t h_r}{\lambda}$$

and

$$P_r(d) = \frac{P_t G_t G_r}{d^4} h_t^2 h_r^2$$

$G_t = 3\,dB = 2$

$G_r = 10\,dB = 10$

So

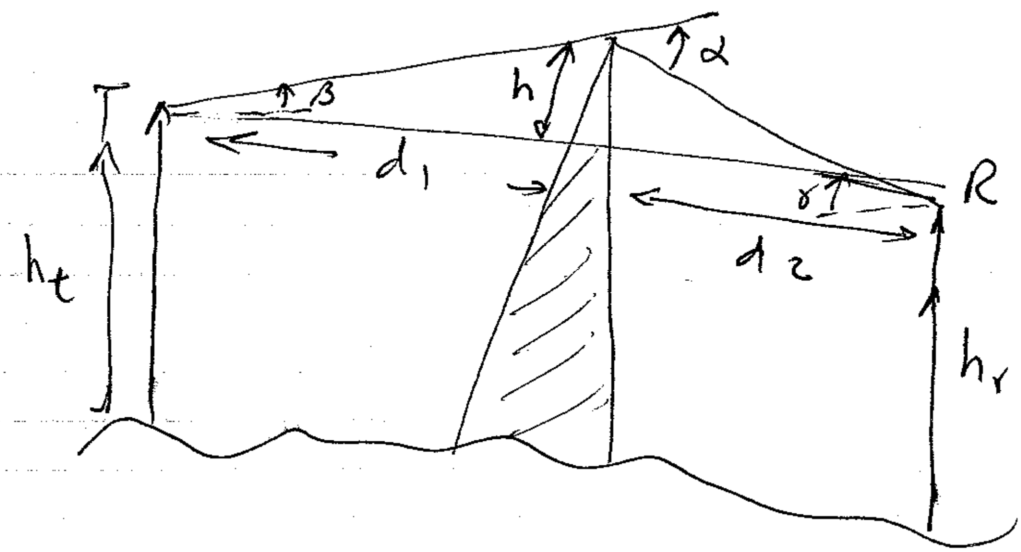
$$P_r(d) = \frac{50 \times 10^{-3} \times 2 \times 10}{(10,000)^4} \times (20)^2 (1.5)^2 \doteq 9 \times 10^{-10}$$

or

$P_r(d) = -90.46$ dBW $= -60.46$ dBm.

Fresnel Zone Geometry:

Another loss mechanism is diffraction. This happens when an obstacle, a hill, for example, stands between transmitter and receiver:



Assuming that $h \ll d_1$ and d_2 and $h \gg \lambda$ we have the <u>excess path length</u> as

$$\Delta \cong \frac{h^2}{2} \frac{(d_1+d_2)}{d_1 d_2}$$

The phase difference between two paths is:

$$\phi = \frac{2\pi\Delta}{\lambda} \simeq \frac{2\pi}{\lambda} \frac{h^2}{2} \frac{(d_1+d_2)}{d_1 d_2}$$

~~Let~~ $\nu = h\sqrt{\frac{2\, d_1\, d_2}{\lambda \cancel{(d_1+d_2)}}}$

The parameter:

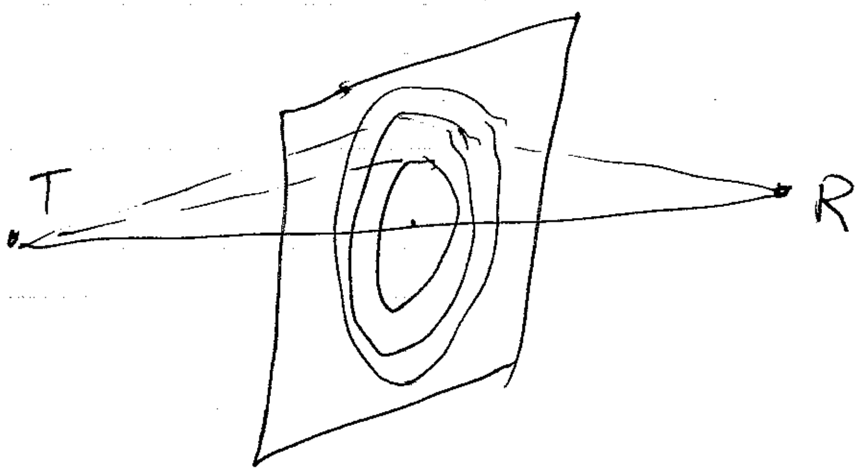
$$\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}}$$

is called the Fresnel-Kirkhoff diffractio[n]

parameter, Then

$$\Phi = \frac{\pi}{2}\nu^2$$

Visualize having a screen between transmitter and receiver.



Fresnel zones represents regions where secondary wave has a phase difference of $n\pi$ ~~from~~ with the direct ray, or ~~have~~ ~~a~~ differ in length by $\frac{n\lambda}{2}$ from the primary path.

The radius of the n th Fresnel ~~zone~~ circle is

$$r_n = \sqrt{\frac{n \lambda d_1 d_2}{d_1 + d_2}}$$

The so called gain (in fact loss) is given as

$$G_d \,(dB) = 20 \log |F(v)|$$

Where,

$$F(v) = \frac{E_d}{E_0} = \frac{1+j}{2} \int_v^{\infty} e^{-j\frac{\pi t^2}{2}} dt$$

$G_d\,(dB)$ has been tabulated and also empirical formulas for it exist, e.g.,

$$G_d\,(dB) = 0 \qquad v \le -1$$

$$G_d\,(dB) = 20 \log (0.5 - 0.62\, v) \qquad -1 \le v \le 0$$

$$G_d\,(dB) = 20 \log (0.5 \exp(-0.95\, v)) \qquad 0 \le v \le 1$$

$$G_d\,(dB) = 20 \log \left(0.4 - \sqrt{0.1184 - (0.38 - 0.1 v)^2}\right) \qquad 1 \le v \le 2.4$$

$$G_d\,(dB) = 20 \log \frac{0.225}{v} \qquad v \ge 2.4$$

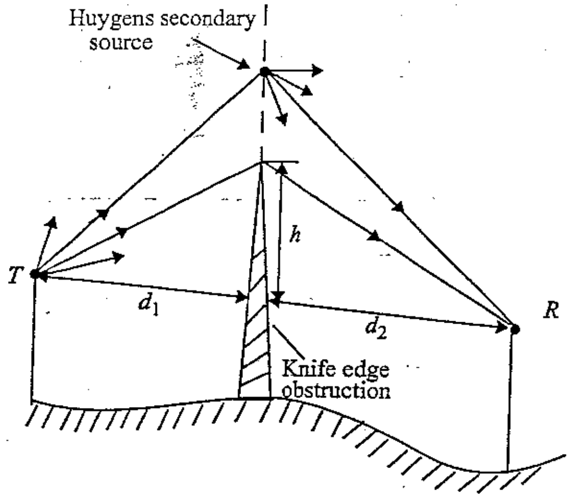


Figure 3.13
Illustration of knife-edge diffraction geometry. The receiver R is located in the shadow region.

The diffraction gain due to the presence of a knife edge, as compared to the free space E-field, is given by

$$G_d\,(\mathrm{dB}) = 20\log|F(v)| \tag{3.60}$$

In practice, graphical or numerical solutions are relied upon to compute diffraction gain. A graphical representation of $G_d\,(\mathrm{dB})$ as a function of v is given in Figure 3.14. An approximate solution for equation (3.60) provided by Lee [Lee85] as

$$G_d\,(dB) = 0 \qquad v \le -1$$

$$G_d\,(\mathrm{dB}) = 20\log(0.5 - 0.62v) \qquad -1 \le v \le 0$$

$$G_d\,(\mathrm{dB}) = 20\log(0.5\exp(-0.95v)) \qquad 0 \le v \le 1$$

$$G_d\,(\mathrm{dB}) = 20\log\left(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2}\right) \qquad 1 \le v \le 2.4$$

$$G_d\,(\mathrm{dB}) = 20\log\left(\frac{0.225}{v}\right) \qquad v > 2.4$$

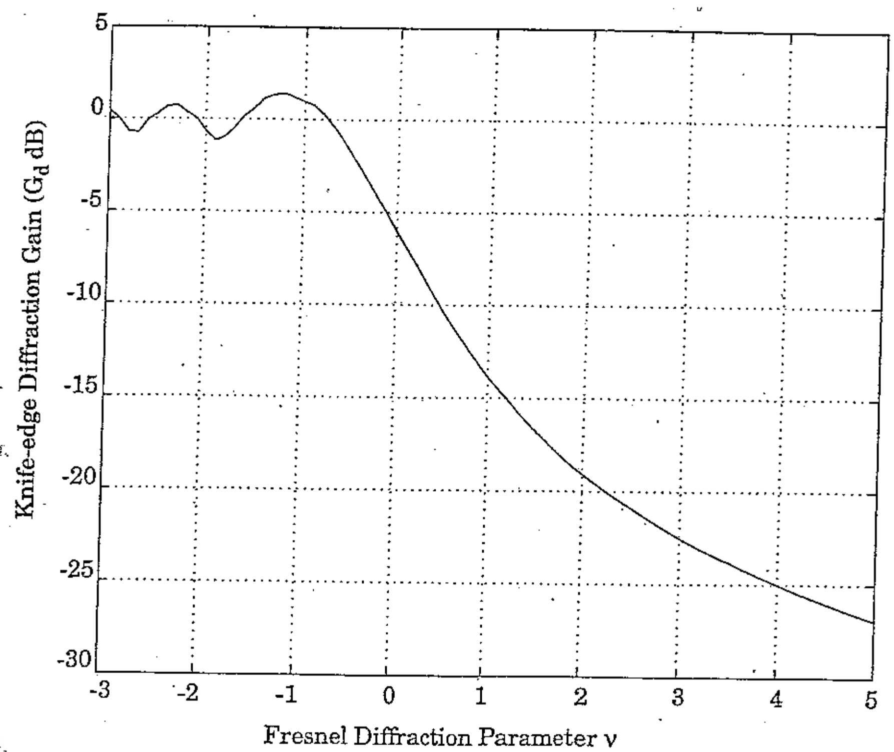


Figure 3.14
Knife-edge diffraction gain as a function of Fresnel diffraction parameter v.

Example: For $\lambda = \frac{1}{3}$ m, $d_1 = 1$ km, $d_2 = 1$ km

and a) $h = 25$ m b) $h = 0$, c) $h = -25$ m

find the diffraction loss:

a)
$$v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = 25\sqrt{\frac{2(1000+1000)}{\frac{1}{3}\times 1000\times 1000}} = 2.74$$

also specify the Fresnel zone where tip of obstruction

$$G_d(dB) = 20\log\left(\frac{0.225}{v}\right) = -21.7\,dB \quad (21.7\,dB \text{ loss})$$

from curve 22 dB

~~b) h=0 ...~~

$$\Delta \simeq \frac{h^2}{2}\frac{(d_1+d_2)}{d_1 d_2} = \frac{25^2}{2}\times\frac{1000+1000}{1000\times 1000} = 0.625 \text{ m}$$

$$\Delta = n\frac{\lambda}{2} \qquad \lambda = \frac{1}{3}$$

$$0.625 = n \times \frac{1}{6} \Rightarrow n = 3.75$$

So, tip of the obstruction obstructs first three (3) Fresnel zones.

b) $h = 0 \Rightarrow v = 0$

$$G_d(dB) = 20\log(0.5 - 0.62v) = 20\log 0.5 = -6\,dB$$

$h=0$, $\Delta = 0$ so, tip of obstruction is in the middle of 1st Fresnel z

c) $h = -25 \Rightarrow v = -2.74$

$G_d(dB) = 0$ dB (from Graph 1 dB)

Empirical Path Loss Models:

Log-distance Path Loss Model:

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n$$

or

$$\overline{PL}(dB) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)$$

Typical values of the path-loss exponent n:

Environment	Path Loss Exponent (n)
Free Space	2
Urban Area Cellular	2.7 to 3.5
Shadowed Urban Area Cellular	3 to 5
In-building LOS	1.6 to 1.8
Obstructed in building	4 to 6
obstructed in Factories	2 to 3

Log-normal Shadowing:

$$PL(d)_{dB} = \overline{PL(d)} + X_\sigma = \overline{PL(d_0)} + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

and

$$P_r(d)_{dBm} = P_{t,dBm} - PL(d)_{dB} + G_t + G_r$$

X_σ is a zero-mean Normal (Gaussian) random variable.

Note: the deviation from <u>mean</u> in <u>dB</u> is Normal, so, the name <u>log-normal</u> shadowing

$$P_r[P_r(d) > \gamma] = \int_{\gamma}^{\infty} p(x)\,dx$$

$P_r(d)$

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left[-\frac{(x-\overline{P_r(d)})^2}{2\sigma^2}\right]$$

$$P_r[P_r(d) > \gamma] = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{\gamma}^{\infty}\exp\left[-\frac{(x-\overline{P_r(d)})^2}{2\sigma^2}\right]dx$$

let

$$\frac{x - \overline{P_r(d)}}{\sigma} = u$$

$$P_r[P_r(d) > \gamma] = \frac{1}{\sqrt{2\pi}} \int_{\frac{\gamma - \overline{P_r(d)}}{\sigma}}^{\infty} \exp[-\frac{u^2}{2}]\, du$$

$$P_r[P_r(d) > \gamma] = Q\left[\frac{\gamma - \overline{P_r(d)}}{\sigma}\right]$$

where

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_{y}^{\infty} e^{-u^2/2}\, du$$

The path loss exponent n and σ are found by curve fitting the measured data.

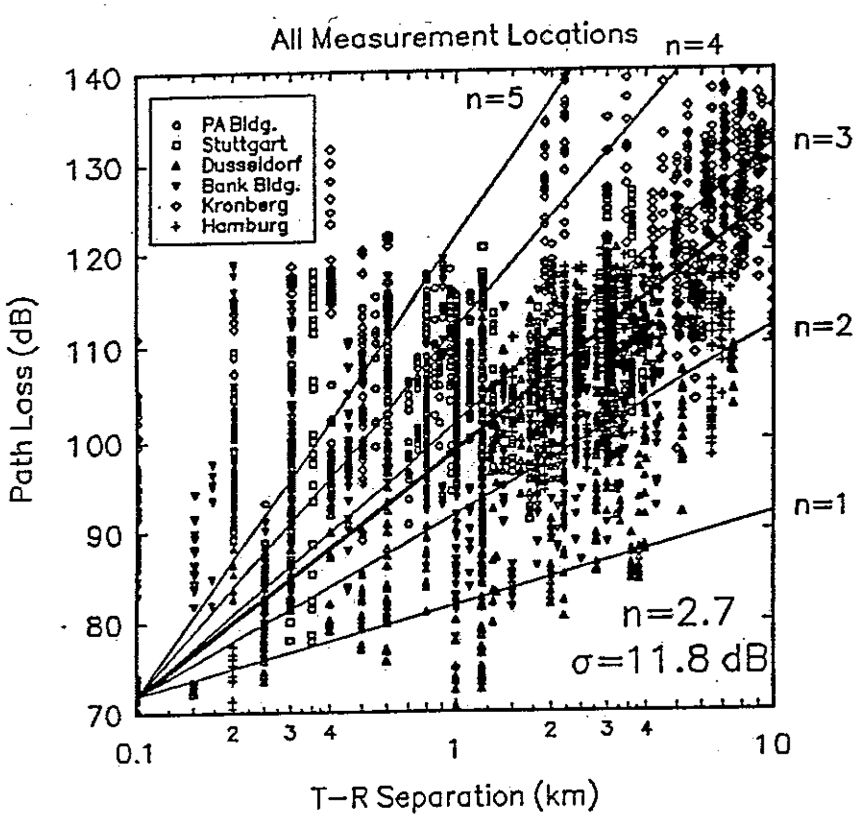


Figure 3.17
Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germ
For this data, $n = 2.7$ and $\sigma = 11.8$ dB [From [Sei91] © IEEE].