Lecture 8

Modulation Techniques

Why modulation?

- Convinience of Transmissio
- Sharing of the bandwid

A baseband signal is not suitable for transmission as its wavelength is very large. An antenna should be comparable with the wavelength of the signal to be transmitted, e.g., quarter wavelength.

Take a speech signal. The frequency is 3 kHz. If it is turned into an electromagnetic wave the wavelength will be:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^3} = 10^5 = 100 \text{ km. } !!$$

So, we need to translate the signal to higher frequen

Also, we need to separate the different signals, blonging to different customers by assigning them different frequency slots.

Different modulation Techniques:

Analog $\begin{cases} \bullet \text{ Amplitude Modulation} \\ \bullet \text{ Angle modulation} \end{cases}$

Digital Modulation $\begin{cases} \text{PSK} \\ \text{QAM} \\ \text{FSK} \end{cases}$

AM : Amplitude Modulation

$$S_{AM}(t) = A_c[1 + m(t)] \cos(2\pi f_c t)$$

modulation index let the baseband signal be

$S(t) = A_m \cos(2\pi f_m t)$

Then:

$$S_{AM}(t) = A_c \cos 2\pi f_c t + A_m \cos(2\pi f_m)t \cos(2\pi f_c t)$$

$$= A_c\left[1 + \frac{A_m}{A_c}\cos(2\pi f_m t)\right] \cos(2\pi f_c t)$$

$$= A_c\,[1 + \underbrace{k \cos(2\pi f_m)t}_{m(t)}] \cos(2\pi f_c t)$$

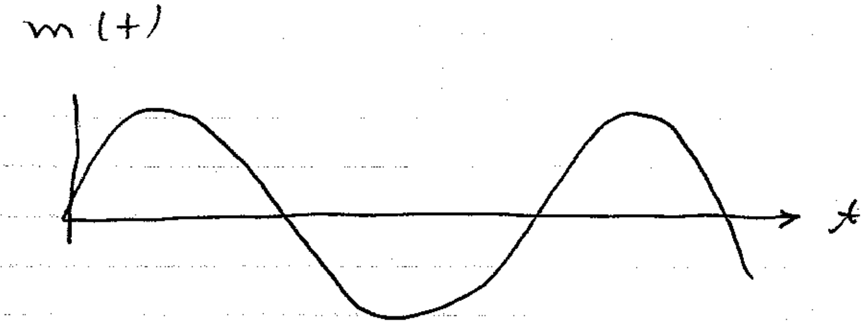
where

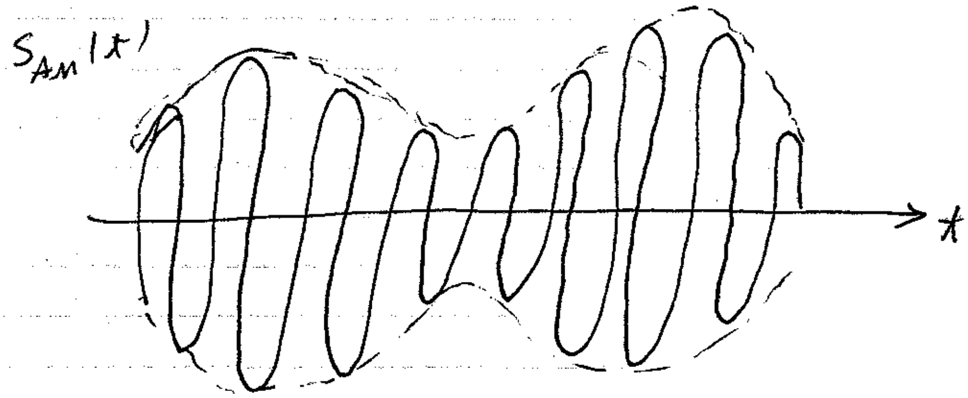
$$k = \frac{A_m}{A_c}$$

is called the modulation index.

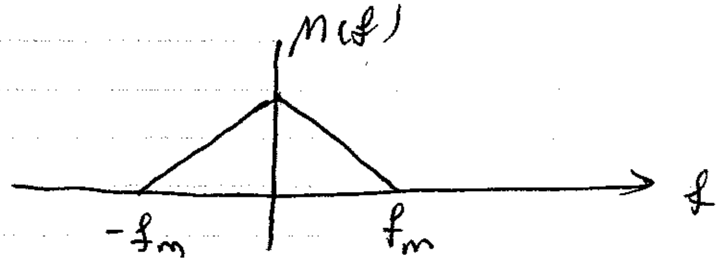
In order to be able to use envelope detection,

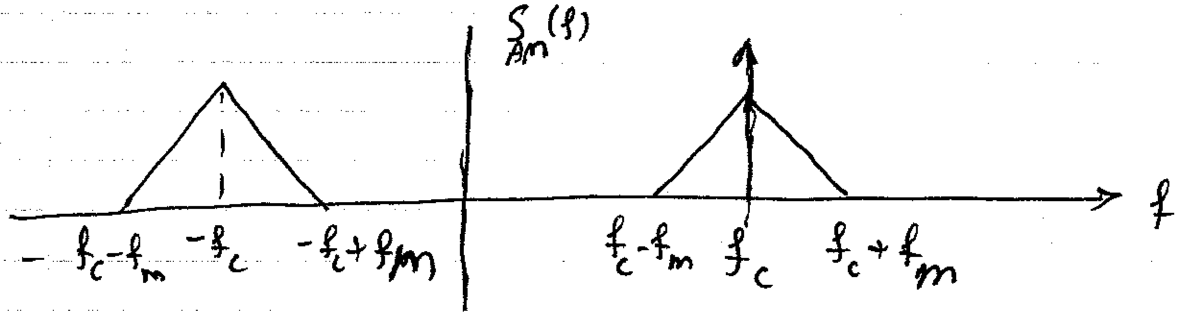
$k \leq 1 \Rightarrow A_m \leq A_c$





$$S_{AM}(f) = \frac{1}{2} A_c \left[\delta(f-f_c) + M(f-f_c) + \delta(f+f_c) + M(f+f_c) \right]$$





$$B_{AM} = 2 f_m$$

$$P_{AM} = \frac{1}{2} A_c^2 \left[1 + 2\overline{m(t)} + \overline{m^2(t)} \right]$$

When $m(t) = k \cos(2\pi f_m t)$

$$P_{AM} = \frac{1}{2} A_c^2 \left[1 + P_m \right] = P_c \left[1 + \frac{k^2}{2} \right]$$

Modulation efficiency

$$\eta = \frac{P_m}{P_{AM}}$$

for a sinusoidal signal :

$$\eta = \frac{k^2/2}{1 + \frac{k^2}{2}}$$

$$\eta_{max} = \frac{1/2}{1+1/2} = \frac{1}{3} = 33\ \%$$

Example : A zero-mean sinusoidal signal is applied to a transmitter that radiates AM signal with 10 kW power. The modulation index is 0.6. Find the power in the carrier, the power in each sideband and the ~~ex~~ efficiency.

$$P_{AM} = P_c \left(1 + \frac{k^2}{2}\right) = 10\ kW$$

$$P_c = \frac{10}{1 + \frac{k^2}{2}} = \frac{10}{1 + \frac{0.36}{2}} = 8.47\ kW$$

$$\frac{10 - 8.47}{2} = 0.765\ kW \qquad \text{Power / Sideband}$$

$$\eta = \frac{0.6^2/2}{1 + 0.6^2/2} = 0.15 \text{ or } 15\ \%$$

in fact efficiency is half this since the two sidebands ~~are~~ convey the same information. Single sideband (SSB) is used to increase ~~the efficiency~~ to save the bandwidth.

Frequency Modulation (FM)

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau\right]$$

$$\phi(t) = 2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\,d\tau$$

$$f_i(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} = f_c + k_f\, m(t)$$

If we have $m(t) = A_m \cos(2\pi f_m t)$

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m}\sin(2\pi f_m t)\right]$$

The modulation index is $\beta = \frac{k_f A_m}{f_m}$

The bandwidth requirement for FM is given by Carson's rule:

$$B_T = 2(\beta+1) f_m = 2(\beta+1) B$$

Tradeoff between SNR and bandwidth

$$SNR_{out} = 3\beta^2(\beta+1)\, SNR_{in}$$

$$SNR_{in} = \frac{A_c^2/2}{2N_0(\beta+1)B}$$

So,

$$SNR_{out} = 3\beta^2 \frac{A_c^2/2}{2N_0 B} = 3\beta^2 SNR_{in,AM}$$

where $SNR_{in,AM} = \dfrac{A_c^2/2}{2N_0 B}$

is the SNR at the input of an AM demodulator

$\frac{A_c^2}{2}$ is the received power P_r, usually referred to Carrier power C. Carrier to noise ratio $\frac{C}{N_0}$ is a measure of the ratio of the received power to receiver noise. We can express SNR_{out} for FM a

$$SNR_{out} = \frac{3\beta^2}{2B} \cdot \frac{C}{N_0}$$

Before starting to talk about digital modulation schemes, let's do an example that shows superiority of the digital modulation over analog. Here, I count on your prior knowledge of digital communications.

Example: Assume that we want to compare the performance of analog and digital transmission for speech. Assume that we want to have 48 dB of SNR. To get this much of quantization noise, we need 8 bits per samples ~~pe~~ if we use 8 pulse code modulation (PCM). That is we take 8000 samples per second and quantiz each sample ~~and~~ using $2^8 = 256$ levels.

This means a bit rate of 64 kbps. Similar quality can be achieved (as evidenced by subjective tests: see Table 8.3 on page 444) with CELP with 14.4 kbps. In this scheme, segments of 20 ms long (160 samples ~~bits~~ each) are encoded into $14.4 \times 10^{3} \times 20 \times 10^{-3} = 288$ bits that represent that segment. If we modulate this 14.4 kb stream using QPSK with $\alpha = 0.5$, we need

$$B = \frac{14400}{2}(1.5) = 10{,}800 \text{ Hz. of bandwidth}$$

If we use an $\frac{E_b}{N_o}$ of 10 dB, we will have a BER of 4×10^{-6}. The probability of ~~er~~ segment error is $\simeq 288 \times 4 \times 10^{-6} \simeq 10^{-3}$, i.e., one out of each 1000 frames will be affected. The required $\frac{C}{N_o}$ will be

$$\frac{C}{N_o} = \frac{P_r}{N_o} = \frac{E_b}{N_o} R = 10 \times 14400 = 144{,}000 = 51.6 \text{ dB}$$

Let's now try to see what we need with FM

$$B_{FM} = 2(\beta + 1)B$$

$$10\,800 = 2(\beta+1)3000$$

$$\beta = 0.8$$

To get 48 dB, with this β, we need

$$10^{4.8} = \frac{3(0.8)^2}{2\times 3000}\cdot\frac{C}{N_0}$$

$$\frac{C}{N_0} = \frac{10^{4.8}\times 2\times 3\,000}{3\times(0.8)^2} = 197{,}174{,}170 \simeq 83\,dB$$

So, we need $83 - 51.6 = 31.4$ dB

This translates into 1000 times more transmission power or 1000 times less receiver noise or a combination of both resulting in 1000 times higher $\frac{C}{N_0}$.

Even if we take 64 kbps using QPSK, the BW is

$$B = \frac{64000}{2}(1+0.5) = 48000\ Hz.$$

If we accept BER $= 10^{-3}$, i.e., out of each hundred and twenty five samples one become corrupted; as the symbol error rate will be $\simeq 0.008$, then equivalent FM, will have β of

$$48000 = 2(\beta+1)\times 3000 \Rightarrow \beta = 7$$

and

$$10^{4.8} = \frac{3 \times 7^2}{2 \times 3000} \cdot \frac{C}{N_o}$$

or

$$\frac{C}{N_o} = 2.5753 \times 10^6 = 64.1 \text{ dB}$$

while ~~the~~ an $\frac{E_b}{N_0} = 6.8$ would be enough to get a BER of 10^{-3}. So,

$$\frac{C}{N_0} = \frac{E_b}{N_0} \cdot R \Rightarrow \frac{C}{N_0} = 6.8 + 10 \log 64000$$
$$= 54.86 \text{ dB}$$

That is a saving of

$$64.1 - 54.86 = 9.24 \text{ dB}$$

can be achieved.

Advantages of digital modulation over analog modulation:

- Better power and bandwidth efficiency.
- possibility of regenerative repeating
- possibility of error correction
- ease of implementation with high accuracy and with high speed using DSP.

limits of digital communication:

Channel Capacity:

Bandwidth efficiency $\eta = \frac{R}{B}$ bps/Hz.

Shannon defined a quantity called channel capacity. Channel capacity is the maximum rate that can be transmitted through the channel ~~while~~ without any error. (in fact an error rate that vanishes with the ~~length~~ duration of transmission).

$C = B \log(1 + \frac{S}{N})$ is the channel capacity ~~pe~~ in terms of bits per second. It gives the ultimate bandwidth efficiency

$$\eta_{max} = \frac{C}{B} = \log(1 + \frac{S}{N}) \quad \text{in bits/use.}$$

Example: The SNR of a wireless system is 20dB ~~an~~ and the bandwidth is 30 kHz. What is the maximum theoretically possible bit rate. Compare it with US Digital Cellular standard.

$$\frac{S}{N} = 20\ dB = 100$$

$C = 30{,}000 \log(1+100) = 199.75$ kbps

The data rate ~~of~~ USDC system is 48.6 kbps which is about 25% of the theoretical limit.

Let's relate this to the $\frac{E_b}{N_o}$.

$$S = E_b R \quad \text{and} \quad N = N_o B$$

So

$$\frac{S}{N} = \frac{E_b}{N_o} \cdot \frac{R}{B}$$

at $R = C$

$$\eta = \frac{C}{B} = \log_2\left(1 + \frac{E_b}{N_o}\frac{C}{B}\right)$$

or

$$\eta = \log_2\left(1 + \frac{E_b}{N_o}\eta\right)$$

or

$$\frac{E_b}{N_o} = \frac{2^{\eta} - 1}{\eta}$$

So, for a given bandwidth efficiency, we can find the $\frac{E_b}{N_o}$ required to achieve zero error probability.

Now, let's take the example of BPSK modulation. The band with efficiency of BPSK is $\eta = 1$.

So, the required $\frac{E_b}{N_0} = 1$ or 0 dB

But, this $\frac{E_b}{N_0}$ results in close to 50% probability of error in BPSK. Say, we accept 10^{-5} as no error case, Then we need an $\frac{E_b}{N_0}$ of 9.6 dB to get this BER. So, BPSK is off from the theoretical result by 9.6 dB.

Digital Modulation Techniques:

Binary Shift Keying (BPSK)

At this point, we present a simple view of BPSK, in the sense that we do not bother with the channel bandwidth constraint and, as a result, do not worry about pulse shaping. We assume that every T_b seconds (T_b is the bit duration), we would like to transmit either a 1 or a 0. Assume that for a 1, we transmit a sinusoid with frequency f_c for the duration of T_b seconds,

$$s_1(t) = A\cos(2\pi f_c t\ ), \quad kT_b \le t \le (k+1)T_b \text{ for } b_k = 1$$

and for a zero, we transmit a 180° phase shifted version of this, i.e.,