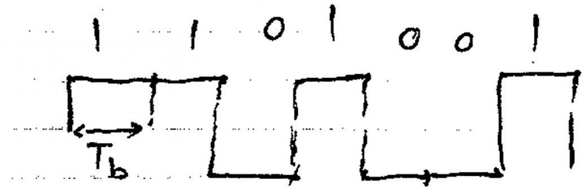
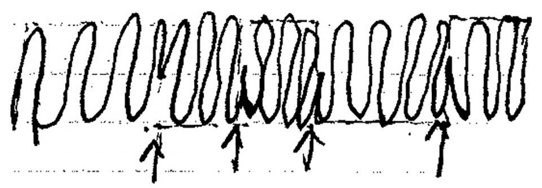
$$s_0(t) = A\cos(2\pi f_c t + \pi) \qquad kT_b \leq t \leq (k+1)T_b$$
$$= -A\cos 2\pi f_c t \qquad \text{for } b_k = 0$$

This operation corresponds to ~~[illegible]~~ mapping 1 and to +1 and −1 and then multiplying the resulting baseband waveform (square wave pulse stream) to $A\cos 2\pi f_c t$.

For example for a sequence of 1101001 we have the equivalent baseband waveform as:

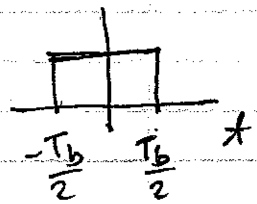


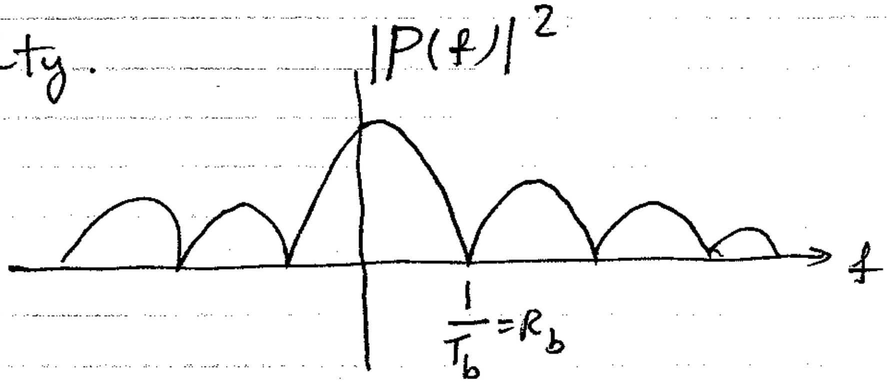
and the modulated waveform is



Note that abrupt phase changes occur at boundaries between bit intervals. In a real-life physical system changes like this are not possible and the constant envelope property of the BPSK disappears when the modulated signal goes through ~~the~~ bandlimi circuits.

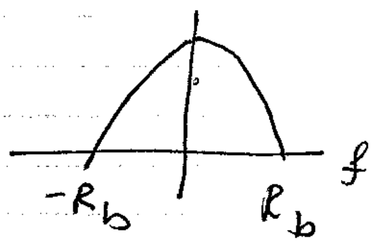
If we look at the frequency spectrum of the pulse shaping ~~...~~ that we have used (we have used a square wave) we notice that it extends to infinity.



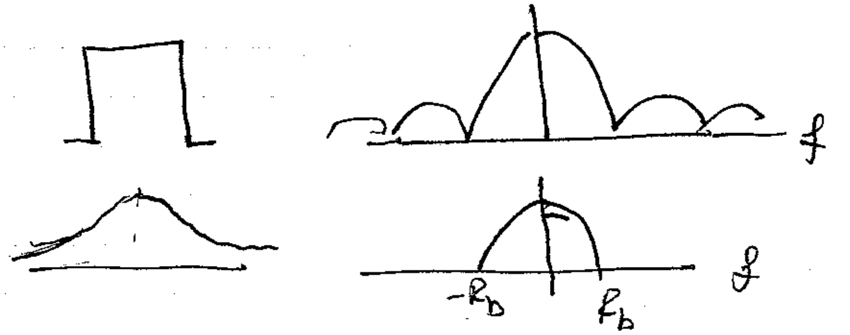


Of course we cannot let this go unfiltered over the channel (of course if we do not want it to interfere with any other transmission at any frequency; or at least at those frequencies not far from it).

But filtering this pulse shape, say, only keeping the first lobe



results in broadening of the pulse in time



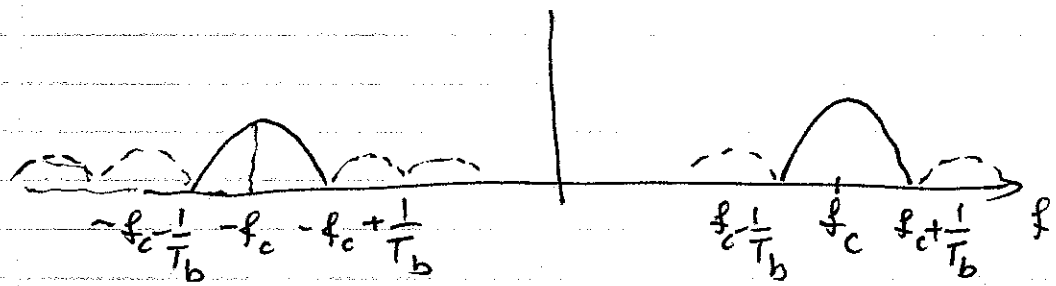
This results in Intersymbol Interference (ISI).

Before discussing more intelligent pulse shaping techniques, let's relate the bit rate to bandwidth requirement of BPSK (in this unfiltered version).

If we keep only the first lobe.

$$B = \frac{1}{T_b} = R_b$$

of course this is at the baseband before translati to carrier frequency after upconversion, we have



So

$$B = 2R_b \quad \text{(for rectangular pulse)}$$

Pulse Shaping for no ISI:

In order not to have ISI, we require:

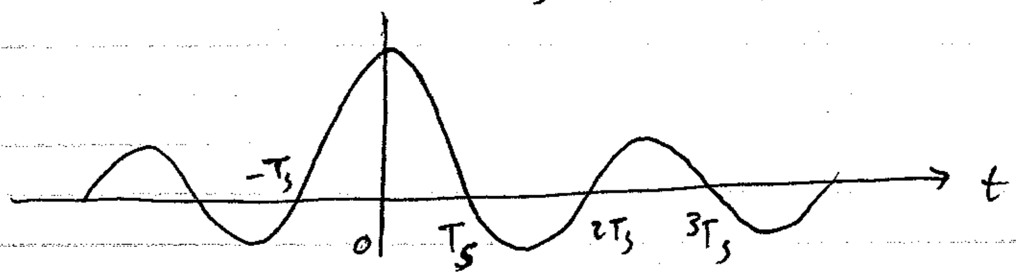
$$h_{eff}(nT_s) = \begin{cases} K & n = 0 \\ 0 & n \neq 0 \end{cases}$$

where

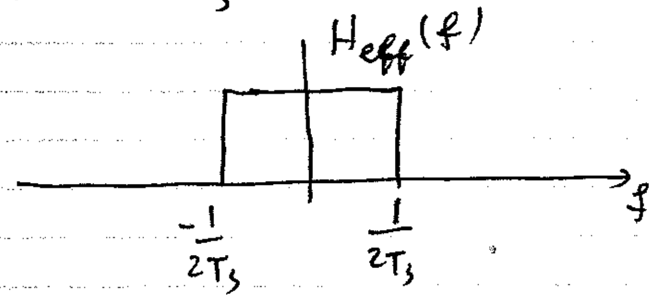
$$h_{eff}(t) = p(t) * h_c(t) * h_r(t)$$

one waveform that has this property is the Nyquist pulse:

$$h_{eff}(t) = \frac{\sin(\pi t / T_s)}{\frac{\pi t}{T_s}}$$



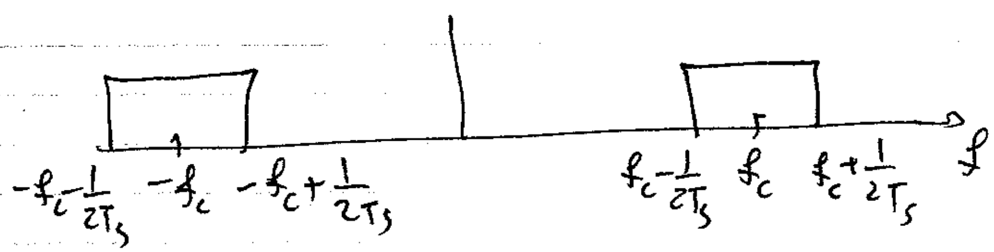
$$H_{eff}(f) = \frac{1}{f_s} \text{rect}\left(\frac{f}{f_s}\right) \quad \text{where } f_s = \frac{1}{2T_s}$$



Note that using this pulse shape the bandwidth requirement is

$$B = \frac{1}{2T_s} = R_{s/2} = R_{b/2} \quad (\text{for BPSK})$$

of Course, after up conversion



$$B = R_s = R_b$$

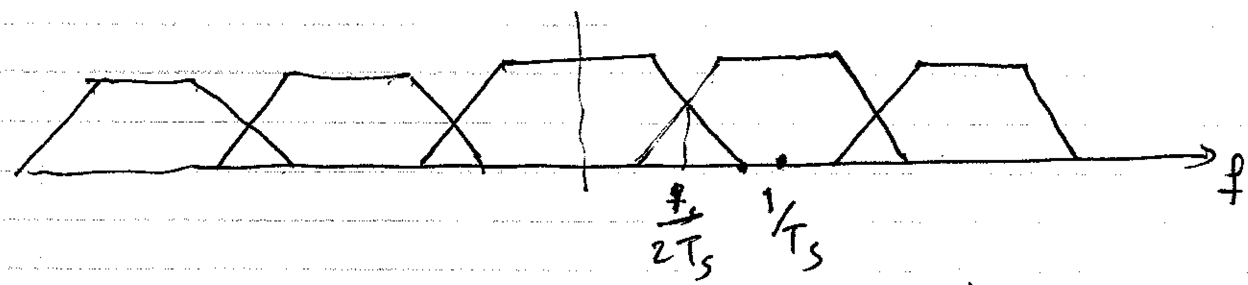
The problem with Nyquist pulse is:

- It is non-causal (not physically realizable)
- It decays slowly (as $\frac{1}{t}$), this makes it sensitive to miss-timing and also difficult to approximate.

Nyquist Condition:

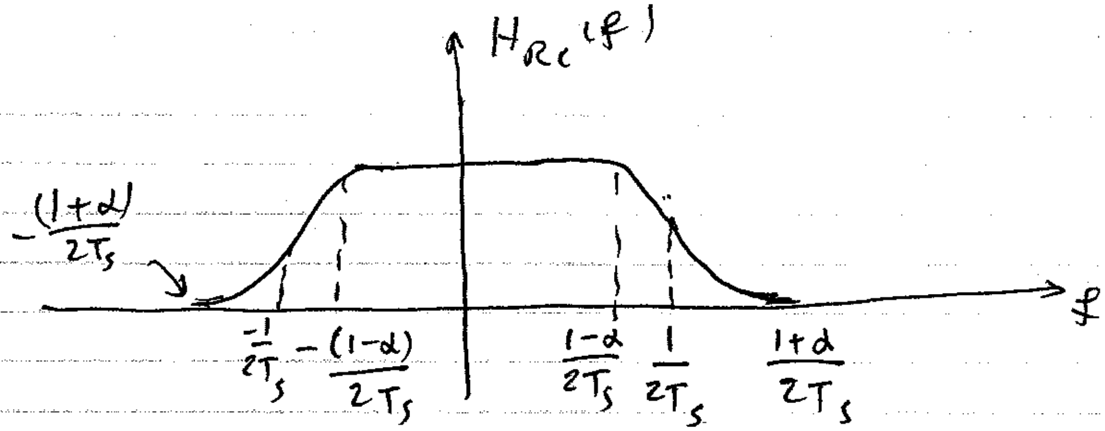
if $$Z(f) = \sum_{n=-\infty}^{\infty} H\left(f - \frac{n}{T_s}\right) = K$$

then $H(f)$ is a pulse shape that avoids ISI



One choice is ~~cos~~ raised-cosine filter

$$H_{RC}(f) = \begin{cases} 1 & 0 \leq |f| \leq \frac{1-\alpha}{2T_s} \\ \frac{1}{2}\left[1 + \cos\left[\frac{\pi |f| \cdot 2T_s - 1 + \alpha}{2\alpha}\right]\right] & \frac{1-\alpha}{2T_s} \leq f \leq \frac{1+\alpha}{2T_s} \\ 0 & |f| > \frac{1+\alpha}{2T_s} \end{cases}$$



The impulse response (the pulse shape) of this filter is:

$$h_{RC}(t) = \frac{\sin\frac{\pi t}{T_s}}{\pi t} \frac{\cos\frac{\pi\alpha t}{T_s}}{1-\left(\frac{4\alpha t}{2T_s}\right)^2}$$

The bandwidth requirement of raised cosine pulse shape is

$$B = \frac{1+\alpha}{2T_s} = (1+\alpha)\frac{R_s}{2}$$ before frequency translation ~~and~~

and

$$B = R_s(1+\alpha)$$ after up conversion

for BPSK it is:

$$B = R_b(1+\alpha)$$

and for QPSK, $R_s = \frac{R_b}{2}$

$$B = \frac{R_b}{2}(1+\alpha)$$ in general $B = \frac{R_b}{\log_2 M}(1+\alpha)$

$$P_{\mathrm{BPSK}} = \frac{E_b}{2}\left[\left(\frac{\sin\pi(f-f_c)T_b}{\pi(f-f_c)T_b}\right)^2 + \left(\frac{\sin\pi(-f-f_c)T_b}{\pi(-f-f_c)T_b}\right)^2\right]$$

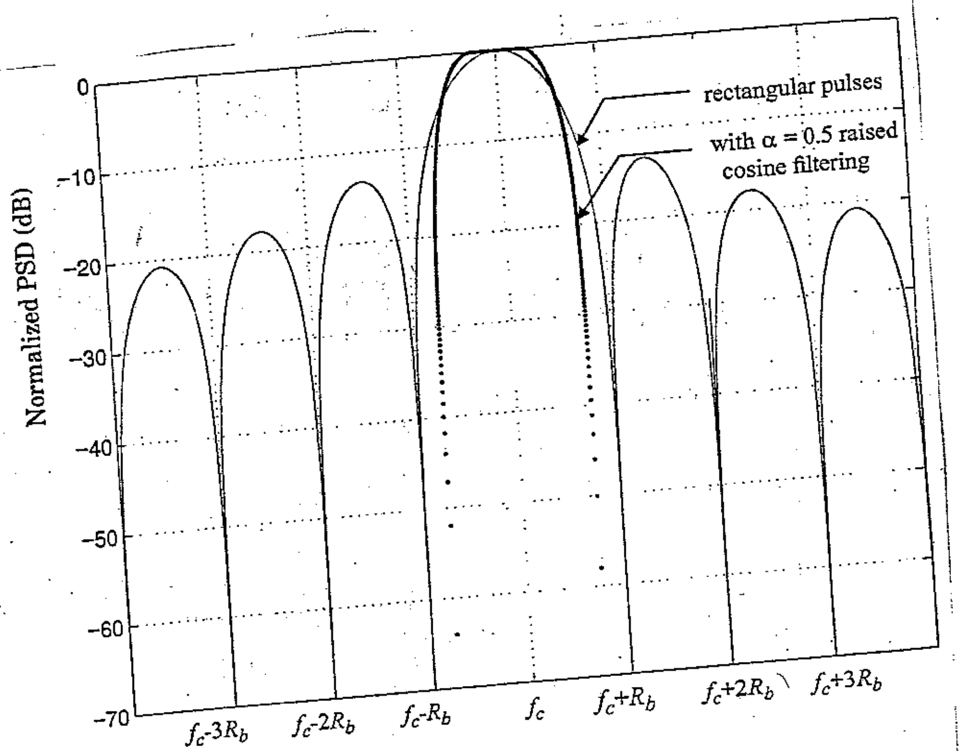


Figure 5.22
Power Spectral Density (PSD) of a BPSK signal.

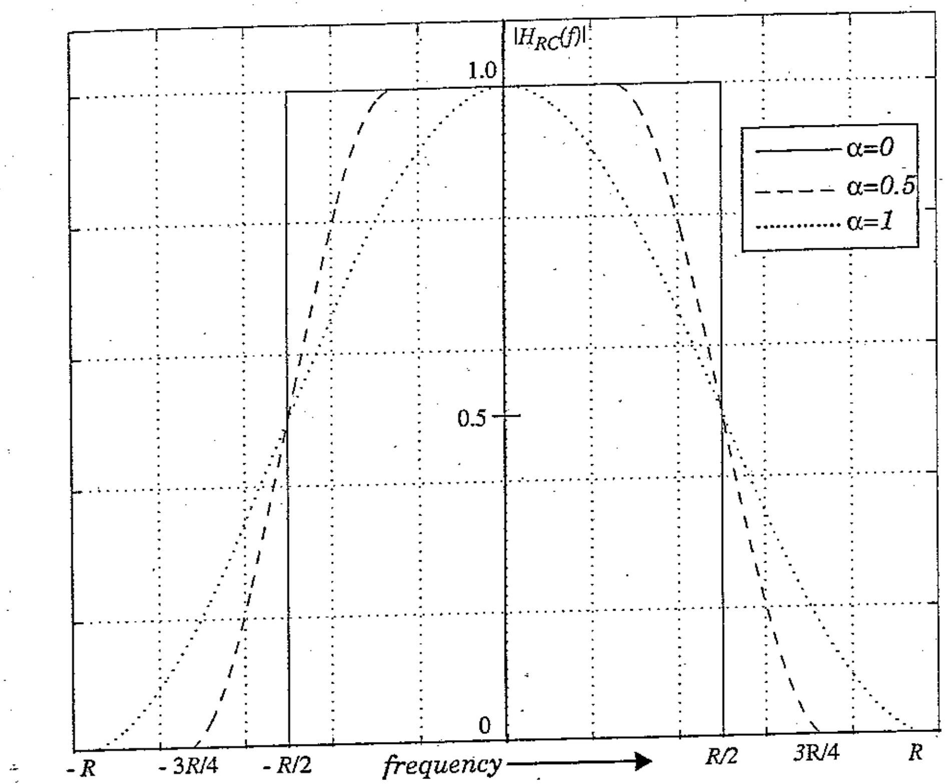


Figure 5.17
Magnitude transfer function of a raised cosine filter.

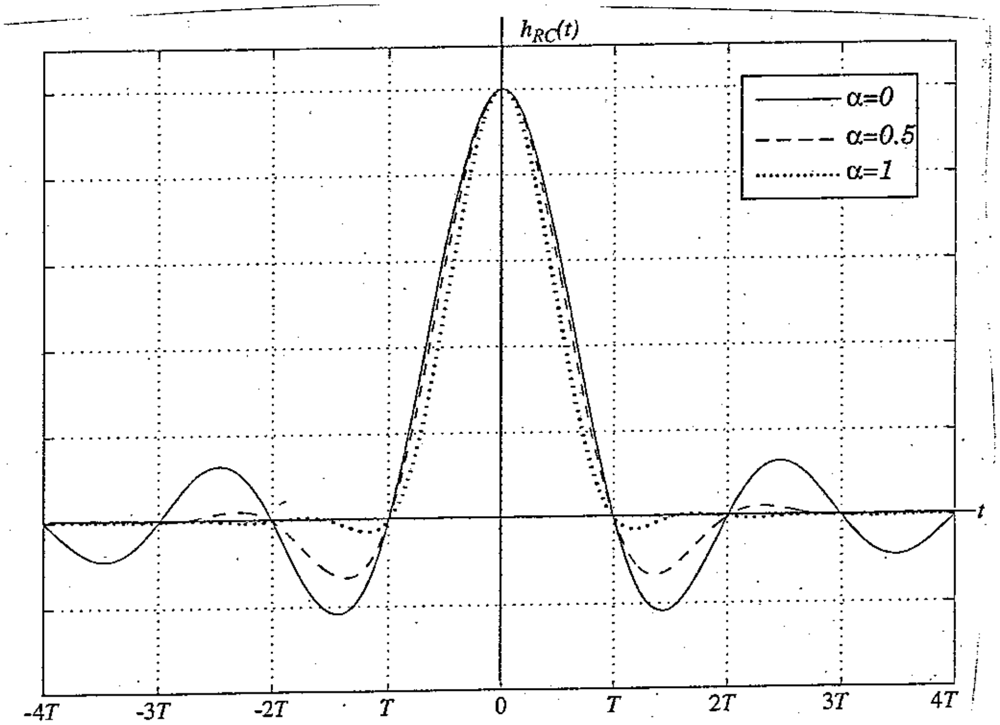
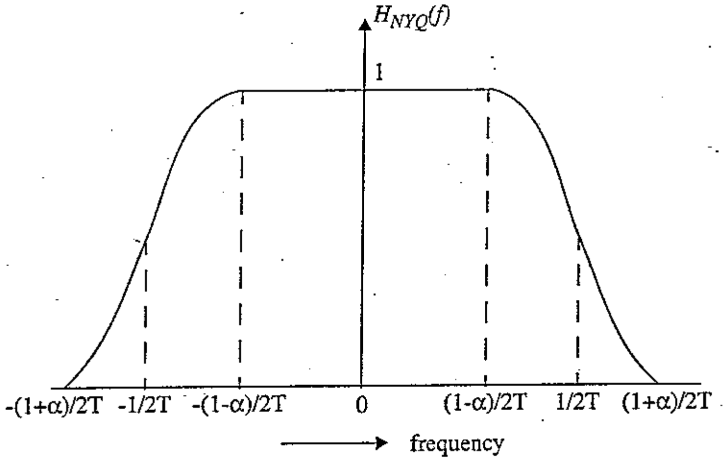


Figure 5.18
Impulse response of a raised cosine rolloff filter.



$$H_{RC}(f) = \begin{cases} 1 & 0 \le |f| \le (1-\alpha)/2T_s \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi[(2T_s|f|) - 1 + \alpha]}{2\alpha}\right)\right] & (1-\alpha)/2T_s < |f| \le (1+\alpha)/2T_s \\ 0 & |f| > (1+\alpha)/2T_s \end{cases}$$

$$h_{RC}(t) = \left(\frac{\sin(\pi t/T_s)}{\pi t}\right)\left(\frac{\cos(\pi\alpha t/T_s)}{1-(4\alpha t/(2T_s))^2}\right)$$

The symbol rate R_s that can be passed through a baseband raised cosine rolloff filter is given by

$$R_s = \frac{1}{T_s} = \frac{2B}{1+\alpha}$$

where B is the absolute filter bandwidth. For RF systems, the RF passband bandwidth doubles and

$$R_s = \frac{B}{1+\alpha}$$

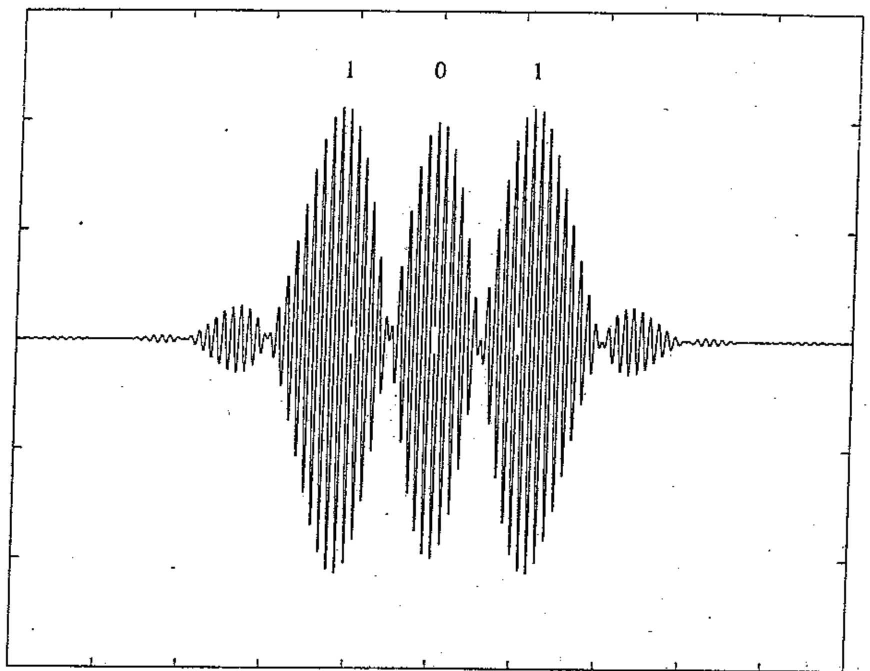


Figure 5.19

Raised cosine filtered ($\alpha = 0.5$) pulses corresponding to 1, 0, 1 data stream for a BPSK signal. Notice that the decision points (at $4T_s$, $5T_s$, $6T_s$) do not always correspond to the maximum values of the RF waveform.