Lecture 7

Small-Scale fading

Small-scale fading describes ~~w~~ variations of amplitude or phase over short distances or over a short period of time.

Fading is caused by interference between several versions of transmitted signal arriving at the receiver.

The cause of fading is the height of objects surrounding the receiver antenna. They cause the multiple receptions by the antenna. These multiple receptions have randomly distributed amplitude, phase and angles of arrival. These multipath components combine at the receiver antenna.

The factors influencing ~~the~~ fading are;

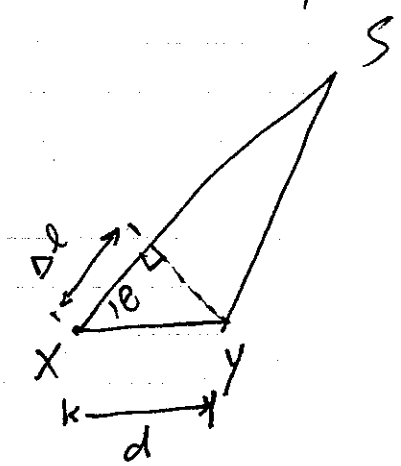
1) Presence of objects reflecting and scattering waves arriving at the receiver.
2) Speed of the receiver
3) Speed of the surrounding objects.

4) Transmission bandwidth of the signal.

Doppler Shift

Doppler Shift is the change in frequency as a result of movement of the receiver.

Assume that a transmitter at point S transmits to points X and Y and mobile is going at speed v from X to Y.



The wave ~~takes~~ travels

$\Delta l = d \cos\theta$

more distance to get to X than Y.

The ~~change~~ difference in phase will be:

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi d \cos\theta}{\lambda} = \frac{2\pi\, v\Delta t}{\lambda}\cos\theta$$

The change in frequency, i.e., Doppler shift, is

$$f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{\cancel{2\pi}\, v\, \cancel{\Delta t}}{\cancel{2\pi}\,\lambda\,\cancel{\Delta t}}\cos\theta = \frac{v\cos\theta}{\lambda}$$

where

$$\lambda = \frac{c}{f_c}$$

So: $f_d = \frac{v}{c} f_c \cos\theta$

Example: A mobile is travelling at a speed of 100km, and transmit at a frequency of 1950 MHz. What is the range of received frequencies

$$f_d = \frac{v}{c} f_c \cos\theta = \frac{100/3600}{300{,}000} \times 1950 \times 10^6 \cos\theta$$

$$= 180.5 \cos\theta \text{ Hz.}$$

The range of frequencies is

$$f_c \pm 180.5 = 1950 * 10^6 \pm 180.5$$

$$= [1949.99982, 1950.000181]$$

Parameters of a multipath fading Channel:

The impulse response of a multipath channel is:

$$h(\tau) = \sum_{i=0}^{N-1} a_i e^{j\theta_i} \delta(\tau - \tau_i)$$

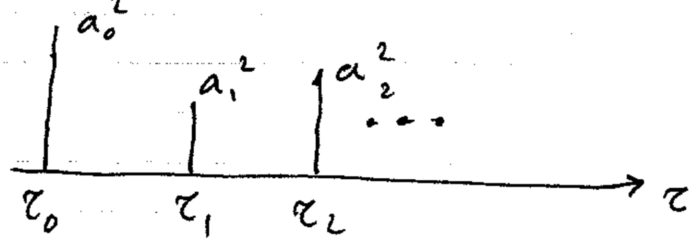
This means that a signal transmitted at time t will be received at times $t+\tau_0$ (the main component), $t+\tau_1$, $t+\tau_2$, ... with amplitudes $a_0, a_1, \ldots, a_{N-1}$ and phases $\theta_0, \theta_1, \ldots \theta_{N-1}$.

To measure the channel, one can transmit a narrow pulse (approximating ~~the~~ an impulse) and measure the received signal ~~at~~ over a period of time.

The power received will be

$$P = \sum_{i=0}^{N-1} a_i^2 .$$

The <u>Power</u>-<u>delay</u> profile looks like this.



There are a few parameters to quantify the multiple delays:

<u>mean</u> <u>excess</u> <u>delay</u>:

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\,\tau_k}{\sum_k P(\tau_k)}$$

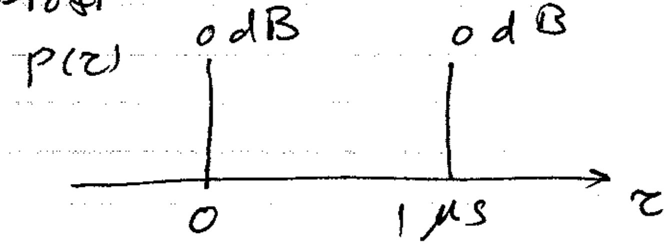
The <u>rms</u> delay spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$$

where

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum a_k^2} = \frac{\sum_k P(\tau_k)\,\tau_k^2}{\sum_k P(\tau_k)}$$

Example: A channel has the following power delay profile



a) Find the rms delay spread of the channel.

b) If QPSK modulation is used, what is the maximum bit rate that can be transmitted over this channel without needing an equalizer?

Hint: ~~Assume that~~ ~~no~~ No equalizer is needed if the symbol duration is much longer thand the rms delay spread of the channel (say 10 times the rms delay spread).

a) $$\bar{\tau} = \frac{(1)(0) + (1)(1)}{1+1} = \frac{1}{2} = 0.5 \ \mu sec.$$

$$\overline{\tau^2} = \frac{(1)(0) + (1)(1)^2}{1+1} = \frac{1}{2} = 0.5 \ \mu sec^2$$

$$\sigma_\tau = \sqrt{0.5 - 0.5^2} = 0.5 \ \mu sec.$$

b) Since a delay spread less than 10% of the symbol duration is not harmful, we need $T_s \geq 10\,\sigma_\tau$.

or

$$T_s \geq 10 \times 0.5 = 5 \ \mu \text{sec.}$$

or

$$R_s = \frac{1}{T_s} \leq 200 \ \text{k symbols/sec.}$$

or

$$R_b \leq 400 \ \text{kbps}$$

Coherence Bandwidth

Coherence bandwidth is a measure of the range of frequencies over which the channel is considered to be flat. By flat, we mean different frequencies being faded "more or less" the same.

For example, we can assume that the range of frequencies over which the correlation between different components is higher than 90% to be the coherence bandwidth. In this case

$$B_c \simeq \frac{1}{50\sigma_\tau}$$

For many applications, this criterion is to restrictive and results in conservative designs i.e., under utilization of the network resource

If we relax the definition and ~~choose~~ accepts the range of frequencies over which the correlation is above 0.5 as the coherence bandwidth then:

$$B_c \simeq \frac{1}{5\sigma_\tau}$$

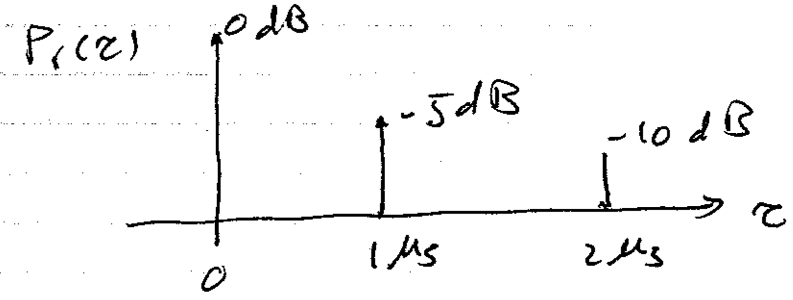
If the bandwidth of the signal is less than the coherence bandwidth, the fading is <u>flat</u> and if $W_s \geq B_c$ then the fading is frequency selective.

Orthogonal Frequency Division Multiplexing (OFDM) is used to improve the performance in frequency selective channels. In OFDM the data stream is divided into lower ~~freq~~ rate streams each being sent over a narrower frequency slot. This way, each sub-channel is flat for ~~d~~ the sub-stream it is carrying.

Equalization is another alternative to compensate for amplitude variations.

Example:

The power delay profile of a fading channel is:



A mobile transmits at a rate of 500 kbps using QPSK with a raised cosine filter with roll-off factor of $\alpha = 0.5$ over this channel. Does this system need equalization.

$$\bar{\tau} = \frac{0 \times 1 + 1 \times 10^{-0.5} + 2 \times 10^{-1}}{1 + 10^{-0.5} + 10^{-1}} = 0.34509 \ \mu s$$

$$\overline{\tau^2} = \frac{0 \times 1 + 1^2 \times 10^{-0.5} + 2^2 \times 10^{-1}}{1 + 10^{-0.5} + 10^{-1}} = 0.5057 \ \mu s^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = 0.610625 \ \mu s$$

$$B_c \simeq \frac{1}{5\sigma_\tau} = 327.5 \ kHz.$$

$$B_s = \frac{500}{2}(1 + 0.5) = 375 \ kHz.$$

$B_s > B_c \Rightarrow$ Equalization is needed.

Doppler Spread and Coherence Time

Coherence time is a measure of how long the channel can be considered constant. So, it dictates the interval between two channel information updates. For example, if a system uses equalization, the frequency of equalizer parameter updates is dictated by the coherence time.

Coherence time is inversely proportional to the Doppler spread of the ~~signal~~ transmitted signal. If a sinusoid with a frequency f_c is transmitted the received signal will be in the range of $f_c - f_d$ to $f_c + f_d$ where

$$f_d = \frac{v}{c} f_c$$

is the Doppler Shift. That is, there is a Doppler spread of $B_d = 2 f_d$. If the signal has a bandwidth much greater than B_d, then the effect of Doppler spread is negligible.

In this case, fading is called Slow fading.
~~This~~ In other words, if the symbol rate (the inverse of transmission bandwidth) is much less than the inverse of the Doppler spread, we have slow fading. Else, we have fast fading.
A quantity proportional to:
the inverse of the Doppler spread is called the Coherence Time. One simple choice is

$$T_c = \frac{1}{f_d}$$

However, this is very wide time interval over which the fading coefficient may vary significantly. A more restrictive result is

$$T_c \simeq \frac{9}{16\pi f_d}$$

which is the time duration over which the correlation function is above 0.5.
A popular choice is the geometric mean of ~~the two,~~ two, i.e.,

$$T_c = \sqrt{\frac{9}{16\pi f_d} \cdot \frac{1}{f_d}} = \frac{0.423}{f_d}$$

Example:

A mobile is travelling at a speed of 50 km./hr. and transmitting at a bit rate of 2 Mbps in the 1800 MHz. band.

a) How often does the equalizer need to be updated?

b) How many bits can be transmitted between two updates?

a)
$$f_d = \frac{50,000}{3600} \cdot \frac{1}{3\times10^8} * 1.8\times10^9 = 83.33\,Hz.$$

$$T_c = \frac{0.423}{83.3} = 0.005076 = 5.076 \text{ m sec.}$$

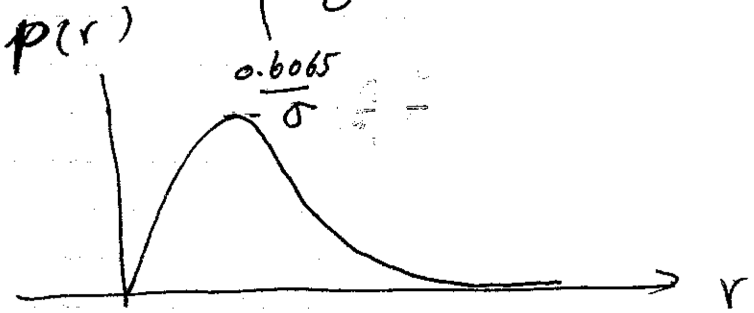
b)
$$N = R\,T_c = 2\times10^6 \times 0.005076 = 10,152 \text{ bits.}$$

Rayleigh Fading Distribution

The statistical time varying nature of the received envelope of a signal.

It is assumed that fading is the result of envelope variation of two quadrature Gaussians:

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} e^{-r^2/2\sigma^2} & 0 \le r \le \infty \\ 0 & r < 0 \end{cases}$$



$$P(R) = P_r(r \le R) = \int_0^R p(r)\,dr = 1 - e^{-R^2/2\sigma^2}$$

and

$$P_r(r > R) = e^{-R^2/2\sigma^2}$$

$$r_{mean} = \int_0^\infty r\,p(r)\,dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\,\sigma$$

$$\sigma_r^2 = E[r^2] - E[r]^2 = \int_0^\infty r^2 p(r)\,dr - \frac{\sigma^2\pi}{2} = \sigma^2\left(2 - \frac{\pi}{2}\right)$$
$$= 0.4292\,\sigma^2$$